Version 1.0

General Certificate of Education (A-level)
January 2012

Mathematics MPC4

(Specification 6360)

Pure Core 4

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from: aqa.org.uk

Set and published by the Assessment and Qualifications Alliance.

The Assessment and Qualifications Alliance (AQA) is a company limited by guarantee registered in England and Wales (company number 3644723) and a registered charity (registered charity number 1073334).
Registered address: AQA, Devas Street, Manchester M15 6EX.

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC4: January 2012 - Mark scheme

Q	Solution	Marks	Total	Comments
1(a)	$2x+3=A(2x+1)+B(2x-1)$	M1		
	$x=\frac{1}{2}\quad x=-\frac{1}{2}$	m1		Use two values of x to find A and B
	$A=2\quad B=-1$	A1	3	Both
(b)	$4x^2-1\overline{)12x^3-7x-6}$ quotient $3x$; $12x^3-\underline{3x}$; $-4x-6$	M1		Complete division leading to values for C and D
	$C=3$	A1		$C=3\quad D=-2$ stated or written in expression.
	$D=-2$	A1	3	SC B1 $C=3$, D not found or wrong; $D=-2$, C not found or wrong.
(c)	$\int 3x-2\left(\frac{2}{2x-1}-\frac{1}{2x+1}\right)\,dx$	M1		Use parts (a) and (b) to obtain integrable form
	$3\frac{x^2}{2}$	A1ft		ft on C
	$-2\left(\ln(2x-1)-\frac{1}{2}\ln(2x+1)\right)$	A1ft		Both correct; ft on A, B and D Condone missing brackets
	$\frac{3}{2}(4-1)-2\left(\left(\ln 3-\frac{1}{2}\ln 5\right)-\left(\ln 1-\frac{1}{2}\ln 3\right)\right)$	m1		Correct substitution of limits
	$\frac{9}{2}-3\ln 3+\ln 5=\frac{9}{2}+\ln\left(\frac{5}{27}\right)$	A1	5	$p=\frac{9}{2}\quad q=\frac{5}{27}$
		Total	**11**	

(a) Condone poor algebra for M1 if continues correctly.

(b) Complete division for M1; obtain a value for C (Cx) and a remainder $ax+b$

(c) Form $\int Cx+\left(\frac{P}{2x-1}+\frac{Q}{2x+1}\right)\,dx$ using candidate's P, Q, C for M1. Condone missing dx.

$\int Cx\,dx=C\frac{x^2}{2}$ for A1ft ISW extra terms eg $\frac{12}{4x^2-1}$ for first three terms only; max 3/5

Candidate's C; must have a value.

$\int\frac{4x+6}{4x^2-1}\,dx=\int\frac{4x}{4x^2-1}+\frac{6}{4x^2-1}\,dx$ is an integrable form, as $\int\frac{1}{x^2-a^2}\,dx=\frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right)$ is in the formula book, but they **must** try to integrate to show they know this, **or** use partial fractions again with $\frac{6}{4x^2-1}=\frac{3}{2x-1}-\frac{3}{2x+1}$ for M1

Substitute limits into $C\frac{x^2}{2}+m\ln(2x-1)+n\ln(2x+1)$, or equivalent, for m1;

substitution must be completely correct.

