

**Mathematics**

Advanced GCE

Unit **4724**: Core Mathematics 4

**Mark Scheme for January 2011**

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- 1 (i) First two terms are  $1 - \frac{1}{2}x$ ..... B1
- Third term =  $\frac{\frac{1}{2} \cdot -\frac{1}{2}}{2} [(-x)^2 \text{ or } x^2 \text{ or } -x^2]$  M1
- =  $-\frac{1}{8}x^2$  A1 3  $-\frac{1}{8}x^2$  without work → M1 A1
- (ii) Attempt to replace  $x$  by  $2y - 4y^2$  or  $2y + 4y^2$  M1 or write as  $1 - (2y - 4y^2 \text{ or } 2y + 4y^2)$
- First two terms are  $1 - y$  B1
- Third term =  $+\frac{3}{2}y^2$  or  $\sqrt{(4b+2)}y^2$  A1√ 3 where  $b = cf(x^2)$  in part (i)

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- 2 (i)  $A(x-2) + B = 7 - 2x$  M1 or  $A(x-2)^2 + B(x-2) = (7-2x)(x-2)$
- $A = -2$  A1
- $B = 3$  A1 3
- (ii)  $\int \frac{A}{x-2} dx = \left( A \text{ or } \frac{1}{A} \right) \ln(x-2)$  B1 Accept  $\ln|x-2|, \ln|2-x|, \ln(2-x)$
- $\int \frac{B}{(x-2)^2} dx = -\left( B \text{ or } \frac{1}{B} \right) \cdot \frac{1}{x-2}$  B1 Negative sign is required
- Correct f.t. of A & B;  $A \ln(x-2) - \frac{B}{x-2}$  B1√ Still accept lns as before
- Using limits =  $-2 \ln 3 + 2 \ln 2 + \frac{1}{2}$  ISW B1 4 No indication of  $\ln(\text{negative})$

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- 3 (i) State/imply  $\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right)$  or  $\frac{d}{dx}(\cos x)^{-1}$  B1 Not just  $\sec x = \frac{1}{\cos x}$
- Attempt quotient rule or chain rule to power  $-1$  M1 Allow  $\frac{u dv - v du}{v^2}$  & wrong trig signs
- Obtain  $\frac{\sin x}{\cos^2 x}$  or  $-(\sin x)(\cos x)^{-2}$  A1 No inaccuracy allowed here
- Simplify with suff evid to **AG** e.g.  $\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$  A1 4 Or vice versa. Not just =  $\sec x \tan x$
- (ii) Use  $\cos 2x = +/-1 +/- 2 \cos^2 x$  or  $+/-1 +/- 2 \sin^2 x$  M1 or  $\pm(\cos^2 x - \sin^2 x)$
- Correct denominator =  $\sqrt{2 \cos^2 x}$  A1  $\sqrt{2 - 2 \sin^2 x}$  needs simplifying
- Evidence that  $\frac{\tan x}{\cos x} = \sec x \tan x$  or  $\int \frac{\tan x}{\cos x} dx = \sec x$  B1 irrespective of any const multiples
- $\frac{1}{\sqrt{2}} \sec x$  (+ c) A1 4 Condone  $\theta$  for  $x$  except final line

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- 4 (i) Attempt to use  $\frac{dy}{dx} \cdot \frac{dx}{dt}$  or  $\frac{dy}{dt} \cdot \frac{dt}{dx}$  M1 Not just quote formula
- $\frac{4}{2t}$  or  $\frac{2}{t}$  A1 2
- (ii) Subst  $t = 4$  into their (i), invert & change sign M1  
 Subst  $t = 4$  into  $(x,y)$  & use num grad for tgt/normal M1  
 $y = -2x + 52$  AEF CAO (no f.t.) A1 3 Only the eqn of normal accepted
- (iii) Attempt to eliminate  $t$  from the 2 given equations M1  
 $x = 2 + \frac{y^2}{16}$  or  $y^2 = 16(x-2)$  AEF ISW A1 2 Mark at earliest acceptable form.
- 7**
- 5 (i) Attempt to connect  $dx$  and  $du$  M1 Including  $\frac{du}{dx} =$  or  $du = \dots dx$  ; not  $dx = du$
- $5 - x = 4 - u^2$  B1 perhaps in conjunction with next line
- Show  $\int \frac{4-u^2}{2+u} \cdot 2u \, du$  reduced to  $\int 4u - 2u^2 \, du$  AG A1 In a fully satisfactory & acceptable manner
- Clear explanation of why limits change B1 e.g. when  $x = 2$ ,  $u = 1$  and when  $x = 5$ ,  $u = 2$
- $\frac{4}{3}$  B1 5 not dependent on any of first 4 marks
- (ii)(a)  $5 - x$  \*B1 1 Accept  $4 - x - 1 = 5 - x$  (this is not AG)
- (b) Show reduction to  $2 - \sqrt{x-1}$  dep\*B1
- $\int \sqrt{x-1} \, dx = \frac{2}{3}(x-1)^{\frac{3}{2}}$  B1 Indep of other marks, seen anywhere in (b)
- $\left(10 - \frac{2}{3} \cdot 8\right) - \left(4 - \frac{2}{3}\right) = \frac{4}{3}$  or  $4\frac{2}{3} - 3\frac{1}{3} = \frac{4}{3}$  B1 3 Working must be shown
- 9**
- 6 (i) Work with correct pair of direction vectors M1  
 Demonstrate correct method for finding scalar product M1 Of any two 3x3 vectors rel to question  
 Demonstrate correct method for finding modulus M1 Of any vector relevant to question  
 24, 24.0 (24.006363..) (degrees) 0.419 (0.41899..) (rad) A1 4 Mark earliest value, allow trunc/rounding
- (ii) Attempt to set up 3 equations M1 Of type  $3 + 2s = 5, 3s = 3 + t, -2 - 4s = 2 - 2t$   
 Find correct values of  $(s, t) = (1, 0)$  or  $(1, 4)$  or  $(5, 12)$  A1 Or 2 diff values of  $s$  (or of  $t$ )  
 Substitute their  $(s, t)$  into equation not used M1 and make a relevant deduction  
Correctly demonstrate failure A1 4 dep on all 3 prev marks
- (iii) Subst their  $(s, t)$  from first 2 eqns into new 3<sup>rd</sup> eqn M1 New 3<sup>rd</sup> eqn of type  $a - 4s = 2 - 2t$   
 $a = 6$  A1 2
- 10**

