## 4766 Statistics 1

## **Section A**

Q1 (i)	(With $\sum fx = 7500$ and $\sum f = 10000$ then arriving at the		
( )	mean)		
	(i) £0.75 scores (B1, B1)	B1 for numerical mean	
	(ii) 75p scores (B1, B1)	(0.75 or 75 seen)	
	(iii) 0.75p scores (B1, B0) (incorrect units)	B1dep for correct units	
	(iv) £75 scores (B1, B0) (incorrect units)	attached	
	After B0, B0 then sight of $\frac{7500}{10000}$ scores SC1. SC1or an answer		
	in the range £0.74 - £0.76 or $74p - 76p$ (both inclusive) scores		
	SC1 (units essential to gain this mark)		
	Standard Deviation: (CARE NEEDED here with close proximity	B2 correct s.d.	
	of answers)	(B1) correct rmsd	
	• 50.2(0) using divisor 9999 scores B2 (50.20148921)		
	• 50.198 (= 50.2) using divisor 10000 scores B1( <i>rmsd</i> )	(B2) default	
	• If divisor is <u>not</u> shown (or calc used) and only an answer	(22) 0010010	
	of 50.2 (i.e. not coming from 50.198) is seen then award		
	B2 on b.o.d. (default)		
	After B0 scored then an attempt at $S_{xx}$ as evident by either	$\sum fx^2 = 25,205,000$	
	$S_{xx} = (5000 + 200000 + 25000000) - \frac{7500^2}{10000} $ (= 25199375)	Beware $\sum x^2 = 25,010,100$	
	or	After B0 scored then	_
	$S_{xx} = (5000 + 200000 + 25000000) - 10000(0.75)^2$	(M1) or M1f.t. for	4
	$S_{xx} = (5000 + 2000000 + 25000000) - 10000(0.75)$	attempt at $S_{xx}$	
	scores (M1) or M1ft 'their 7500 <sup>2</sup> ' or 'their 0.75 <sup>2</sup> '	NB full marks for correct	
	NB The <u>structure</u> must be correct in both above cases with a max of <u>1 slip only after applying the f.t.</u>	results from recommended method which is use of calculator functions	

(ii)	P(Two £10 or two £100)		
	$= \frac{50}{10000} \times \frac{49}{9999} + \frac{20}{10000} \times \frac{19}{9999}$ $= 0.0000245 + 0.0000038 = (0.00002450245 + 0.00000380038)$ $= 0.000028(3) \text{ o.e.} = (0.00002830283)$ $\frac{\text{After M0, M0}}{10000} \text{ then } \frac{50}{10000} \times \frac{50}{10000} + \frac{20}{10000} \times \frac{20}{10000} \text{ o.e.}$ Scores SC1 (ignore final answer but SC1 may be implied by sight of $2.9 \times 10^{-5}$ o.e.) Similarly, $\frac{50}{10000} \times \frac{49}{10000} + \frac{20}{10000} \times \frac{19}{10000} \text{ scores SC1}$	M1 for either correct product seen (ignore any multipliers) M1 sum of both correct (ignore any multipliers) A1 CAO (as opposite with no rounding)  (SC1 case #1)  (SC1 case #2) <u>CARE</u> answer is also 2.83 × 10 <sup>-5</sup>	3
		TOTAL	7
Q2 (i)	Either P(all correct) = $\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} \times \frac{1}{1} = \frac{1}{720}$ or P(all correct) = $\frac{1}{6!} = \frac{1}{720} = 0.00139$	M1 for 6! Or 720 (sioc) or product of fractions A1 CAO (accept 0.0014)	2
(ii)	Either P(picks T, O, M) = $\frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} = \frac{1}{20}$ or P(picks T, O, M) = $\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times 3! = \frac{1}{20}$	M1 for denominators M1 for numerators or 3! A1 CAO	2
	or P(picks T, O, M) = $\frac{1}{\binom{6}{3}} = \frac{1}{20}$	Or M1 for $\binom{6}{3}$ or 20 sioc M1 for $1/\binom{6}{3}$ A1 CAO	3
		TOTAL	5
Q3 (i)	p = 0.55	B1 cao	1
(ii)	$E(X) = 0 \times 0.55 + 1 \times 0.1 + 2 \times 0.05 + 3 \times 0.05 + 4 \times 0.25 = 1.35$	M1 for $\Sigma rp$ (at least 3 non zero terms correct) A1 CAO(no 'n' or 'n-1' divisors)	
	$E(X^{2}) = 0 \times 0.55 + 1 \times 0.1 + 4 \times 0.05 + 9 \times 0.05 + 16 \times 0.25$ $= 0 + 0.1 + 0.2 + 0.45 + 4$ $= (4.75)$	M1 for $\Sigma r^2 p$ (at least 3 non zero terms correct)  M1dep for – their E( X )² provided Var( X ) > 0	
	$Var(X) = \text{ 'their' } 4.75 - 1.35^2 = 2.9275 \text{ awfw } (2.9275 - 2.93)$	A1 cao (no 'n' or 'n-1' divisors)	5
(iii)	P(At least 2 both times) = $(0.05+0.05+0.25)^2 = 0.1225$ o.e.	M1 for (0.05+0.05+0.25) <sup>2</sup> or 0.35 <sup>2</sup> seen A1cao: awfw (0.1225 - 0.123) or 49/400	2

