

General Certificate of Education
January 2009
Advanced Level Examination



MATHEMATICS
Unit Statistics 2B

MS2B

Thursday 29 January 2009 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
 - the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MS2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 Fortune High School gave its students a wider choice of subjects to study. The table shows the number of students, of each gender, who chose to study each of the additional subjects during the school year 2007/08.

	Bulgarian	Climate Change	Finance	Polish
Male	7	31	25	40
Female	2	24	22	19

Assuming that these data form a random sample, use a χ^2 test, at the 10% level of significance, to test whether the choice of these subjects is independent of gender.

(11 marks)

- 2 A group of estate agents in a particular area claimed that, after the introduction of a new search procedure at the Land Registry, the mean completion time for the purchase of a house in the area had not changed from 8 weeks.

- (a) A random sample of 9 house purchases in the area revealed that their completion times, in weeks, were as follows.

6 7 10 12 9 11 7 8 14

Assuming that completion times in the area are normally distributed with standard deviation 2.5 weeks, test, at the 5% level of significance, the group's claim. *(7 marks)*

- (b) It was subsequently discovered that, after the introduction of the new search procedure at the Land Registry, the mean completion time for the purchase of a house in the area remained at 8 weeks.

Indicate whether a Type I error, a Type II error or neither has occurred in carrying out your hypothesis test in part (a). Give a reason for your answer. *(2 marks)*

- 3 Joe owns two garages, Acefit and Bestjob, each specialising in the fitting of the latest satellite navigation device.

The daily demand, X , for the device at Acefit garage may be modelled by a Poisson distribution with mean 3.6.

The daily demand, Y , for the device at Bestjob garage may be modelled by a Poisson distribution with mean 4.4.

(a) Calculate:

(i) $P(X \leq 3)$; *(1 mark)*

(ii) $P(Y = 5)$. *(2 marks)*

(b) The total daily demand for the device at Joe's two garages is denoted by T .

(i) Write down the distribution of T , stating any assumption that you make. *(2 marks)*

(ii) Determine $P(6 < T < 12)$. *(3 marks)*

(iii) Calculate the probability that the total demand for the device will exceed 14 on each of two consecutive days. Give your answer to one significant figure. *(4 marks)*

(iv) Determine the minimum number of devices that Joe should have in stock if he is to meet his total demand on at least 99% of days. *(2 marks)*

Turn over for the next question

Turn over ►

4 The continuous random variable X has the cumulative distribution function

$$F(x) = \begin{cases} 0 & x < -c \\ \frac{x+c}{4c} & -c \leq x \leq 3c \\ 1 & x > 3c \end{cases}$$

where c is a positive constant.

(a) Determine $P\left(-\frac{3c}{4} < X < \frac{3c}{4}\right)$. *(2 marks)*

(b) Show that the probability density function, $f(x)$, of X is

$$f(x) = \begin{cases} \frac{1}{4c} & -c \leq x \leq 3c \\ 0 & \text{otherwise} \end{cases} \quad (2 \text{ marks})$$

(c) Hence, or otherwise, find expressions, in terms of c , for:

(i) $E(X)$; *(1 mark)*

(ii) $\text{Var}(X)$. *(1 mark)*

- 5 Jane, who supplies fruit to a jam manufacturer, knows that the weight of fruit in boxes that she sends to the manufacturer can be modelled by a normal distribution with unknown mean, μ grams, and unknown standard deviation, σ grams.

Jane selects a random sample of 16 boxes and, using the t -distribution, calculates correctly that a 98% confidence interval for μ is (70.65, 80.35).

- (a) (i) Show that the sample mean is 75.5 grams. *(1 mark)*
- (ii) Find the width of the confidence interval. *(1 mark)*
- (iii) Calculate an estimate of the standard error of the mean. *(3 marks)*
- (iv) Hence, or otherwise, show that an unbiased estimate of σ^2 is 55.6, correct to three significant figures. *(2 marks)*
- (b) Jane decides that the width of the 98% confidence interval is too large.
- Construct a 95% confidence interval for μ , based on her sample of 16 boxes. *(2 marks)*
- (c) Jane is informed that the manufacturer would prefer the confidence interval to have a width of at most 5 grams.
- (i) Write down a confidence interval for μ , again based on Jane's sample of 16 boxes, which has a width of 5 grams. *(1 mark)*
- (ii) Determine the percentage confidence level for your interval in part (c)(i). *(3 marks)*

Turn over for the next question

Turn over ►

- 6 A small supermarket has a total of four checkouts, at least one of which is always staffed. The probability distribution for R , the number of checkouts that are staffed at any given time, is

$$P(R = r) = \begin{cases} \frac{2}{3} \left(\frac{1}{3}\right)^{r-1} & r = 1, 2, 3 \\ k & r = 4 \end{cases}$$

- (a) Show that $k = \frac{1}{27}$. (2 marks)
- (b) Find the probability that, at any given time, there will be at least 3 checkouts that are staffed. (1 mark)
- (c) It is suggested that the total number of customers, C , that can be served at the checkouts per hour may be modelled by

$$C = 27R + 5$$

Find:

- (i) $E(C)$; (3 marks)
- (ii) the standard deviation of C . (4 marks)

- 7 The continuous random variable X has the probability density function given by

$$f(x) = \begin{cases} \frac{1}{16}x^3 & 0 \leq x \leq 2 \\ \frac{1}{6}(5-x) & 2 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Sketch the graph of f . (3 marks)
- (b) Prove that the cumulative distribution function of X for $2 \leq x \leq 5$ can be written in the form

$$F(x) = 1 - \frac{1}{12}(5-x)^2 \quad (4 \text{ marks})$$

- (c) Hence, or otherwise, determine $P(X \geq 3 | X \leq 4)$. (5 marks)

END OF QUESTIONS

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