# LEVEL 2 CERTIFICATE
# Further Mathematics

8360/1 – Paper 1 Non-calculator

Mark scheme

June 2018

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

## Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

| | |
|---|---|
| **M** | Method marks are awarded for a correct method which could lead to a correct answer. |
| **M dep** | A method mark dependent on a previous method mark being awarded. |
| **A** | Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied. |
| **B** | Marks awarded independent of method. |
| **B dep** | A mark that can only be awarded if a previous independent mark has been awarded. |
| **ft** | Follow through marks. Marks awarded following a mistake in an earlier step. |
| **SC** | Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth. |
| **oe** | Or equivalent. Accept answers that are equivalent.<br>eg, accept 0.5 as well as $\frac{1}{2}$ |
| **[a, b]** | Accept values between *a* and *b* inclusive. |
| **3.14…** | Accept answers which begin 3.14 eg 3.14, 3.142, 3.1416 |

Examiners should consistently apply the following principles.

***Diagrams***

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

***Responses which appear to come from incorrect methods***

Whenever there is doubt as to whether a candidate has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the candidate. In cases where there is no doubt that the answer has come from incorrect working then the candidate should be penalised.

***Questions which ask candidates to show working***

Instructions on marking will be given but usually marks are not awarded to candidates who show no working.

***Questions which do not ask candidates to show working***

As a general principle, a correct response is awarded full marks.

***Misread or miscopy***

Candidates often copy values from a question incorrectly. If the examiner thinks that the candidate has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

***Further work***

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

***Choice***

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

***Work not replaced***

Erased or crossed out work that is still legible should be marked.

***Work replaced***

Erased or crossed out work that has been replaced is not awarded marks.

***Premature approximation***

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

***Continental notation***

Accept a comma used instead of a decimal point (for example, in measurements or currency), provided that it is clear to the examiner that the candidate intended it to be a decimal point.

