AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

# A-level MATHEMATICS

## Unit Pure Core 3

Wednesday 14 June 2017 Morning Time allowed: 1 hour 30 minutes

**Materials**

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

| For Examiner's Use | |
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| Question | Mark |
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**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

**1 (a)** Given that $y = (\sin 4x)(\sec 3x)$, use the product rule to find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

**[2 marks]**

**(b)** Find $\int \frac{6x}{2x^2 + 3} \mathrm{d}x$ .

**[2 marks]**

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**2 (a)** Use the mid-ordinate rule with five strips to find an estimate for $\int_{0.5}^{1.5} e^{3x-x^3}\,dx$, giving your answer to three decimal places.

**[4 marks]**

**(b)** A curve has equation $y = e^{3x-x^3}$. Find the exact values of the coordinates of the stationary points of the curve and determine the nature of these stationary points.

**[7 marks]**

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**3** Use the substitution $u = \cos 2x$ to find

$$\int \cos^2 2x \sin^3 2x \, dx$$

**[5 marks]**

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**4** The line $y = x$ and the curve with equation $y = \ln\left(\frac{3x+10}{3x+1}\right)$, where $x > 0$, intersect at a single point where $x = \alpha$.

**(a)** Show that $\alpha$ lies between $1$ and $2$.

**[2 marks]**

**(b) (i)** Use the iterative formula

$$x_{n+1} = \ln\left(\frac{3x_n + 10}{3x_n + 1}\right)$$

with $x_1 = 2$ to find the values of $x_2$ and $x_3$, giving your answers to three decimal places.

**[2 marks]**

**(ii)** **Figure 1**, on the opposite page, shows a sketch of parts of the graphs of $y = \ln\left(\frac{3x+10}{3x+1}\right)$ and $y = x$, and the position of $x_1$.

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of $x_2$ and $x_3$ on the $x$-axis.

**[2 marks]**

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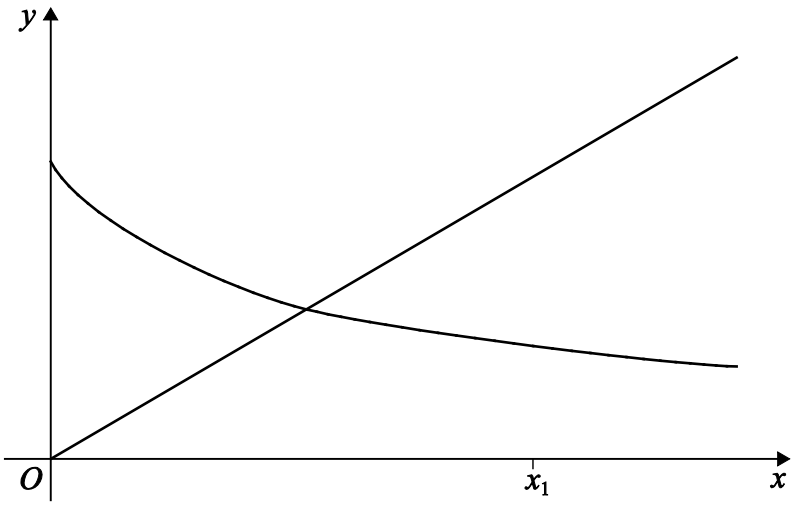
**Figure 1**



**5** The function $\mathrm{f}$ is defined by

$$\mathrm{f}(x) = \ln(3x+1), \text{ for } x \geqslant 0$$

The function $\mathrm{g}$ is defined by

$$\mathrm{g}(x) = \frac{\mathrm{d}}{\mathrm{d}x}(\mathrm{f}(x)), \text{ for } x \geqslant 0$$

The inverse of $\mathrm{f}$ is $\mathrm{f}^{-1}$.

**(a)** Find expressions for $\mathrm{f}^{-1}(x)$ and $\mathrm{g}(x)$.

**[4 marks]**

**(b)** Show that the equation $\mathrm{f}^{-1}(x) = \mathrm{g}(x)$ can be rearranged into the form

$$x = \ln\left(\frac{3x+10}{3x+1}\right)$$

**[2 marks]**

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**6** Use integration by parts to find the value of $\int_1^5 \frac{3x}{\sqrt{2x-1}}\,\mathrm{d}x$.

**[6 marks]**

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**7** You are given that $k$ is a positive constant.