	TOTAL	8

Q4	$X \sim B(50, 0.03)$		
(i)	(A) $P(X = 1) = {50 \choose 1} \times 0.03 \times 0.97^{49} = 0.3372$	M1 $0.03 \times 0.97^{49}$ or $0.0067(4)$	
	$(\mathbf{B})  \mathbf{P}(\mathbf{X} = 0) = 0.97^{50} = 0.2181$	M1 $\binom{50}{1} \times pq^{49}$ (p+q =1) A1 CAO (awfw 0. 337 to 0. 3372) or 0.34(2s.f.) or 0.34(2d.p.)	3
	P(X > 1) = 1 - 0.2181 - 0.3372 = 0.4447	but not just 0.34  B1 for $0.97^{50}$ or $0.2181$ (awfw 0.218 to 0.2181)  M1 for $1 - ($ 'their' $p(X = 0) + $ 'their' $p(X = 1)$ )  must have both probabilities  A1 CAO (awfw 0.4447 to 0.445)	3
(ii)	Expected number = $np = 240 \times 0.3372 = 80.88 - 80.93 = (81)$ Condone $240 \times 0.34 = 81.6 = (82)$ but for M1 A1f.t.	M1 for 240× prob (A) A1FT	2
	. , ,	TOTAL	8
Q5 (i)	$P(R) \times P(L) = 0.36 \times 0.25 = 0.09 \neq P(R \cap L)$ Not equal so not independent. (Allow $0.36 \times 0.25 \neq 0.2$ or $0.09 \neq 0.2$ or $0.2 = 0.2$ or $0.09 \neq 0.2$	M1 for 0.36 × 0.25 or 0.09 seen A1 (numerical justification needed)	2
(ii)	R .16 (0.2) 0.05	G1 for two overlapping circles labelled G1 for 0.2 and either 0.16 or 0.05 in the correct places G1 for all 4 correct probs in the correct	
	0.59	places (including the 0.59)  The last two G marks are independent of the labels	3
(iii)	$P(L \mid R) = \frac{P(L \cap R)}{P(R)} = \frac{0.2}{0.36} = \frac{5}{9} = 0.556 \text{ (awrt 0.56)}$	M1 for 0.2/0.36 o.e. A1 cao	
	This is the probability that Anna is late given that it is raining. (must be in context)	E1 (indep of M1A1) Order/structure <u>must</u> be correct i.e. no reverse	3
	Condone 'if' or 'when' or 'on a rainy day' for 'given that' but <u>not</u> the words 'and' or 'because' or 'due to'	statement	

## **Section B**

Q6 (i)	Median = 4.06 - 4.075  (inclusive)	B1cao	
	$Q_1 = 3.8$ $Q_3 = 4.3$	B1 for Q <sub>1</sub> (cao) B1 for Q <sub>3</sub> (cao)	
	Inter-quartile range = $4.3 - 3.8 = 0.5$	B1 ft for IQR must be using t-values not locations to earn this mark	4
(ii)	Lower limit 'their $3.8' - 1.5 \times$ 'their $0.5' = (3.05)$ Upper limit 'their $4.3' + 1.5 \times$ 'their $0.5' = (5.05)$ Very few if any temperatures below $3.05$ (but not zero) None above $5.05$ 'So few, if any outliers' scores SC1	B1ft: must have -1.5 B1ft: must have +1.5 E1ft dep on -1.5 and Q <sub>1</sub> E1ft dep on+1.5 and Q <sub>3</sub> Again, must be using t- values NOT locations to earn these 4 marks	4
(iii)	Valid argument such as 'Probably not, because there is nothing to suggest that they are not genuine data items; (they do not appear to form a separate pool of data.')  Accept: exclude outlier – 'measuring equipment was wrong' or 'there was a power cut' or ref to hot / cold day  [Allow suitable valid alternative arguments]	E1	1
(iv)	Missing frequencies 25, 125, 50	B1, B1, B1 (all cao)	3
(v)	Mean = $(3.2 \times 25 + 3.6 \times 125 + 4.0 \times 243 + 4.4 \times 157 + 4.8 \times 50)/600$ = $2432.8/600 = 4.05(47)$	M1 for at least 4 midpoints correct and being used in attempt to find $\sum ft$	2
		A1cao: awfw (4.05 – 4.055) ISW or rounding	
(vi)	New mean = 1.8 × 'their 4.05(47)' + 32 = 39.29(84) to 39.3 New s = 1.8 × 0.379 = 0.682	B1 FT M1 for 1.8 × 0.379 A1 CAO awfw (0.68 – 0.6822)	3
		TOTAL	17