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 1 | $\frac{6x^5}{2}$ or $3x^5$ or $\frac{4x^3}{4}$ or $x^3$ | M1 | oe eg $\frac{12x^5}{4}$ |
| | $3x^5 + x^3$ | A1 | or a correct factorised version<br>eg $x^3(3x^2 + 1)$ |
| | **Additional Guidance** | | |
| | Do not ignore further work, eg correct answer followed by $4x^8$ scores M1 A0<br>They must use the powers of $x$ as given in the question, so no misread possible here | | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 2 | **Alternative method 1** | | |
| | $\frac{6 + (-12)}{2}$ or $\frac{-2 + b}{2}$ | M1 | oe<br>eg 18 ÷ 2 = 9 **and** 6 − 9<br>eg 6 × 2 = 12 **and** −2 + 12<br>These come from distances of 18 and 6, as seen in a diagram and used correctly |
| | $a = -3$ | A1 | |
| | $b = 10$ | A1 | |
| | **Alternative method 2** | | |
| | $a - (-12) = 6 - a$ or<br>$4 - b = -2 - 4$ | M1 | oe eg $4 + 4 - -2$ |
| | $a = -3$ | A1 | |
| | $b = 10$ | A1 | |
| | **Alternative method 3** | | |
| | eg $6 - (-12) = 2(a - -12)$<br>eg $-2 - b = 2(-2 - 4)$ | M1 | for using an equation relating the "gap" between the points |
| | $a = -3$ | A1 | |
| | $b = 10$ | A1 | |
| | **Additional guidance** | | |
| | Either answer correct, but no working, implies the M mark, eg $a = -3$, $b = 6$ scores M1 A1 A0<br>Correct answer seen with no working scores full marks<br>$a = 10$ and $b = -3$ (correct values but the wrong way round) with no working scores SC1 | | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 3 | $\begin{pmatrix} 4 & 16 \\ -8 & 17 \end{pmatrix}$ | B2 | B1 for any two or three correct elements in the correct position in a 2 × 2 matrix |
| | **Additional Guidance** | | |
| | Correct answer followed by further work, eg $\begin{pmatrix} 20 \\ 9 \end{pmatrix}$ scores B1 only<br>Matrices multiplied the wrong way round can score SC1 if correct<br>BA = $\begin{pmatrix} 14 & -14 \\ 7 & 7 \end{pmatrix}$<br>Condone no brackets around the numbers in their 2 × 2 matrix<br>Ignore any commas that appear in their 2 × 2 matrix<br>Do not follow through on any misreads of the numbers in the given matrices | | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 4 | **Alternative method 1** | | |
| | $1.25 \times 4x$ or $5x$ | M1 | oe |
| | $0.6 \times 7x$ or $4.2x$ | M1 | oe |
| | their $5x$ − their $4.2x = 28$<br>or $0.8x = 28$ | M1dep | oe eg their $5x$ = their $4.2x + 28$<br>dep upon at least one of previous M marks earned |
| | $x = 35$ | A1 | |
| | **Alternative method 2** | | |
| | two numbers in the ratio 4 : 7 | M1 | |
| | correct increase by 25% and decrease by 40% calculations **and** comparison with 28 | M1dep | If difference is not 28, then first numbers must be clearly rejected |
| | second trial with correct calculations and comparison | M1dep | correct first trial means 2nd and 3rd M marks scored automatically |
| | $x = 35$ | A1 | |
| | **Additional Guidance** | | |
| | Mark the better of their two versions if they try both methods. | | |
| | In alt 2 ... for the 2nd M1 (dep on 1st M1) ... the % calculations must be correct. If the difference is not 28 they must reject them. Attempting another two % calculations is sufficient evidence of this. | | |
| | In alt 2 ... for the 3rd M1 (dep on the first two M1's) ... the difference must be closer than their first attempt. They can have more than one attempt at this so as to eventually score the 3rd M1. To score this mark they need to indicate clearly that this further attempt is better than their first attempt. | | |
| | In alt 2 ... if it isn't clear in which order they have done their attempts (eg very untidy working written all over the page) and they do not indicate which is the better attempt, then they can score a maximum of 2 marks. | | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 5 | $(x^3) + 5x^2 + kx - 3x^2 - 15x\ (-\ 3k)$ | M1 | allow one sign error in the $x^2$ or $x$ terms |
| | their (5 – 3) = their ($k$ – 15) | M1dep | $5x^2 - 3x^2 = kx - 15x$ on its own, is not enough for M1dep |
| | 17 | A1 | |
| | **Additional Guidance** | | |
| | For the first M1, we do not need to see the $x^3$ term or the $-\ 3k$ term, but we do need to see the other 4 terms (3 terms, if they combine the $x^2$ terms).<br>The terms of the expansion might appear in a grid, which can score the first M1<br>Mark positively ... terms in a grid might differ from terms written as a string of terms ... mark the better version. | | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 6 | $(x + 1)^2 + (y - 2)^2 = 25$ | B1 | tick in 3rd box |
| | **Additional Guidance** | | |
| | | | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 7 | **Alternative method 1** | | |
| | reflex angle *AOC* = $2 \times 2x$ or $4x$ | M1 | |
| | their $4x + x + 75 = 360$ | M1dep | oe<br>If they start with this equation, the first M1, for reflex angle *AOC* = $4x$, is implied |
| | $(x =)$ 57 | A1 | |
| | **Alternative method 2** | | |
| | reflex angle *AOC* = $360 - (x + 75)$<br>or $285 - x$ | M1 | oe |
| | $360 - (x + 75) = 2(2x)$<br>or their $285 - x = 2(2x)$ | M1dep | oe |
| | $(x =)$ 57 | A1 | |
| | **Alternative method 3** | | |
| | angle at circumference = $180 - 2x$ | M1 | creating a cyclic quadrilateral |
| | $x + 75 = 2(180 - 2x)$<br>or $x + 75 = 360 - 2(2x)$ | M1dep | oe |
| | $(x =)$ 57 | A1 | |
| | **Alternative method 4** | | |
| | angle at circumference = $\frac{x + 75}{2}$ | M1 | oe<br>creating a cyclic quadrilateral |
| | $\frac{x + 75}{2} + 2x = 180$ | M1dep | oe<br>$\frac{x}{2} + \frac{\text{their } 75}{2} + 2x = 180$ scores this mark |
| | $(x =)$ 57 | A1 | |
| | **Additional Guidance** | | |
| | $4x = x + 75$ (ans $x = 25$) and $x + 75 + 2x = 180$ (ans $x = 35$) both score 0 marks | | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 8 | $4 - \sqrt{5} + 8\sqrt{5} - 2\sqrt{5}\sqrt{5}$ | M1 | oe<br>allow one incorrect term in a four term expansion |
| | $-6 + 7\sqrt{5}$ | A1 | |
| | **Additional Guidance** | | |
| | Any incorrect further work loses the A mark, so they can only score M1 A0 | | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 9 | **Alternative method 1** | | |
| | $14 - (2x)^2$ or $14 - 4x^2$ | M1 | or $14 - (2x)^2 = 5$ or $14 - 4x^2 = 5$ |
| | $14 - 5 = (2x)^2$ or $9 = 4x^2$<br>or $9 - 4x^2 = 0$ or $4x^2 - 9 = 0$<br>or $(2x + 3)(2x - 3) = 0$ | M1dep | |
| | $(x =)\ \frac{3}{2}$ or 1.5 | A1 | |
| | $(x =)\ -\frac{3}{2}$ or $-1.5$ | A1 | |
| | **Alternative method 2** | | |
| | $14 - x^2 = 5$ and $x = \pm 3$ | M1 | |
| | $2x = \pm 3$ | M1dep | |
| | $(x =)\ \frac{3}{2}$ or 1.5 | A1 | |
| | $(x =)\ -\frac{3}{2}$ or $-1.5$ | A1 | |
| | **Additional Guidance** | | |
| | A final answer of $\sqrt{\frac{9}{4}}$ scores M1 M1 A0 A0<br>A final answer of $\pm\sqrt{\frac{9}{4}}$ scores M1 M1 A1 A0 | | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 10 | A correct first step using algebra | M1 | Here are some of the possible alternatives<br>$\frac{1}{x} = y\left(4 - \frac{3}{y}\right)$ multiplying through by y<br>$1 = xy\left(4 - \frac{3}{y}\right)$ multiplying through by xy<br>$1 = 4xy - \frac{3xy}{y}$ multiplying through by xy<br>$y = 4xy^2 - 3xy$ multiplying through by $xy^2$<br>$\frac{1}{xy} = \frac{4y - 3}{y}$ making the RHS an algebraic fraction<br>$\frac{1 + 3x}{xy} = 4$ rearranging **and** making the LHS an algebraic fraction |
| | Further correct algebra which leads to an equation that is one step from the final answer. | M1dep | Following two of the above alternatives ...<br>$y = 4xy^2 - 3xy$<br>$y = x(4y^2 - 3y)$ M1dep gained<br>$\frac{1 + 3x}{xy} = 4$<br>$1 + 3x = 4xy$<br>$1 = 4xy - 3x$<br>$1 = x(4y - 3)$ M1dep gained |
| | A correct final answer in **any** form | A1 | $x = \frac{1}{4y - 3}$ $x = \frac{-1}{3 - 4y}$<br>$x = \frac{y}{4y^2 - 3y}$ $x = \frac{-y}{3y - 4y^2}$<br>$x = \frac{1}{y\left(4 - \frac{3}{y}\right)}$ $x = \frac{-1}{y\left(\frac{3}{y} - 4\right)}$<br>$x = \frac{1}{\left(4 - \frac{3}{y}\right)} \div y$ |
| | | | |

