Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS
METHODS FOR ADVANCED MATHEMATICS, C3

4753

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF 2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, Centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You **may** use a graphical or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

Section A (36 marks)

1 It is suggested that the function $\mathrm{f}(x) = (x+1)^2$ is even.
Prove this is false. **[2]**

2 Find $\int x \sin 2x \, \mathrm{d}x$. **[4]**

3 Make t the subject in $P = P_0 \mathrm{e}^{0.1(t-3)}$. **[5]**

4 Sketch the graph of $y = |2x+3|$.
Hence, or otherwise, solve the equation $|2x+3| = 2 - x$. **[5]**

5 Using the substitution $u = 2x - 1$, or otherwise, calculate the exact value of $\int_0^{0.5} 4x(2x-1)^7 \, \mathrm{d}x$. **[5]**

6 Differentiate $\sqrt{2x+1}$ with respect to x and show that $\frac{\mathrm{d}}{\mathrm{d}x}(x^2\sqrt{2x+1}) = \frac{5x^2+2x}{\sqrt{2x+1}}$. **[7]**

7 The function f(x) is defined as $\mathrm{f}(x) = \frac{\cos x}{\mathrm{e}^x}$ for $-\pi \le x \le \pi$.

Show that $\mathrm{f}(x) \ge 0$ for $\frac{-\pi}{2} \le x \le \frac{\pi}{2}$.

State the values of x for which f(x) = 0.

Show, using calculus, that the maximum value of f(x) is 1.55, correct to 2 decimal places. **[8]**

Section B (36 marks)

8 Fig. 8.1 shows a sketch of the graph $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \sqrt{4 - x}$ for $0 \le x \le 4$.

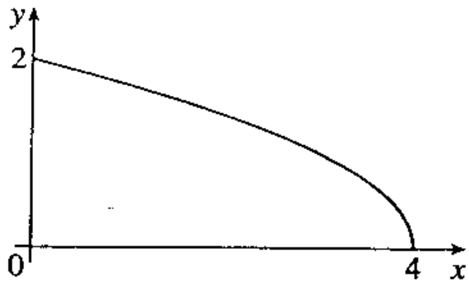

Fig. 8.1

(i) Write down the domain and range of $\mathrm{f}(x)$. **[2]**

(ii) **(*A*)** Find the inverse function $\mathrm{f}^{-1}(x)$. **[3]**

(*B*) Copy Fig 8.1 and draw the graph of $y = \mathrm{f}^{-1}(x)$ on the same diagram.
What is the connection between the graph of $y = \mathrm{f}(x)$ and the graph of $y = \mathrm{f}^{-1}(x)$? **[2]**

(iii) Figs. 8.2, 8.3 and 8.4 below show the graph of $y = \mathrm{f}(x)$, together with the graphs of $y = \mathrm{f}_1(x)$, $y = \mathrm{f}_2(x)$ and $y = \mathrm{f}_3(x)$ respectively, each of which is a simple transformation of the graph $y = \mathrm{f}(x)$.
Find expressions in terms of *x* for each of the functions $\mathrm{f}_1(x)$, $\mathrm{f}_2(x)$ and $\mathrm{f}_3(x)$.

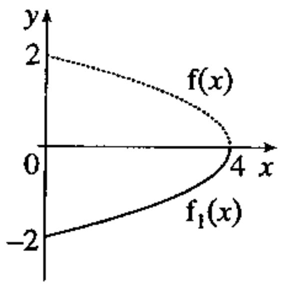

Fig. 8.2

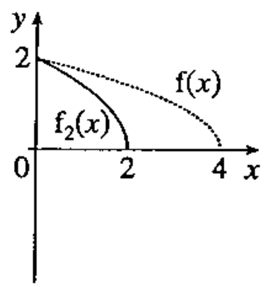

Fig. 8.3

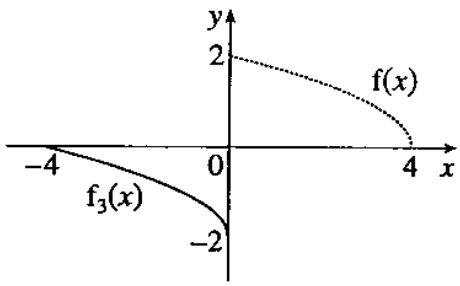

Fig. 8.4 **[4]**

(iv) The function $\mathrm{g}(x)$ is defined in such a way that the composite function $\mathrm{gf}(x)$ is given by $\mathrm{gf}(x) = x - 4$.
Find the functions $\mathrm{g}(x)$ and $\mathrm{g}^2(x)$. **[3]**

(v) State the range of the function $\mathrm{f}^2(x)$.
Hence show that the equation $\mathrm{f}^2(x) = x$ must have a solution.
[You are **not** required to solve the equation.] **[4]**

9 Fig. 9 shows a sketch of the graph $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{\ln x}{x}$ $(x > 0)$.

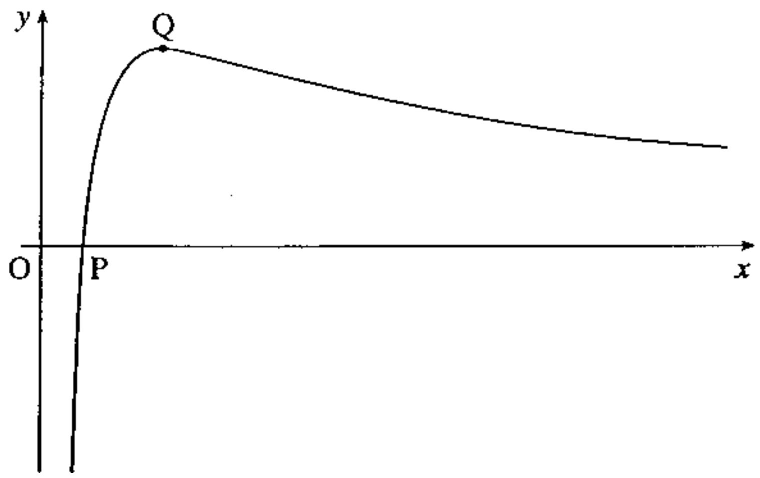

Fig. 9

The graph crosses the x-axis at the point P and has a turning point at Q.

(i) Write down the x-coordinate of P. **[2]**

(ii) Find the first and second derivatives $\mathrm{f}'(x)$ and $\mathrm{f}''(x)$, simplifying your answers as far as possible. **[5]**

(iii) **(*A*)** Hence show that the x-coordinate of Q is e. **[2]**

(*B*) Find the y-coordinate of Q in terms of e. **[1]**

(*C*) Find $\mathrm{f}''(\mathrm{e})$ and use this result to verify that Q is a maximum point. **[2]**

(iv) Find the exact area of the finite region between the graph $y = \mathrm{f}(x)$, the x-axis, and the line $x = 2$. **[6]**