Q7 (i)	$(A) \ Either \ P(X = 8) = {10 \choose 8} \times 0.8^8 \times 0.2^2 = 0.3020 \ (awrt)$ $or \qquad P(X = 8) = P(X \le 8) - P(X \le 7)$ $= 0.6242 - 0.3222 = 0.3020$ $(B) \ Either \ P(X \ge 8) = 1 - P(X \le 7)$ $= 1 - 0.3222 = 0.6778$ $or \qquad P(X \ge 8) = P(X = 8) + P(X = 9) + P(X = 10)$ $= 0.3020 + 0.2684 + 0.1074 = 0.6778$	M1 $0.8^8 \times 0.2^2$ or $0.00671$ M1 $\binom{10}{8} \times p^8 q^2$ ; (p +q =1)  Or $45 \times p^8 q^2$ ; (p +q =1)  A1 CAO (0.302) not 0.3  OR: M2 for $0.6242 - 0.3222$ A1 CAO  M1 for $1 - 0.3222$ (s.o.i.)  A1 CAO awfw $0.677 - 0.678$ or  M1 for sum of 'their'  p(X=8) plus correct expressions for p(x=9) and p(X=10)  A1 CAO awfw $0.677 - 0.678$	3
(ii)			
(II)	Let $X \sim B(18, p)$ Let $p =$ probability of delivery (within 24 hours) (for population) $H_0$ : $p = 0.8$ $H_1$ : $p < 0.8$ $P(X \le 12) = 0.1329 > 5\%$ ref: [pp =0.0816]	B1 for definition of <i>p</i> B1 for H <sub>0</sub> B1 for H <sub>1</sub> M1 for probability 0.1329  M1dep strictly for comparison of 0.1329 with 5% (seen or clearly implied)	
	So not enough evidence to reject $H_0$ Conclude that there is not enough evidence to indicate that less than 80% of orders will be delivered within 24 hours	A1dep on both M's  E1dep on M1,M1,A1 for conclusion in context	
	Note: use of critical region method scores M1 for region {0,1,2,,9, 10} M1dep for 12 does not lie in critical region then A1dep E1dep as per scheme	Conclusion in Context	7

(iii)	Let $X \sim B(18, 0.8)$ $H_1: p \neq 0.8$ LOWER TAIL $P(X \leq 10) = 0.0163 < 2.5\%$ $P(X \leq 11) = 0.0513 > 2.5\%$ UPPER TAIL $P(X \geq 17) = 1 - P(X \leq 16) = 1 - 0.9009 = 0.0991 > 2.5\%$ $P(X \geq 18) = 1 - P(X \leq 17) = 1 - 0.9820 = 0.0180 < 2.5\%$ So critical region is $\{0,1,2,3,4,5,6,7,8,9,10,18\}$ o.e. Condone $X \leq 10$ and $X \geq 18$ or $X = 18$ but <b>not</b> $P(X \leq 10)$ and $P(X \geq 18)$ Correct CR without supportive working scores SC2 max after the 1 <sup>st</sup> B1 (SC1 for each fully correct tail of CR)	B1 for H <sub>1</sub> B1 for 0.0163 or 0.0513 seen  M1dep for either correct comparison with 2.5% (not 5%) (seen or clearly implied)  A1dep for correct lower tail CR (must have zero)  B1 for 0.0991 or 0.0180 seen  M1dep for either correct comparison with 2.5% (not 5%) (seen or clearly implied)  A1dep for correct upper tail CR	7
		TOTAL	19