| | Additional Guidance |
|---|---|
| | There are many ways of scoring the first M mark. They do not need to give any reasons but you need to check that what they do is valid.<br>For the M1dep mark you must check that their algebra is correct and will lead to a result that is one step from the final answer. 'One step from ...' means that when they divide through, they have a correct version where $x$ is the subject.<br>Some of the final answers are more compact than others, but we didn't ask for any simplification so we have to accept a correct answer in any form.<br><br>... and, finally, one to look out for ... correct answer from wrong working ... 0 marks<br>$\frac{1}{xy} = 4 - \frac{3}{y} \rightarrow xy = \frac{1}{4} - \frac{y}{3} \rightarrow x = \frac{1}{4y} - \frac{1}{3} \rightarrow x = \frac{1}{4y-3}$ (creative thinking !) |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 11 | $4x + 3$ or gradient $= -5$ seen | M1 | |
| | $4x + 3 = -5$ | M1dep | |
| | $x = -2$ | A1 | |
| | $y = -7$ | A1ft | ft their $x$ only if M2 earned |
| | **Additional Guidance** | | |
| | | | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 12 | $\frac{5}{3} \times 15$<br>or<br>25 seen as the length of *OB* or the coordinates of *B* | M1 | |
| | gradient *AB* $= \frac{0 - \text{their } 25}{15 - 0}$ or $-\frac{5}{3}$ | M1 | oe |
| | gradient *BC* $= -1 \div (\text{their } -\frac{5}{3})$ or $\frac{3}{5}$ | M1 | oe |
| | $y = \frac{3}{5}x + 25$ | A1 | oe eg $y = \frac{15}{25}x + 25$ or $5y = 3x + 125$ |
| | **Additional Guidance** | | |
| | We must see y = .......... for A1 (or any other correct equation)<br>Look for this in their working if it isn't written on the answer line.<br><br>A sign error in their gradient *AB*, after a correct expression, can be recovered.<br><br>eg gradient *AB* $= \frac{0 - 25}{15 - 0} = \frac{25}{15} = \frac{5}{3}$<br><br>gradient *BC* $= \frac{3}{5}$ (positive gradient because they can see it from the diagram)<br>equation *BC* is $y = \frac{3}{5}x + 25$ ... this scores 4 marks<br><br>similarly, recovery can be from ...<br><br>gradient *AB* $= \frac{25}{15} = \frac{5}{3}$ ... without seeing $\frac{0 - 25}{15 - 0}$<br>... and can still lead to 4 marks | | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 13 | **Alternative method 1** | | |
| | $3x + 5 = \frac{2}{x}$ or $x(3x + 5) = 2$ | M1 | oe |
| | $3x^2 + 5x - 2$ (= 0) or $3x^2 + 5x = 2$ | M1dep | |
| | $(3x + a)(x + b)$ (= 0) | M1dep | $ab = -2$ or $a + 3b = 5$ |
| | $(3x - 1)(x + 2)$ (= 0) | A1 | |
| | $x = \frac{1}{3}$ $x = -2$ or $x = \frac{1}{3}$ $y = 6$ or $x = -2$ $y = -1$ | A1 | |
| | $x = \frac{1}{3}$ $x = -2$ $y = 6$ $y = -1$ or $x = \frac{1}{3}$ $y = 6$ $x = -2$ $y = -1$ | A1 | either correct $x$'s and correct $y$'s or correct coordinate pairs |
| | **Alternative method 2** | | |
| | $3x + 5 = \frac{2}{x}$ or $x(3x + 5) = 2$ | M1 | oe |
| | $3x^2 + 5x - 2$ (= 0) or $3x^2 + 5x = 2$ | M1dep | |
| | $x = \frac{-5 \pm \sqrt{[(5)^2 - 4(3)(-2)]}}{2(3)}$ | M1dep | allow one sign error ... but the 2 × 3 term must be beneath the full numerator |
| | $x = \frac{-5 \pm 7}{6}$ | A1 | |
| | $x = \frac{1}{3}$ $x = -2$ or $x = \frac{1}{3}$ $y = 6$ or $x = -2$ $y = -1$ | A1 | |
| | $x = \frac{1}{3}$ $x = -2$ $y = 6$ $y = -1$ or $x = \frac{1}{3}$ $y = 6$ $x = -2$ $y = -1$ | A1 | either correct $x$'s and correct $y$'s or correct coordinate pairs |