By sketching the graphs of $y = |5x - 3k|$ and $y = 3|x + 4k|$ on the same axes, solve the inequality

$$|5x - 3k| \geqslant 3|x + 4k|$$

**[5 marks]**

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**8 (a)** By using a suitable trigonometrical identity, solve the equation

$$\tan^2\left(2x-\frac{\pi}{6}\right)=11-\sec\left(2x-\frac{\pi}{6}\right)$$

giving all values of $x$ in radians to two decimal places in the interval $0\leqslant x\leqslant\pi$.

**[7 marks]**

**(b)** Describe a sequence of **two** geometrical transformations that maps the graph of $y=\mathrm{f}\left(2x-\frac{\pi}{6}\right)$ onto the graph of $y=\mathrm{f}(x)$.

**[4 marks]**

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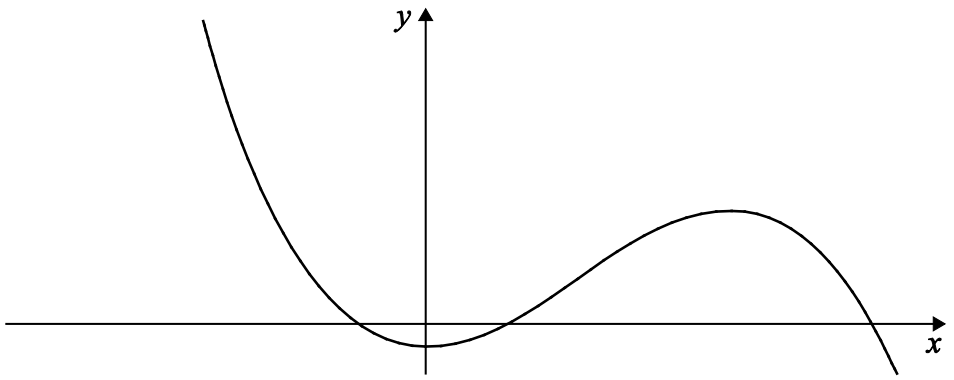
**9** **Figure 2** shows part of the curve with equation $y = \mathrm{f}(x)$.

**Figure 2**




**(a)** On **Figure 3** below, sketch the curve with equation $y = |\mathrm{f}(x)|$.

**[3 marks]**

**(b)** On **Figure 4** opposite, sketch the curve with equation $y = -\mathrm{f}(|x|)$.

**[2 marks]**

**(c)** The curve with equation $y = \mathrm{f}(x)$ has a minimum point at $(0, b-2)$ and a maximum point at $(a, 9b)$, where $0 < b < 2$.

**(i)** Find the coordinates of the minimum point of the curve with equation $y = \mathrm{f}(x + a) + 2b$.

**[2 marks]**

**(ii)** Find the coordinates of the maximum point of the curve with equation $y = 3\mathrm{f}(2x)$.

**[2 marks]**

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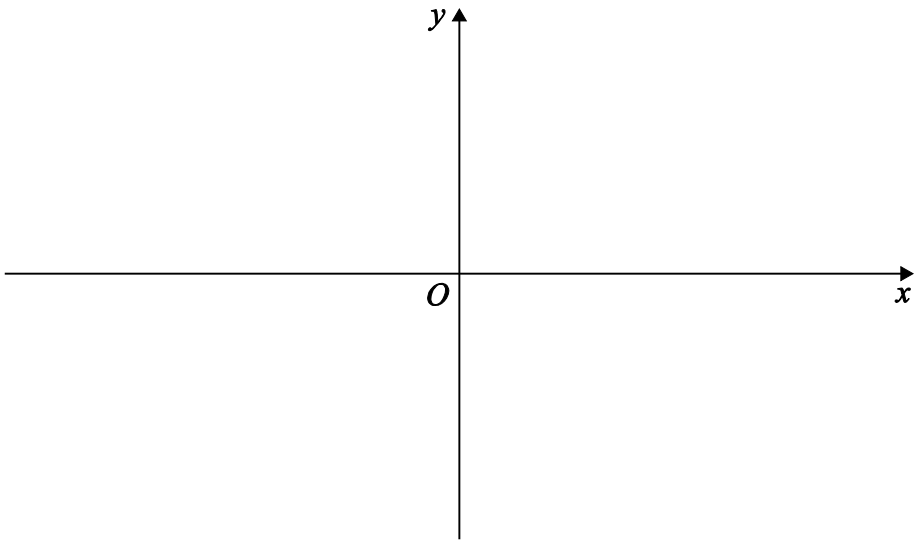
**Figure 3**