- 7 Attempt parts with  $u = x^2 + 5x + 7$ ,  $dv = \sin x$  M1 as far as  $f(x) + / - \int g(x) dx$
- 1<sup>st</sup> stage =  $-(x^2 + 5x + 7)\cos x + \int (2x + 5)\cos x dx$  A1 signs need not be amalgamated at this stage
- $\int (2x + 5)\cos x dx = (2x + 5)\sin x - \int 2 \sin x dx$  B1 indep of previous A1 being awarded
- =  $(2x + 5)\sin x + 2 \cos x$  B1
- $I = -(x^2 + 5x + 7)\cos x + (2x + 5)\sin x + 2 \cos x$  A1 WWW
- (Substitute  $x = \pi$ )  $-(\text{Substitute } x = 0)$  M1 An attempt at subst  $x = 0$  must be seen
- $\pi^2 + 5\pi + 10$  WWW AG A1 7
- 7**
- 8 (i)  $\frac{d}{dx}(y^2) = 2y \frac{dy}{dx}$  B1
- $\frac{d}{dx}(-5xy) = (-)(5)x \frac{dy}{dx} + (-)(5)y$  M1 i.e. reasonably clear use of product rule
- LHS completely correct  $4x - 5x \frac{dy}{dx} - 5y + 2y \frac{dy}{dx} (= 0)$  A1 Accept " $\frac{dy}{dx} =$ " provided it is not used
- Substitute  $\frac{dy}{dx} = \frac{3}{8}$  or solve for  $\frac{dy}{dx}$  & then equate to  $\frac{3}{8}$  M1 Accuracy not required for "solve for  $\frac{dy}{dx}$ "
- Produce  $x = 2y$  WWW AG (Converse acceptable) A1 5 Expect  $17x = 34y$  and/or  $\frac{dy}{dx} = \frac{5y - 4x}{2y - 5x}$
- (ii) Substitute  $2y$  for  $x$  or  $\frac{1}{2}x$  for  $y$  in curve equation M1
- Produce either  $x^2 = 36$  or  $y^2 = 9$  A1
- AEF of  $(\pm 6, \pm 3)$  A1 3 ISW Any correct format acceptable
- 8**
- 9 (i) Attempt to sep variables in the form  $\int \frac{P}{(x-8)^{1/3}} dx = \int q dt$  M1 Or invert as  $\frac{dt}{dx} = \frac{r}{(x-8)^{1/3}}$ ;  $p, q, r$  const
- $\int \frac{1}{(x-8)^{1/3}} dx = k(x-8)^{2/3}$  A1  $k$  const
- All correct (+ c) A1
- For equation containing 'c'; substitute  $t = 0$ ,  $x = 72$  M1 M2 for  $\int_{72}^{35} = \int_0^t$  or  $\int_{35}^{72} = \int_0^t$
- Correct corresponding value of  $c$  from correct eqn A1
- Subst their  $c$  &  $x = 35$  back into eqn M1
- $t = \frac{21}{8}$  or 2.63 / 2.625 [C.A.O] A1 7 A2:  $t = \frac{21}{8}$  or 2.63 / 2.625 WWW
- (ii) State/imply in some way that  $x = 8$  when flow stops B1
- Substitute  $x = 8$  back into equation containing numeric 'c' M1
- $t = 6$  A1 3

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