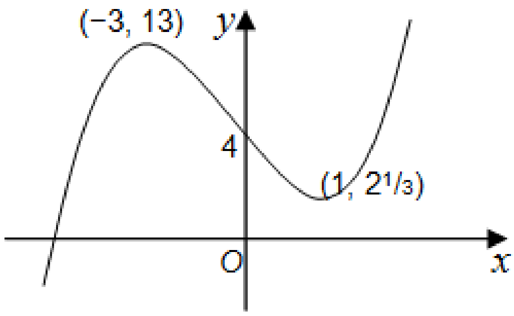
| | | | |
|---|---|---|---|
| | **Alternative method 3** | | |
| | $3x + 5 = \frac{2}{x}$ or $x(3x + 5) = 2$ | M1 | oe |
| | $3x^2 + 5x - 2\ (= 0)$ or $3x^2 + 5x = 2$ | M1dep | |
| | $(3 \times)\ (x + {}^5/_6)^2$ ............. | M1dep | |
| | $x + {}^5/_6 = \pm\ {}^7/_6$ | A1 | |
| | $x = \frac{1}{3}$ $x = -2$ or $x = \frac{1}{3}$ $y = 6$ or $x = -2$ $y = -1$ | A1 | |
| | $x = \frac{1}{3}$ $x = -2$ $y = 6$ $y = -1$ or $x = \frac{1}{3}$ $y = 6$ $x = -2$ $y = -1$ | A1 | either correct x's and correct y's or correct coordinate pairs |
| | **Alternative method 4** | | |
| | $y = 3\left(\frac{2}{y}\right) + 5$ or $\frac{y(y-5)}{3} = 2$ | M1 | oe |
| | $y^2 - 5y - 6 = 0$ or $y^2 - 5y = 6$ | M1dep | |
| | $(y + a)(y + b)\ (= 0)$ | M1dep | $ab = -6$ or $a + b = -5$ |
| | $(y - 6)(y + 1)\ (= 0)$ | A1 | |
| | $y = 6$ $y = -1$ or $y = 6$ $x = \frac{1}{3}$ or $y = -1$ $x = -2$ | A1 | |
| | $x = \frac{1}{3}$ $x = -2$ $y = 6$ $y = -1$ or $x = \frac{1}{3}$ $y = 6$ $x = -2$ $y = -1$ | A1 | either correct x's and correct y's or correct coordinate pairs |