Condone $\frac{9}{2}-\ln\left(\frac{27}{5}\right)$ for A1

Q	Solution	Marks	Total	Comments
1 (a)	Alternative; equating coefficients $2x+3=A(2x+1)+B(2x-1)$ x term $\quad 2=2A+2B$ constant $\quad 3=A-B$ $A=2 \quad B=-1$	M1 m1 A1	 3	 Set up simultaneous equations and solve. Both
	Alternative; cover up rule $x=\frac{1}{2} \quad A=\frac{2\times\frac{1}{2}+3}{2\times\frac{1}{2}+1} \quad \left(=\frac{4}{2}\right)$ $x=-\frac{1}{2} \quad B=\frac{2\times\left(-\frac{1}{2}\right)+3}{2\times\left(-\frac{1}{2}\right)-1} \quad \left(=\frac{2}{-2}\right)$ $A=2 \quad B=-1$	M1 A1A1	 3	$x=\frac{1}{2}$ and $x=-\frac{1}{2}$ used to find A and B SC NMS A and B both correct 3/3 One of A or B correct 1/3
1 (b)	Alternative $\frac{12x^3-7x-6}{4x^2-1}=\frac{12x^3-3x-4x-6}{4x^2-1}$ $=3x-\frac{2(2x+3)}{4x^2-1}$ $C=3$ $D=-2$	M1 A1 A1	 3	 $C=3 \quad D=-2$ stated or written in expression SC B1 $C=3$, D not found or wrong; $D=-2$, C not found or wrong.
	Alternative $12x^3-7x-6=4Cx^3-Cx+2Dx+3D$ $C=3$ $D=-2$	M1 A1 A1	 3	Complete method for C and D $C=3$, $D=-2$ stated or written in expression. SC B1 $C=3$, D not found or wrong; $D=-2$, C not found or wrong.
	Alternative $x=0 \qquad x=1$ $6=-3D \qquad -\frac{1}{3}=C+\frac{5}{3}D$ $C=3$ $D=-2$	M1 A1 A1	 3	Use two values of x to set up simultaneous equations $C=3 \quad D=-2$ stated or written in expression. SC B1 $C=3$, D not found or wrong; $D=-2$, C not found or wrong.

Q	Solution	Marks	Total	Comments
2(a)(i)	$\tan\alpha = \frac{4}{3}$	B1	1	Fraction required Allow 1.333 …. (recurring)
(ii)	$1,\ 2,\ \sqrt{3}$ seen (from Pythagoras) or $4 = 1 + \cot^2\beta$	M1		Use $\operatorname{cosec}^2\beta = 1 + \cot^2\beta$
	$\tan\beta = -\frac{1}{\sqrt{3}}$	A1	2	SC B1 $\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$
(b)	$\tan(\alpha+\beta) = \dfrac{\frac{4}{3} - \frac{1}{\sqrt{3}}}{1 - \frac{4}{3}\left(-\frac{1}{\sqrt{3}}\right)}$	M1		Use $\tan(\alpha+\beta)$ formula
	Remove fractions within fractions	m1		Correct manipulation to form $\frac{a+b\sqrt{3}}{c+d\sqrt{3}}$ $a\ b\ c\ d$ integers
	$= \dfrac{4\sqrt{3}-3}{3\sqrt{3}+4}$	A1	3	$m = 4 \quad n = 3$ or any multiple
		Total	**6**	
(b)	Alternative $\tan(\alpha+\beta)$ $= \dfrac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \dfrac{\frac{4}{5}\times\left(-\frac{\sqrt{3}}{2}\right) + \frac{3}{5}\times\frac{1}{2}}{\frac{3}{5}\times\left(-\frac{\sqrt{3}}{2}\right) - \frac{4}{5}\times\frac{1}{2}}$	M1		Use formulae for $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$
	Remove fractions within fractions	m1		Correct manipulation to form $\frac{a+b\sqrt{3}}{c+d\sqrt{3}}$ $a\ b\ c\ d$ integers
	$= \dfrac{-4\sqrt{3}+3}{-3\sqrt{3}-4}$ $\left(= \dfrac{4\sqrt{3}-3}{3\sqrt{3}+4} \right)$	A1		$m = -4 \quad n = -3$ or any multiple

(a)(ii) Special case B1 for $\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$

(b) M1 for substituting candidates values for $\tan\alpha$ and $\tan\beta$ into correct formula.
Completely correct or completely correct ft on $\tan\alpha$, $\tan\beta$.
Special case answer is $\dfrac{12+3\sqrt{3}}{9-4\sqrt{3}}$ or $\times\dfrac{a}{a}$ where a is integer or $\sqrt{3}$ for M1m1A0

