GCE

Further Mathematics B (MEI)

Y434/01: Numerical methods

Advanced GCE

Mark Scheme for Autumn 2021

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

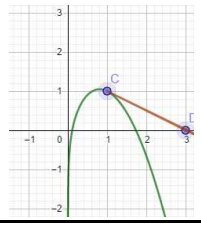
Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>AOs</th><th></th><th>Guidance</th></tr>
<tr><td>1</td><td>(a)</td><td>i</td><td>$\frac{1.414214-\sqrt{2}}{\sqrt{2}}$ or $\frac{1.414214^2-2}{2}$ oe soi
0.000000309449 isw
0.000000618898 isw</td><td>M1
A1
A1</td><td>1.1a
1.1
1.1</td><td>ignore modulus signs
to 2 sf or more
to 2 sf or more</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>[3]</td><td></td><td></td><td></td></tr>
<tr><td>1</td><td>(a)</td><td>ii</td><td>the second relative error is double the first relative error oe</td><td>B1</td><td>2.2a</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>[1]</td><td></td><td></td><td></td></tr>
<tr><td>1</td><td>(b)</td><td></td><td>Ben is wrong because the spreadsheet stores 1.414214 to a higher precision than is displayed (and so when the square of this number is calculated, 2 is returned) isw</td><td>B1</td><td>2.4</td><td>or 1.414214 is an approximation to $\sqrt{2}$ so $1.414214^2 \neq 2$ oe</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>[1]</td><td></td><td></td><td></td></tr>
<tr><td>2</td><td>(a)</td><td></td><td>
<table>
<tr><th>x</th><th>f(x)</th><th>Δ</th><th>Δ^2</th></tr>
<tr><td>1</td><td>–0.65</td><td></td><td></td></tr>
<tr><td></td><td></td><td>0.3</td><td></td></tr>
<tr><td>2</td><td>–0.35</td><td></td><td>1.82</td></tr>
<tr><td></td><td></td><td>2.12</td><td></td></tr>
<tr><td>3</td><td>1.77</td><td></td><td>1.82</td></tr>
<tr><td></td><td></td><td>3.94</td><td></td></tr>
<tr><td>4</td><td>5.71</td><td></td><td>1.82</td></tr>
<tr><td></td><td></td><td>5.76</td><td></td></tr>
<tr><td>5</td><td>11.47</td><td></td><td></td></tr>
</table>
</td><td>M1
A1</td><td>1.1
1.1</td><td>finds 4 Δ values, allow one error
all correct</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>[2]</td><td></td><td></td><td></td></tr>
</table>

Question			Answer	Marks	AOs		Guidance
2	(b)		the second differences are constant **oe**	**B1**	**1.1**	allow the 3rd differences are zero	
				[1]			
2	(c)		$-0.65 + 0.3(x-1) + 1.82\times\frac{(x-1)(x-2)}{2!}$	**M1**	**1.1**	must be correct form; allow 1 substitution error	
			$[P_2(x) =]\ 0.91x^2 - 2.43x + 0.87$	**A1**	**1.1**	two of three terms correct	
				A1	**1.1**	all correct	
				[3]			
3	(a)		$\sinh x^2 - x^3 - 2 = 0$	**B1**	**1.1**	must see = 0	
				[1]			
3	(b)		=IF(H5>0,G5,E5)	**B1**	**1.1**	or =IF(H5<0,E5,G5)	must see =
				[1]			
3	(c)		$\frac{1.48719\times17.2899-2\times-0.77825}{17.2899--0.77825}$ **oe**	**M1**	**3.1a**	may be implied by 1.509…	
			awrt 1.50928	**A1**	**1.1**	NB f(1.50928) = –0.6111 to 4 sf	
			awrt 1.52603	**A1**	**1.1**		
				[3]			

Question			Answer	Marks	AOs		Guidance
3	(d)		the ratios are decreasing which suggests the convergence is (slightly) faster than 1st order the ratios are close to 1 which suggests the convergence is slow	B1 B1	2.2b 2.2b	allow between 1st and 2nd order	do not allow eg not first order
				[2]			
4	(a)		$\frac{4.2472072-4}{0.1}$ or $\frac{4.0239468-4}{0.01}$ or $\frac{4.0023871-4}{0.001}$ or $\frac{4.0002386-4}{0.0001}$ 2.472072 (with $h = 0.1$) 2.39468 (with $h = 0.01$) 2.3871 (with $h = 0.001$) 2.386 (with $h = 0.0001$)	M1 A1 A1 A1	3.1a 1.1 1.1 1.1	use of forward difference method any two correct any three correct all four correct	may be implied by one correct answer
				[4]			
4	(b)		comparison of last two estimates 2.39 is secure **or** 2.386 is possible	M1 A1	1.1 2.2b	if **M0** allow **SC1** for 2.39 is secure **or** 2.386 is possible regardless of justification	
				[2]			
5	(a)		48×0.5 **soi** £24	M1 A1	3.3 3.4		
				[2]			
5	(b)		consistent because 1.77 < 24	B1	2.4	allow consistent because error < mpe	
				[1]			