| | | | |
|---|---|---|---|
| | **Alternative method 5** | | |
| | $y = 3\left(\frac{2}{y}\right) + 5$ or $\frac{y(y-5)}{3} = 2$ | M1 | oe |
| | $y^2 - 5y - 6 = 0$ or $y^2 - 5y = 6$ | M1dep | |
| | $y = \frac{5 \pm \sqrt{[(-5)^2 - 4(1)(-6)]}}{2(1)}$ | M1dep | allow one sign error ... but the 2 × 1 term must be beneath the full numerator |
| | $y = \frac{5 \pm 7}{2}$ | A1 | |
| | $y = 6$  $y = -1$ or $y = 6$  $x = \frac{1}{3}$ or $y = -1$  $x = -2$ | A1 | |
| | $x = \frac{1}{3}$  $x = -2$  $y = 6$  $y = -1$ or $x = \frac{1}{3}$  $y = 6$  $x = -2$  $y = -1$ | A1 | either correct $x$'s and correct y's or correct coordinate pairs |
| | **Alternative method 6** | | |
| | $y = 3\left(\frac{2}{y}\right) + 5$ or $\frac{y(y-5)}{3} = 2$ | M1 | oe |
| | $y^2 - 5y - 6 = 0$ or $y^2 - 5y = 6$ | M1dep | |
| | $(y - {}^5/_2)^2$ .............. | M1dep | |
| | $y - {}^5/_2 = \pm\, {}^7/_2$ | A1 | |
| | $y = 6$  $y = -1$ or $y = 6$  $x = \frac{1}{3}$ or $y = -1$  $x = -2$ | A1 | |
| | $x = \frac{1}{3}$  $x = -2$  $y = 6$  $y = -1$ or $x = \frac{1}{3}$  $y = 6$  $x = -2$  $y = -1$ | A1 | either correct $x$'s and correct y's or correct coordinate pairs |