Q	Solution	Marks	Total	Comments
3 (a)	$(1+6x)^{\frac{2}{3}} = 1+\frac{2}{3}\times 6x + kx^2$	M1		
	$= 1+4x-4x^2$	A1	2	Simplified coefficients required
(b)	$(8+6x)^{\frac{2}{3}} = 8^{\frac{2}{3}}\left(1+\frac{6}{8}x\right)^{\frac{2}{3}}$	B1		OE
	$\left(1+\frac{6}{8}x\right)^{\frac{2}{3}} = 1+4\left(\frac{x}{8}\right)-4\left(\frac{x}{8}\right)^2$	M1		x replaced by $\frac{x}{8}$ in answer to (a) Condone missing brackets, allow one error.
	$(8+6x)^{\frac{2}{3}} = 4+2x-\frac{1}{4}x^2$	A1	3	Simplified coefficients required.
(c)	$(100 = 10^2 \quad 8+6x = 10 \quad x=\frac{1}{3})$ $4+2\times\frac{1}{3}-\frac{1}{4}\times\left(\frac{1}{3}\right)^2$	M1		Use $x=\frac{1}{3}$ in binomial expansion from part (b)
	$= \frac{167}{36}$	A1	2	$\sqrt[3]{100} \approx \frac{167}{36}$
		Total	**7**	
3 (b)	**Alternative** $(8+6x)^{\frac{2}{3}} = 8^{\frac{2}{3}}\left(1+\frac{6}{8}x\right)^{\frac{2}{3}}$			OE
	$\left(1+\frac{6}{8}x\right)^{\frac{2}{3}} = 1+\frac{2}{3}\left(\frac{6}{8}x\right)+\frac{2}{3}\left(\frac{2}{3}-1\right)\frac{1}{2}\left(\frac{6}{8}x\right)^2$			Condone missing brackets, allow one error.
	$(8+6x)^{\frac{2}{3}} = 4+2x-\frac{1}{4}x^2$			
	Alternative $8^{\frac{2}{3}}+\frac{2}{3}\times 8^{-\frac{1}{3}}\times 6x+\frac{2}{3}\left(\frac{2}{3}-1\right)\frac{1}{2}\times 8^{-\frac{4}{3}}\times(6x)^2$ $4+2x-\frac{1}{4}x^2$			Use binomial formula; condone one error and missing brackets.
(a)(b)	Condone $1^{\frac{2}{3}}$ for 1 for M1			

Q	Solution	Marks	Total	Comments
4 (a)	$P = 500e^{\frac{1}{8}\times 60}$ $= 904\ 000$	M1 A1	2	Must use $t = 60$ Nearest thousand required 904000 only
(b)(i)	$\left(e^{\frac{1}{8}t}\right)^2 = \frac{500000}{500}$ $t = 8\ln\sqrt{1000}$ $t = 27.6$ (minutes)	M1 M1 A1	3	OE Take logs correctly leading to expression for t. Accept 27.631
(ii)	$500e^{\frac{1}{8}T} - 500000e^{-\frac{1}{8}T} = 45000$ $\times\frac{e^{\frac{1}{8}T}}{500} \Rightarrow \left(e^{\frac{1}{8}T}\right)^2 - 1000 = 90e^{\frac{1}{8}T}$ $\left(e^{\frac{1}{8}T}\right)^2 - 90e^{\frac{1}{8}T} - 1000 = 0$ $e^{\frac{1}{8}T} = 100$ ($e^{\frac{1}{8}T} = -10$ rejected) $t = 36.8$ (minutes)	M1 A1 M1 A1	4	Set up equation; condone one error; allow in t. Condone inequality. Multiply by $\frac{e^{\frac{1}{8}T}}{500}$ and rearrange to AG, be convinced. Solve quadratic equation (retaining positive root). CAO
		Total	**9**	
4 (b)(i)	Alternative $e^{\frac{1}{8}t} = 1000e^{-\frac{1}{8}t} \Rightarrow e^{\frac{1}{4}t} = \frac{500000}{500}$ $t = 4\ln 1000$ $t = 27.6$ (minutes) Alternative $e^{\frac{1}{8}t} = 1000e^{-\frac{1}{8}t} \Rightarrow \ln\left(e^{\frac{1}{8}t}\right) = \ln 1000 + \ln\left(e^{-\frac{1}{8}t}\right)$ $t = 4\ln 1000$ $t = 27.6$ (minutes)	M1 M1 A1 M1 M1 A1	3 3	Take logs correctly leading to expression for t. Take logs correctly.

