General Certificate of Education
June 2005
Advanced Level Examination

MATHEMATICS **MPC3**
Unit Pure Core 3

Friday 24 June 2005 Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) Find $\frac{dy}{dx}$ when $y = x \sin 2x$. *(3 marks)*

(b) (i) Find $\frac{dy}{dx}$ when $y = (x^2 - 6)^4$. *(2 marks)*

(ii) Hence, or otherwise, find $\int x(x^2 - 6)^3 dx$. *(3 marks)*

2 The functions f and g are defined with their respective domains by

$$f(x) = x - 2 \qquad \text{for all real values of } x$$

$$g(x) = \frac{6}{x + 3} \qquad \text{for real values of } x, \quad x \neq -3$$

The composite function fg is denoted by h.

(a) Find $h(x)$. *(2 marks)*

(b) (i) Find $h^{-1}(x)$, where h^{-1} is the inverse of h. *(3 marks)*

(ii) Find the range of h^{-1}. *(1 mark)*

3 (a) Find $\int e^{4x} dx$. *(1 mark)*

(b) Use integration by parts to find $\int e^{4x}(2x + 1) dx$. *(3 marks)*

(c) By using the substitution $u = 1 + \ln x$, or otherwise, find $\int \frac{1 + \ln x}{x} dx$. *(4 marks)*

4 It is given that $\tan^2 x = \sec x + 11$.

(a) Show that the equation $\tan^2 x = \sec x + 11$ can be written in the form

$$\sec^2 x - \sec x - 12 = 0$$ *(2 marks)*

(b) Hence show that $\cos x = \frac{1}{4}$ or $\cos x = -\frac{1}{3}$. *(3 marks)*

(c) Hence, or otherwise, solve the equation $\tan^2 x = \sec x + 11$, giving all values of x to the nearest degree in the interval $0° < x < 360°$. *(3 marks)*

5 (a) Solve the equation $2e^x = 5$, giving your answer as an exact natural logarithm. *(2 marks)*

(b) (i) By substituting $y = e^x$, show that the equation $2e^x + 5e^{-x} = 7$ can be written as

$$2y^2 - 7y + 5 = 0$$ *(2 marks)*

(ii) Hence solve the equation $2e^x + 5e^{-x} = 7$, giving your answers as exact values of x. *(3 marks)*

6 (a) (i) Sketch the graph of $y = 4 - x^2$, indicating the coordinates of the points where the graph crosses the coordinate axes. *(2 marks)*

(ii) The region between the graph and the x-axis from $x = 0$ to $x = 2$ is rotated through 360° about the x-axis. Find the exact value of the volume of the solid generated. *(4 marks)*

(b) (i) Sketch the graph of $y = |4 - x^2|$. *(2 marks)*

(ii) Solve $|4 - x^2| = 3$. *(3 marks)*

(iii) Hence, or otherwise, solve the inequality $|4 - x^2| < 3$. *(2 marks)*

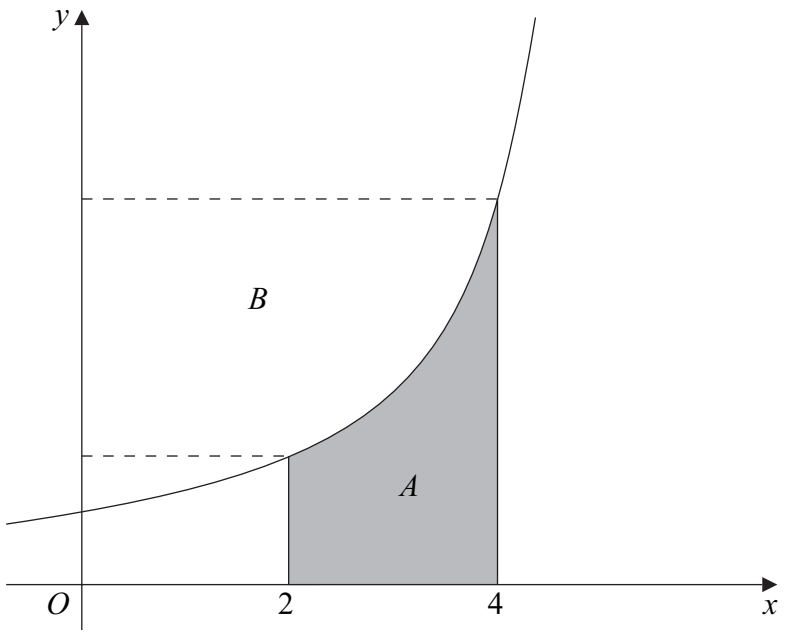
7 (a) Sketch the graph of $y = \tan^{-1}x$. *(2 marks)*

(b) (i) By drawing a suitable straight line on your sketch, show that the equation $\tan^{-1} x = 2x - 1$ has only one root. *(2 marks)*

(ii) Given that the root of this equation is α, show that $0.8 < \alpha < 0.9$. *(2 marks)*

(c) Use the iteration $x_{n+1} = \frac{1}{2}(\tan^{-1} x_n + 1)$ with $x_1 = 0.8$ to find the value of x_3, giving your answer to two significant figures. *(3 marks)*

8 The diagram shows part of the graph of $y = e^{2x} + 3$.



(a) Describe a sequence of two geometrical transformations that maps the graph of $y = e^x$ onto the graph of $y = e^{2x} + 3$. *(4 marks)*

(b) Use the mid-ordinate rule with four strips of equal width to find an estimate for the area of the shaded region A, giving your answer to three significant figures. *(4 marks)*

(c) Find the exact value of the area of the shaded region A. *(4 marks)*

(d) The region B is indicated on the diagram. Find the area of the region B, giving your answer in the form $pe^8 + qe^4$, where p and q are numbers to be determined. *(4 marks)*

END OF QUESTIONS