| | | |
|---|---|---|
| | **Additional Guidance** | |
| | Trial and improvement ... 0 marks     No working shown ..... 0 marks<br>The instructions were clearly stated in the question. | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 14 | **Alternative method 1** | | |
| | $3^{\frac{1}{2}} \times 3^{\frac{1}{2}} + 3^{\frac{1}{2}} \times 3^{\frac{3}{2}} + 3^{\frac{1}{2}} \times 3^{\frac{3}{2}} + 3^{\frac{3}{2}} \times 3^{\frac{3}{2}}$<br>or<br>$\sqrt{3}\sqrt{3} + \sqrt{3}\sqrt{27} + \sqrt{3}\sqrt{27} + \sqrt{27}\sqrt{27}$ | M1 | oe<br>allow an error in one term |
| | 3 or 9 or 27 | M1dep | |
| | 48 | A1 | |
| | **Alternative method 2** | | |
| | $\sqrt{3}$ and $3\sqrt{3}$ | M1 | $3\sqrt{3}$ must come from correct working |
| | $(4\sqrt{3})^2$ | M1dep | |
| | 48 | A1 | |
| | **Alternative method 3** | | |
| | $\left(3^{\frac{1}{2}}\right)^2 (1+3)^2$ | M1 | oe |
| | $3 \times 4^2$ | M1dep | oe |
| | 48 | A1 | |
| | **Additional Guidance** | | |
| | Alt 1 mark scheme ... likely to see a 3 (or 9 or 27) somewhere, so need to be careful that the M1 mark has been earned before awarding A1 | | |
| | In alt 1, for the first M1, we want to see an attempt at the full expansion of the correct terms.<br>Probably 4 terms, but there could be 3 if they combine the middle two terms.<br>eg $(\sqrt{3} + 27)(\sqrt{3} + 27)$ scores M0 because it ought to be $\sqrt{27}$ not 27 | | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 15 | $2 - x$ or $x - 2$ | M1 | do not award M1 if you see evidence of incorrect method for finding a linear expression |
| | $y = 2 - x$ accurately drawn | M1 | |
| | 3.4 | A1 | accept 3.3 to 3.5 |
| | 0.6 | A1 | accept 0.5 to 0.7 |
| | **Additional Guidance** | | |
| | For the first M1, start by looking for evidence of a correct method.<br>eg $x^2 - 4x + 2 + 3x - x^2 = -x + 2$<br>or<br>$x^2 - 4x + 2 = 0 \rightarrow x^2 - 3x - x + 2 = 0 \rightarrow -x + 2 = 3x - x^2$ | | |
| | Attempts to solve $x^2 - 4x + 2 = 0$ by using the quadratic formula or by completing the square or by drawing a new quadratic graph (for $y = x^2 - 4x + 2$) score 0 marks<br>You might see work which uses the quadratic formula or completing the square which leads to answers of $2 \pm \sqrt{2}$ ... and if this follows working using a correct method to find the linear graph, it can be ignored (they could be using it as a check on their answers obtained graphically), but if it looks like it is their main method, then award 0 marks, as stated above..<br>Ignore any $y$ coordinates that might accompany the final $x$ values. | | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 16 | factorising to get<br>$(x + 3)(x - 1)$ (= 0)<br>or<br>completing the square and getting as far as $x + 1 = \pm 2$<br>or<br>using the quadratic formula and getting as far as $x = \dfrac{-2 \pm 4}{2}$ | M1 |  |
| | $x = -3$ and $x = 1$ | A1 | |
| | (−3, 13) as a maximum point and<br>$(1, 2\frac{1}{3})$ as a minimum point, plotted | M1 | |
| | Smooth correct curve which must have the stationary points plotted in the correct quadrants and **must** cross the negative $x$-axis | A1 | |
| | **Additional Guidance** | | |
| | SC1 for a fully correct sketch with the stationary points in the correct quadrants but lacking any detail in terms of the $x$ coordinates of the stationary points, or with incorrect values of the stationary points, and with no evidence of a valid method to obtain $x = -3$ and $x = 1$ | | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 17 (a) | $f(2) = (2)^3 + 8(2)^2 + 5(2) - 50$<br>$= 8 + 32 + 10 - 50 = 0$ | B1 | substitutes $x = 2$ and verifies that $f(2) = 0$<br>... the terms **must** be evaluated |
| | **Additional Guidance** | | |
| | Using the factor theorem is essential. Using long division here scores M0 | | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 17 (b) | **Alternative method 1** | | |
| | $x^3 + 8x^2 + 5x - 50$<br>$\equiv (x - 2)(x^2 + kx + 25)$ | M1 | Sight of a 3 term quadratic with $x^2$ and +25 as the first and last terms |
| | $x^2 + 10x + 25$ | A1 | |
| | $(x - 2)(x + 5)^2$ | A1 | oe |
| | **Alternative method 2** | | |
| | Substitutes another value into the expression and tests for '= 0' | M1 | their value correctly worked out<br>eg $f(1) = -36$ $f(3) = 64$ |
| | $(x + 5)$ | A1 | coming from<br>$f(-5) = -125 + 200 - 25 - 50 = 0$ |
| | $(x - 2)(x + 5)^2$ | A1 | oe |
| | **Alternative method 3** | | |
| | Long division of polynomials getting as far as $x^2 + 10x$ ...... | M1 | |
| | $x^2 + 10x + 25$ | A1 | |
| | $(x - 2)(x + 5)^2$ | A1 | oe |
| | **Alternative method 4** | | |
| | Using synthetic division to arrive at<br>$x^2 + 10x$ ...... | M1 | $\begin{array}{r\|rrrr} 2 & 1 & 8 & 5 & -50 \\ & 0 & 2 & 20 & 50 \\ \hline & 1 & 10 & 25 & 0 \end{array}$ |
| | $x^2 + 10x + 25$ | A1 | |
| | $(x - 2)(x + 5)^2$ | A1 | oe |
| | **Alternative method 5** | | |
| | $x^3 + 8x^2 + 5x - 50$<br>$\equiv (x - 2)(ax^2 + bx + c)$<br>$\equiv ax^3 - 2ax^2 + bx^2 - 2bx + cx - 2c$ | M1 | |