(b)(ii) M1 for solve quadratic equation Let $x = e^{\frac{1}{8}t}$ solve quadratic equation $x^2 - 90x - 1000 = 0$ by
inspection, $x = 100$ seen;
factors $(x-100)(x+10)$ with 100 and 10 seen;
complete square $x = 45 \pm \sqrt{3025}$ all correct
formula $x = \frac{90 \pm \sqrt{90^2 + 4000}}{2}$ all correct

Final answer ; must have $t = 36.8$ for A1

(b)(i) 27.6 as final answer NMS 3/3
27.6 following wrong working AO (FIW) but could still score M mark(s)

Q	Solution	Marks	Total	Comments
5(a)	$xy^2+3y=\left(8t^2-t\right)\left(\frac{3}{t}\right)^2+3\left(\frac{3}{t}\right)$	M1		Substitute and expand
	$=72-\frac{9}{t}+\frac{9}{t}=72$	A1	2	$k=72$
(b)(i)	$\frac{\mathrm{d}x}{\mathrm{d}t}=16t-1 \quad \frac{\mathrm{d}y}{\mathrm{d}t}=-\frac{3}{t^2}$	B1B1		
	$t=\frac{1}{4} \quad \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-\frac{3}{\left(\frac{1}{4}\right)^2}}{16\times\frac{1}{4}-1}$	M1		Use chain rule $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-3}{16t^3-t^2}\right)$ and calculate gradient using $t=\frac{1}{4}$
	$=-16$	A1		
	$t=\frac{1}{4} \quad x=\frac{8}{16}-\frac{1}{4} \quad y=\frac{3}{\frac{1}{4}}$	M1		Calculate x and y using $t=\frac{1}{4}$
	$x=\frac{1}{4} \quad y=12$	A1		Both correct
	tangent $y=-16x+16$	A1	7	ACF CSO $y-12=-16\left(x-\frac{1}{4}\right)$ ISW
(ii)	$y=-16\times\frac{3}{2}+16=-8$	M1		Substitute $x=\frac{3}{2}$ into candidate's tangent; calculate y
	$\frac{3}{2}(-8)^2+3\times(-8)=96-24=72$	A1	2	$y=-8$ used to verify 72
		Total	**11**	
5(a)	Alternative $x=8\left(\frac{3}{y}\right)^2-\frac{3}{y}$	M1		Eliminate t
	$xy^2+3y=72$	A1	2	$k=72$
(b)(i)	Alternative $2xy\frac{\mathrm{d}y}{\mathrm{d}x}+y^2$	M1A1		Product rule attempted; two terms added, one with $\frac{\mathrm{d}y}{\mathrm{d}x}$
	$+3\frac{\mathrm{d}y}{\mathrm{d}x}=0$	B1		
	$t=\frac{1}{4} \quad x=\frac{8}{16}-\frac{1}{4} \quad y=\frac{3}{\frac{1}{4}}$	M1		Calculate x and y using $t=\frac{1}{4}$
	$x=\frac{1}{4} \quad y=12$	A1		Both correct.
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-y^2}{2xy+3}\right) \quad \frac{\mathrm{d}y}{\mathrm{d}x}=-16$	m1		Calculate gradient from candidate's expression.
	tangent $y=-16x+16$	A1	7	ACF CSO $y-12=-16\left(x-\frac{1}{4}\right)$ ISW

Q	Solution	Marks		Total
5(b)(i)	Alternative			
	$x=\dfrac{72-3y}{y^2}$	M1		Correct expression for x from candidate's implicit equation. Quotient rule attempted; y^4 and two terms subtracted.
	$\dfrac{dx}{dy}=\dfrac{y^2(-3)-(72-3y)\times 2y}{y^4}$	A1 A1		Numerator; first term; second term
	$\left(\dfrac{dx}{dy}=\dfrac{3y-144}{y^3}\right)$			
	$t=\dfrac{1}{4}\quad y=\dfrac{3}{\frac{1}{4}}=12$	B1		
	$\dfrac{dx}{dy}=-\dfrac{1}{16}\quad \dfrac{dy}{dx}=-16$	m1		Use $t=\dfrac{1}{4}$ to calculate y
	$t=\dfrac{1}{4}\quad x=\dfrac{8}{16}-\dfrac{1}{4}=\dfrac{1}{4}$	B1		Evaluate and invert.
	$y=-16x+16$	A1		
				Use $t=\dfrac{1}{4}$ to calculate x
	Alternative for $\dfrac{dx}{dy}$			ACF CSO
	$x=\dfrac{72}{y^2}-\dfrac{3}{y}$	M1		
	$\dfrac{dx}{dy}=-\dfrac{144}{y^3}+\dfrac{3}{y^2}$	A1 A1		Correct expression for x from candidate's implicit equation and attempt derivatives
	$\left(\dfrac{dx}{dy}=\dfrac{3y-144}{y^3}\right)$			