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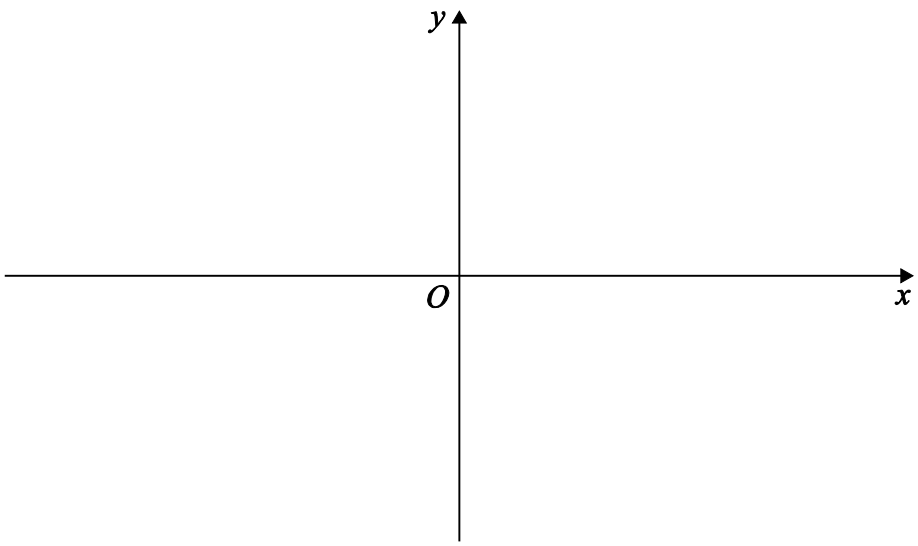
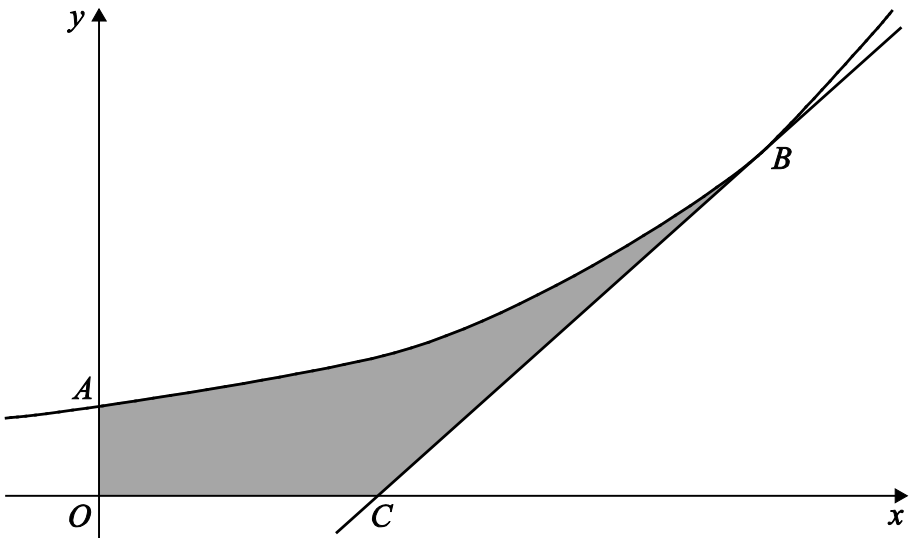
**Answer space for question 9**

**Figure 4**



**10** The diagram shows the curve $y = e^{2x}$, intersecting the $y$-axis at the point $A$, and the tangent to this curve at the point $B$, where $x = \ln 4$, intersecting the $x$-axis at the point $C$.




**(a) (i)** Find an equation of the tangent to the curve at $B$.

**[3 marks]**

**(ii)** Hence show that the coordinates of $C$ are $\left(\ln 4 - \frac{1}{2},\ 0\right)$.

**[1 mark]**

**(b)** The shaded region $OABC$ is rotated through $2\pi$ radians about the $x$-axis to form a solid.

Find the **exact** value of the volume of the solid generated.

(You may assume that the volume of a cone of radius $r$ and height $h$ is $\frac{1}{3}\pi r^2 h$.)

**[8 marks]**

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