| | | | |
|---|---|---|---|
| | **and** any two of $a = 1$, $b = 10$, $c = 25$ | | |
| | $x^2 + 10x + 25$ | A1 | |
| | $(x - 2)(x + 5)^2$ | A1 | oe |
| | **Additional Guidance** | | |
| | This work might appear in 17a ... you can mark it having seen it in 17a unless there is a contradiction with any work in 17b.<br>Also, mark from what you might see in 17a if there is no work in 17b | | |
| | Ignore further work which gives answers of 2, $-5$ and $-5$ (from solving $f(x) = 0$) | | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 18 | Angle *DST* = x | M1 | ' base angles of isosceles triangle *DST*'<br>but we do **not** require a reason for this mark |
| | Angle *DFS* = x angle in alternate segment<br>or<br>Angle *RSF* = x corresponding | M1 | either of these angles **with a correct reason** scores this mark<br><br>no reason or an incorrect reason is M0 |
| | Further evaluation of angles, **with correct reasons**, to arrive at a stage where ...<br><br>either ... it is possible to use the converse of a theorem<br><br>or ... which leads to the fact that *DTSF* is a parallelogram | M1dep | Here is a complete example ...<br><br>angle *DST* = x<br><br>angle *DSR* = 180 − x angles on a straight line<br><br>angle *RSF* = x corresponding<br><br>angle *FDS* = x *FDS* = *RSF*, angle in alternate segment |
| | A statement of the angles, or the values of the angles, that will complete the proof ... the angles must be clearly identified | M1dep | angle *DSR* + angle *FDS*<br>= 180 − x + x<br>= 180 |
| | A statement of the correct reason to accompany these angles, thus completing the proof | A1 | *FD* is parallel to *RST* because these angles add to 180 ... using the (converse) of the co-interior angles theorem |
| | **Additional Guidance** | | |
| | Some methods are much shorter than others.<br>Follow their reasoning to see if it is free from error. | | |
| | The 3rd mark is dependent on both of the previous two M marks.<br>So, if they have **not** scored M1 M1 they cannot score any more marks | | |
| | For the 3rd M mark ... when following their work, stop as soon as there is an error in the value of any angle or in the reason given for any particular angle.<br>The proof breaks down at that point and they can only score a maximum of 2 marks. | | |
| | The 4th mark (M1dep) and the 5th mark (A1) are closely related.<br>They can be thought of as a pair because the 4th mark is for identifying the two angles that make the completion of the proof possible and the 5th mark is for stating the correct reason.<br>Condone the fact that you may not see the word 'converse'. | | |
| | Any sign of them using the result (eg *DST* = *FDS*, alternate angles) means they have compromised the proof immediately and they can only score a maximum of 2 marks. | | |