Q	Solution	Marks	Total	Comments
6(a)	$16\left(\frac{3}{4}\right)^3+11\left(\frac{3}{4}\right)-15$	M1		Evaluate $f\left(\frac{3}{4}\right)$; not long division.
	$=\frac{27}{4}+\frac{33}{4}-15=0\Rightarrow$ factor	A1	2	Processing and conclusion.
(b)	$27\cos\theta\left(2\cos^2\theta-1\right)+$	B1		Use acf of $\cos 2\theta$ formula
	$19\sin\theta\left(2\sin\theta\cos\theta\right)-15=0$	B1		Use acf of $\sin 2\theta$ formula
	$54\cos^3\theta-27\cos\theta+38\left(1-\cos^2\theta\right)\cos\theta$ $-15=0$	M1		All in cosines.
	$16\cos^3\theta+11\cos\theta-15=0$ $x=\cos\theta\Rightarrow 16x^3+11x-15=0$	A1	4	Simplification and substitute $x=\cos\theta$ to obtain AG CSO.
(c)	$16x^3+11x-15=\left(4x-3\right)\left(4x^2+3x+5\right)$	M1A1		Factorise $f\left(x\right)$
	$b^2-4ac=3^2-4\times4\times5 \quad \left(=-71\right)$	m1		Find discriminant of quadratic factor; or seen in formula
	$b^2-4ac<0$, no solution $\left(\text{to } 4x^2+3x+5=0\right)$ $\Rightarrow$ (only) solution is $\cos\theta=\frac{3}{4}$	A1	4	Conclusion; CSO Condone $x=\frac{3}{4}$ is (only) solution
		Total	10	

(a) For A1; minimum processing seen; $16\times\frac{27}{64}+11\times\frac{3}{4}-15=0$; $15-15=0$ and no other working is A0

minimum conclusion $=0$ hence factor

(b) For M1 mark; $\cos 2\theta$ (eventually) in form $a\cos^2\theta+b$; $19\sin\theta\sin 2\theta$ in form $c\cos\theta\sin^2\theta$ and use $\sin^2\theta=1-\cos^2\theta$ to obtain $c\cos\theta\left(1-\cos^2\theta\right)$

(c) M1 $\left(4x-3\right)\left(4x^2+kx\pm5\right)$ A1 fully correct

m1 candidate's values of *a, b, c* used in expression for b^2-4ac

or complete square to obtain $\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}$

A1 b^2-4ac correct or $\left(x+\frac{3}{8}\right)^2=\frac{9}{64}-\frac{5}{4}$ $\left(=-\frac{71}{64}\right)$ and stated to be negative so no solution

or solutions are not real (imaginary)

Accept imaginary solutions from calculator if stated to be imaginary.

Condone $\sqrt{-71}$ is negative, or similar, so no solution.

Conclusion $x=\frac{3}{4}$ is solution, or $\cos\theta=\frac{3}{4}$ is solution

Q	Solution	Marks	Total	Comments
7	$\int \frac{dy}{y^2} = \int x \sin 3x \, dx$	B1		Correct separation and notation; condone missing integral signs
	$\int \frac{dy}{y^2} = -\frac{1}{y}$	B1		
	$\int x \sin 3x \, dx = x\left(-\frac{1}{3}\cos 3x\right)$	M1		Use parts $u = x$ $\frac{dv}{dx} = \sin 3x$ $\frac{du}{dx} = 1$ $v = k \cos 3x$
	$- \int -\frac{1}{3}\cos 3x \, dx$	A1		with correct substitution into formula
	$= -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x$	A1		CAO
	$-\frac{1}{y} = -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x + C$			
	$-1 = -\frac{1}{3} \times \frac{\pi}{6}\cos\left(\frac{\pi}{2}\right) + \frac{1}{9}\sin\left(\frac{\pi}{2}\right) + C$	M1		Use $x = \frac{\pi}{6}$ $y = 1$ to find C
	$C = -\frac{10}{9}$	A1		CAO
	$-\frac{1}{y} = -\frac{1}{9}(3x\cos 3x - \sin 3x + 10)$	m1		And invert to $-y = -\frac{9}{(.....)}$
	$y = \frac{9}{3x\cos 3x - \sin 3x + 10}$	A1	9	CSO, condone first B1 not given
		Total	**9**	

