

General Certificate of Education
June 2008
Advanced Level Examination



MATHEMATICS
Unit Pure Core 3

MPC3

Friday 23 May 2008 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 Find $\frac{dy}{dx}$ when:

(a) $y = (3x + 1)^5$; *(2 marks)*

(b) $y = \ln(3x + 1)$; *(2 marks)*

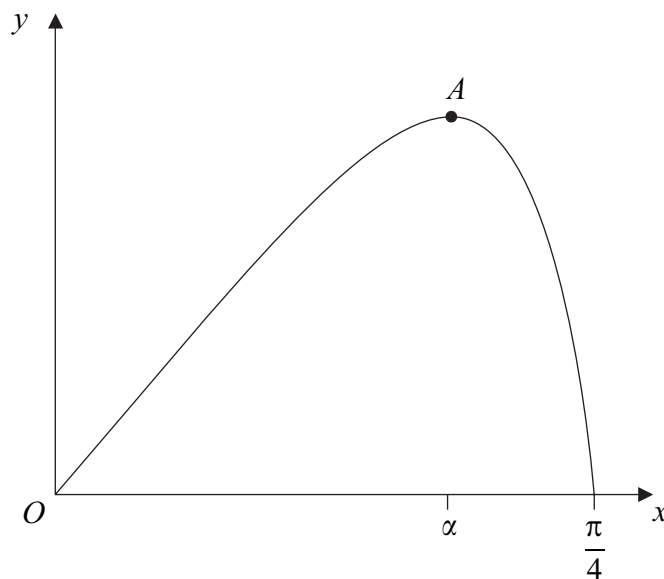
(c) $y = (3x + 1)^5 \ln(3x + 1)$. *(3 marks)*

2 (a) Solve the equation $\sec x = 3$, giving the values of x in radians to two decimal places in the interval $0 \leq x < 2\pi$. *(3 marks)*

(b) Show that the equation $\tan^2 x = 2 \sec x + 2$ can be written as $\sec^2 x - 2 \sec x - 3 = 0$. *(2 marks)*

(c) Solve the equation $\tan^2 x = 2 \sec x + 2$, giving the values of x in radians to two decimal places in the interval $0 \leq x < 2\pi$. *(4 marks)*

- 3 A curve is defined for $0 \leq x \leq \frac{\pi}{4}$ by the equation $y = x \cos 2x$, and is sketched below.



- (a) Find $\frac{dy}{dx}$. (2 marks)
- (b) The point A , where $x = \alpha$, on the curve is a stationary point.
- (i) Show that $1 - 2\alpha \tan 2\alpha = 0$. (2 marks)
- (ii) Show that $0.4 < \alpha < 0.5$. (2 marks)
- (iii) Show that the equation $1 - 2x \tan 2x = 0$ can be rearranged to become $x = \frac{1}{2} \tan^{-1} \left(\frac{1}{2x} \right)$. (1 mark)
- (iv) Use the iteration $x_{n+1} = \frac{1}{2} \tan^{-1} \left(\frac{1}{2x_n} \right)$ with $x_1 = 0.4$ to find x_3 , giving your answer to two significant figures. (2 marks)
- (c) Use integration by parts to find $\int_0^{0.5} x \cos 2x \, dx$, giving your answer to three significant figures. (5 marks)

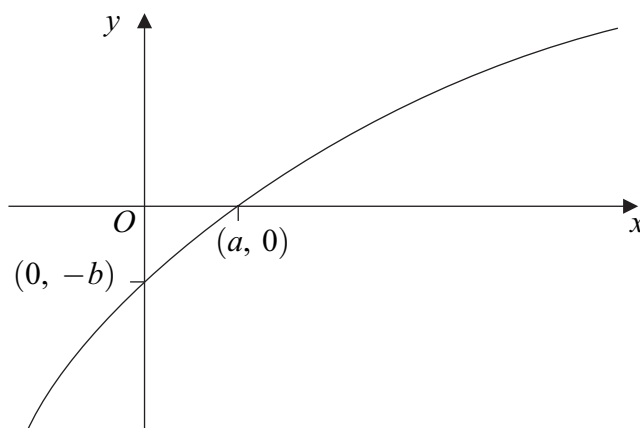
4 The functions f and g are defined with their respective domains by

$$f(x) = x^2, \quad \text{for all real values of } x$$

$$g(x) = \frac{1}{2x-3}, \quad \text{for real values of } x, \quad x \neq \frac{3}{2}$$

- (a) State the range of f . (1 mark)
- (b) (i) The inverse of g is g^{-1} . Find $g^{-1}(x)$. (3 marks)
- (ii) State the range of g^{-1} . (1 mark)
- (c) Solve the equation $fg(x) = 9$. (3 marks)

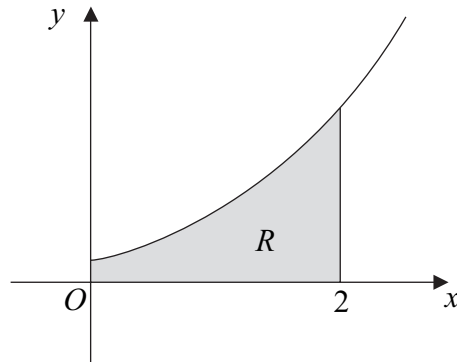
5 (a) The diagram shows part of the curve with equation $y = f(x)$. The curve crosses the x -axis at the point $(a, 0)$ and the y -axis at the point $(0, -b)$.



On separate diagrams, sketch the curves with the following equations. On each diagram, indicate, in terms of a or b , the coordinates of the points where the curve crosses the coordinate axes.

- (i) $y = |f(x)|$. (2 marks)
- (ii) $y = 2f(x)$. (2 marks)
- (b) (i) Describe a sequence of geometrical transformations that maps the graph of $y = \ln x$ onto the graph of $y = 4 \ln(x + 1) - 2$. (6 marks)
- (ii) Find the exact values of the coordinates of the points where the graph of $y = 4 \ln(x + 1) - 2$ crosses the coordinate axes. (4 marks)

- 6 The diagram shows the curve with equation $y = (e^{3x} + 1)^{\frac{1}{2}}$ for $x \geq 0$.



- (a) Find the gradient of the curve $y = (e^{3x} + 1)^{\frac{1}{2}}$ at the point where $x = \ln 2$. (5 marks)
- (b) Use the mid-ordinate rule with four strips to find an estimate for $\int_0^2 (e^{3x} + 1)^{\frac{1}{2}} dx$, giving your answer to three significant figures. (4 marks)
- (c) The shaded region R is bounded by the curve, the lines $x = 0$, $x = 2$ and the x -axis.

Find the exact value of the volume of the solid generated when the region R is rotated through 360° about the x -axis. (4 marks)

- 7 (a) Given that $y = \frac{\sin \theta}{\cos \theta}$, use the quotient rule to show that $\frac{dy}{d\theta} = \sec^2 \theta$. (3 marks)
- (b) Given that $x = \sin \theta$, show that $\frac{x}{\sqrt{1-x^2}} = \tan \theta$. (2 marks)
- (c) Use the substitution $x = \sin \theta$ to find $\int \frac{1}{(1-x^2)^{\frac{3}{2}}} dx$, giving your answer in terms of x . (5 marks)

END OF QUESTIONS

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