Question			Answer	Marks	AOs		Guidance
5	(c)		52×0.495 £25.74	M1 A1	3.3 3.4		
				[2]			
5	(d)		this could happen if a large number of items eg cost less than £1 eg cost £1.99 or £2.99 etc eg more than 50p over the pound eg the mean error per item was 52.38p	B1	3.5a		
				[1]			
5	(e)		mpe = £0.99*n*	B1	3.4	condone omission of units, allow 99*n* pence	
				[1]			
5	(f)		expected error for Nina's model is £0 since you would expect to round half the prices up and half down **oe** **or** expected error in Kareem's model is –£0.495*n* since you would expect the average "chop" to be 49.5p **oe** so new model should be "estimated cost" + £0.495*n*	B1 B1	2.4 3.5c		U6

Question			Answer	Marks	AOs		Guidance
				[2]			
6	(a)		$\frac{1}{2x} - 2x + 1$ seen	M1	2.1		
			$x_{n+1} = x_n - \frac{0.5\ln(x_n) - x_n^2 + x_n + 1}{\frac{1}{2x_n} - 2x_n + 1}$ **oe soi**	M1	1.1	may be implied by correct iterates	condone omission of subscripts
			1 3 2.0791668 1.7783346 1.7360141 1.7351281 1.7351277 1.735128	M1 A1	1.1 1.1	at least three further correct iterates derived from starting at 1 if **M0** allow **SC1** for 1.735128 from N-R method used with different x_0 and at least 3 correct iterates shown	correct to at least 5 sf where appropriate
				[4]			
6	(b)			M1 A1	2.4 1.1	tangent at (1,1) (1,1) to (3,0)	
				[2]			

Question			Answer	Marks	AOs		Guidance
6	(c)		N-R generally has 2nd order convergence whereas fixed point iteration generally has 1st order convergence	B1	2.4	allow eg N-R converges faster allow eg fixed point iteration more likely to fail **oe**	
				[1]			
6	(d)		ln(-0.403) is undefined (so the spreadsheet cannot compute a value)	B1	2.2a		
				[1]			
6	(e)		0.5 1.0739769 1.4524673 1.6245304 1.6932631 1.7194743 1.7293015 converges to β	M1 A1	2.1 2.2a	need to see at least 3 iterates correct to at least 5 sf	
				[2]			

Question			Answer	Marks	AOs		Guidance
6	(f)		0.5 0.4764669 0.4528879 0.4293074 0.4057756 0.3823498 … 0.1116318 0.1111278 0.1110835 0.1110821 0.1110821 0.111082	M1 A1	1.1 2.2a	at least 3 correct iterates derived from starting at 0.5 if **M0** allow **SC1** for 0.111082 from relaxation method used with different x_0 and at least 3 correct iterates shown	iterates correct to at least 5 sf
				[2]			
7	(a)		$\frac{1}{16}$ **isw** or 0.0625 **isw**	B1	2.2a		
				[1]			
7	(b)		by comparison of T_{16} and T_{32} 0.6 is certain **or** 0.63 is probable	B1	2.2b		
				[1]			

Question			Answer	Marks	AOs		Guidance
7	(c)		r appears to be between 0.25 and 0.5 so order of convergence is between 1st and 2nd order	B1 B1	2.2b 2.2b		
			Alternative $r > 0.25$ so convergence slower than 2nd order $r < 0.5$ so convergence faster than 1st order	 B1 B1			
				[2]			
7	(d)		$\frac{2M_n+T_n}{3}$ or $\frac{4T_{2n}-T_n}{3}$ **soi** = (2*O5 + N5)/3 **or** =(4*N6 – N5)/3	M1 A1	1.1 1.1	 must see =	
				[2]			
7	(e)		awrt 0.62658745 awrt 0.00029 awrt 0.354	B1 B1 B1	1.1 1.1 1.1		
				[3]			

Question			Answer	Marks	AOs		Guidance
7	(f)		S_{2n} and difference from table used in extrapolation	M1	3.1a	eg their 0.62658745 and their 0.00029	If **M0** allow **SC2** for awrt 0.626607 obtained from $\frac{16\times0.62658745-0.62629755}{15}$
			awrt 0.62658745 and awrt 0.00029 used	A1	1.1	may see more dp for difference	
			$0.62658745 + 0.00029 \times \frac{r}{1-r}$	A1	1.1	$0.35 \leq r \leq 0.36$	then **SC1 for** 0.627 obtained from comparison with S_{64}
			awrt 0.62674355 to awrt 0.62675058	A1	1.1		
			comparison with their S_{64}	M1	3.2a		
			0.6267 is secure	A1	2.2b	**or** 0.62675 is possible; allow 0.626746	
						the last two **A** marks are only available if answers obtained from extrapolation to infinity from S_{64}	
				[6]			

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