Second M1 finding C; substitute $x = \frac{\pi}{6}$ $y = 1$ into $f(y) = px\cos 3x + q\sin 3x + C$ and evaluate using radians. Must calculate a value of C.

m1 for reaching form $\pm\frac{k}{y} = \frac{1}{9}(Px\cos 3x + Q\sin 3x + R)$ where P and Q are ± 3 or $\pm\frac{1}{3}$ or ± 1

and inverting to $\pm\frac{y}{k} = \frac{9}{(Px\cos 3x + Q\sin 3x + R)}$

Q	Solution	Marks	Total	Comments
8 (a)(i)	$\overrightarrow{AB}=\begin{bmatrix}2\\0\\-1\end{bmatrix}-\begin{bmatrix}4\\-2\\3\end{bmatrix}=\begin{bmatrix}-2\\2\\-4\end{bmatrix}$	M1 A1	2	$\pm\left(\overrightarrow{OB}-\overrightarrow{OA}\right)$ implied by two correct components Allow as $(-2,\ 2,\ -4)$
(ii)	$\begin{bmatrix}1\\5\\-2\end{bmatrix}\bullet\overrightarrow{AB}=-2+10+8=16$	M1 A1ft		ft on $\overrightarrow{AB}$
	$\cos\theta=\frac{16}{\sqrt{24}\sqrt{30}}$	M1		Correct formula for $\cos\theta$ with consistent vectors and correct moduli, in form $\sqrt{a^2+b^2+c^2}$
	$\theta=53°$	A1	4	CSO Accept 53.4°, 53.40°
(b)	$\overrightarrow{AB}\bullet\overrightarrow{BC}=\begin{bmatrix}-2\\2\\-4\end{bmatrix}\bullet\left(\begin{bmatrix}4+p\\-2+5p\\3-2p\end{bmatrix}-\begin{bmatrix}2\\0\\-1\end{bmatrix}\right)$	M1		SC B1 90° following *sp* = 0 Set up scalar product. $\mu=p$ at *C*. Any letter for *p*. Clear attempt to find $\overrightarrow{BC}$ in terms of *p*.
	$\overrightarrow{BC}=\begin{bmatrix}2+p\\-2+5p\\4-2p\end{bmatrix}$	B1		$\overrightarrow{BC}$ or $\overrightarrow{CB}$ correct
	$-4-2p-4+10p-16+8p=0$	m1		
	$16p=24\qquad p=\frac{3}{2}$	A1		Expand scalar product and solve for *p*; ($=0$ possibly implied)
	$\overrightarrow{OD}=\overrightarrow{OA}+\overrightarrow{BC}=\begin{bmatrix}4\\-2\\3\end{bmatrix}+\begin{bmatrix}\frac{7}{2}\\\frac{11}{2}\\1\end{bmatrix}\quad\left(=\begin{bmatrix}\frac{15}{2}\\\frac{7}{2}\\4\end{bmatrix}\right)$	m1		Correct vector expression to find $\overrightarrow{OD}$ written in components
	D is at $\left(\frac{15}{2},\frac{7}{2},4\right)$	A1	6	CAO; condone column vector
		Total	**12**	
	Alternative for last 2 marks $\overrightarrow{OD}=\overrightarrow{OC}+\overrightarrow{BA}=\begin{bmatrix}4\\-2\\3\end{bmatrix}+\frac{3}{2}\begin{bmatrix}1\\5\\-2\end{bmatrix}+\begin{bmatrix}2\\-2\\4\end{bmatrix}$	m1		
	D is at $\left(\frac{15}{2},\frac{7}{2},4\right)$	A1		

Part (b) **NB** $p=\frac{3}{2}$ can come from wrong working where candidate uses $\overrightarrow{OC}$ in place of $\overrightarrow{BC}$.
This is M0 and scores **no further marks**, (unless they happen to find and go on to use it correctly).