| | | | |
|---|---|---|---|
| 19 | **Alternative method 1** | | |
| | sight of $2(x^2 - 8x \ldots\ldots\ldots)$ | M1 | |
| | sight of $2(x - 4)^2 \ldots\ldots\ldots$ | M1dep | |
| | $2[(x - 4)^2 - 16] + 13$<br>or<br>$2(x - 4)^2 - 32 + 13$<br>or<br>$2[(x - 4)^2 - 16 + 6.5]$ | M1dep | |
| | $2(x - 4)^2 - 19$ | A1 | or $a = 2$, $b = -4$, $c = -19$ |
| | **Alternative method 2** | | |
| | $a = 2$ | B1 | |
| | $-16 = 2ab$ or $-16 = 4b$<br>or $13 = ab^2 + c$ or $13 = 2b^2 + c$ | M1 | |
| | $-16 = 2ab$ **and** $13 = ab^2 + c$<br>or<br>$-16 = 4b$ **and** $13 = 2b^2 + c$ | M1dep | oe |
| | $2(x - 4)^2 - 19$ | A1 | or $a = 2$, $b = -4$, $c = -19$ |
| | | | |
| | **Additional Guidance** | | |
| | | | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 20 | **Alternative method 1** | | |
| | sin 45 = $\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$ or sin 60 = $\frac{\sqrt{3}}{2}$ | B1 | look for these in their working ... **one** of them correct will gain the mark at any stage of their working |
| | $\frac{x}{\sin 45} = \frac{6\sqrt{2}}{\sin 60}$ | M1 | oe |
| | (x =) $\frac{6\sqrt{2}\times\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2}}$ or $\frac{6\sqrt{2}\times\frac{\sqrt{2}}{2}}{\frac{\sqrt{3}}{2}}$ | M1dep | oe<br><br>a correct expression or equation coming from **both** surds correct<br><br>eg $\frac{\sqrt{3}}{2}x = 6\sqrt{2}\times\frac{1}{\sqrt{2}}$ or $\frac{6}{x} = \frac{\sqrt{3}}{2}$ |
| | (x =) $\frac{12}{\sqrt{3}}$ or $\sqrt{3}x = 12$ | A1 | |
| | (x =) $4\sqrt{3}$ | A1 | |
| | **Alternative method 2** | | |
| | sin 45 = $\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$ or sin 60 = $\frac{\sqrt{3}}{2}$ | B1 | look for these in their working ... **one** of them correct will gain the mark at any stage of their working |
| | $\frac{AD}{6\sqrt{2}}$ = sin 45 or (AD =) $6\sqrt{2}$ × sin 45 or (AD =) 6 | M1 | where D is the foot of the perpendicular from A to BC |
| | (x =) $\frac{6\sqrt{2}\times\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2}}$ or $\frac{6\sqrt{2}\times\frac{\sqrt{2}}{2}}{\frac{\sqrt{3}}{2}}$ | M1dep | oe<br><br>a correct expression or equation coming from **both** surds correct<br><br>eg $\frac{\sqrt{3}}{2}x = 6\sqrt{2}\times\frac{1}{\sqrt{2}}$ or $\frac{6}{x} = \frac{\sqrt{3}}{2}$ |
| | (x =) $\frac{12}{\sqrt{3}}$ or $\sqrt{3}x = 12$ | A1 | |
| | (x =) $4\sqrt{3}$ | A1 | |
| | **Additional Guidance** | | |
| | If one surd value is incorrect they can only score a maximum of 2 marks | | |