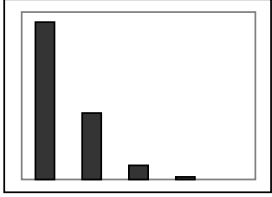



4733 Probability & Statistics 2

1	$U \sim B(800, 0.005) \approx \text{Po}(4)$ $P(U \leq 6)$ $\quad\quad\quad = \mathbf{0.8893}$ $n > 50/\text{large}, np < 5/p \text{ small}$	B1 M1 A1 B1	4 Po(np) stated or implied Tables or formula ± 1 term, e.g. 0.7851, 0.9489, 0.1107, <i>not</i> 1– Answer 0.889 or a.r.t. 0.8893 Both conditions
2	$\frac{23.625 - 23}{5/\sqrt{n}} = 2$ $\sqrt{n} = 16$ $\quad\quad\quad n = \mathbf{256}$	M1 A1 M1 A1	4 Standardise with \sqrt{n} , allow $\sqrt{2}$ errors Equate to 2 or a.r.t. 2.00, signs correct Solve for \sqrt{n} , needs Φ^{-1} , <i>not</i> from $1/n$ 256 only, allow from wrong signs
3 (i)	(a) $e^{-0.42}$ $\quad\quad\quad = \mathbf{0.657}$ (b) $0.42 e^{-0.42} = \mathbf{0.276}$	M1 A1 A1	3 Correct formula for $R = 0$ or 1 P(0), a.r.t. 0.657 P(1), a.r.t. 0.276
(ii)	Po(2.1): $1 - P(\leq 3) = 1 - 0.8386$ $\quad\quad\quad = \mathbf{0.1614}$	M1 M1 A1	3 Po(2.1) stated or implied Tables or formula, e.g. 0.8386 or 0.6496 or 0.9379 or complement; Answer, in range [0.161, 0.162]
(iii)		B2	2 At least 3 separate bars, all decreasing <i>Allow histogram. Allow convex</i> P(0) < P(1) but otherwise OK: B1 Curve: B1 <i>[no hint of normal allowed]</i>
4 (i)	$H_0 : p = 0.14$ $H_1 : p < 0.14$ $B(22, 0.14)$ $P(\leq 2) = .86^{22} + (22 \times .86^{21} \times .14) +$ $(231 \times .86^{20} \times .14^2) = \mathbf{0.3877}$ > 0.1 Do not reject H_0 . Insufficient evidence that company overestimates viewing proportion	B2 M1 A1 A1 B1 M1 A1	8 Both correct. 1 error, B1, but x or r or \bar{x} etc: 0 B(22, 0.14) stated or implied, e.g. N(3.08, 2.6488) or Po(3.08) Correct formula for 2 or 3 terms, <i>or</i> $P(\leq 0) = 0.036$ and CR Correct answer, a.r.t. 0.388, <i>or</i> CR is = 0 Explicitly compare 0.1 or CR with 2, OK from Po but <i>not</i> from N Correct comparison type and conclusion, needs binomial, at least 2 terms, <i>not</i> from $P(< 2)$ Contextualised, some acknowledgement of uncertainty [SR: Normal: B2 M1 A0 B0 M0] [SR: 2-tailed, or $p > 0.14$, $P(\geq 2)$: B1M1A2B0M1A1]
(ii)	Selected independently Each adult equally likely to be chosen	B1 B1	2 Independent <i>selection</i> Choice of sample elements equally likely (no credit if not focussed on selection) [Only “All samples of size n equally likely”: B1 only unless related to Binomial conditions]
5 (i)		B1 B1 B1	3 Horizontal straight line Symmetrical U-shaped curve Both correct, including relationship between the two and not extending beyond $[-2, 2]$, curve through (0,0)
(ii)	S is equally likely to take any value T is more likely at extremities	B2	2 Correct statement about both distributions, $\sqrt{\quad}$ on their graph [Correct for one only, or partial description: B1] <i>Not</i> “probability of S is constant”, etc.
(iii)	$\frac{5}{64} \int_{-2}^2 x^6 dx = \frac{5}{64} \left[\frac{x^7}{7} \right]_{-2}^2 = \left[\frac{20}{7} \right]$ $- 0^2$ $= \frac{20}{7}$	M1 A1 B1 A1	4 Integrate $x^2 g(x)$, limits $-2, 2$ Correct indefinite integral [= $5x^7/448$] 0 or 0^2 subtracted or $E(X) = 0$ seen, <i>not</i> $\int x^2 f(x) dx - \int x f(x) dx$ Answer $\frac{20}{7}$ or $2\frac{6}{7}$ or a.r.t. 2.86, don't need 0

6 (i)	$50.0 \pm 1.96 \sqrt{\frac{20.25}{81}} = 50.0 \pm 0.98$ $= 49.02, 50.98$ $\bar{W} < 49.02 \text{ and } \bar{W} > 50.98$	M1 B1 A1A1 A1√ 5	$50.0 \pm z\sqrt{(1.96/81)}$, allow one sign only, allow $\sqrt{\quad}$ errors $z = 1.96$ in equation (<i>not</i> just stated) Both critical values, min 4 SF at some stage (if both 3SF, A1) CR, allow \leq / \geq , don't need \bar{W} , $\sqrt{\quad}$ on their CVs, can't recover [Ans 50 ± 0.98 : A1 only] [SR: 1 tail, M1B0A0; 50.8225 or 49.1775: A1]
6 (ii)	$\frac{50.98 - 50.2}{0.5} = 1.56$ $\frac{49.02 - 50.2}{0.5} = -2.36$ $\Phi(1.56) - \Phi(-2.36) = \mathbf{0.9315}$	M1 A1 A1 M1 A1 5	Standardise one limit with same SD as in (i) A.r.t. 1.56, allow $-$ } Can allow $\sqrt{\quad}$ here A.r.t. -2.36, allow $+$ } if very unfair Correct handling of tails for Type II error Answer in range [0.931, 0.932] [SR 1-tail M1: -1.245 or 2.045 A1: 0.893 or 0.9795 A1]
6 (iii)	It would get smaller	B1 1	No reason needed, but withhold if definitely wrong reason seen. Allow from 1-tail
7 (i)	$\hat{\mu} = \bar{t} = 13.7$ $\frac{12657.28}{64} - 13.7^2 [=10.08]; \times \frac{64}{63}$ $= \mathbf{10.24}$ $H_0: \mu = 13.1, H_1: \mu > 13.1$ $\frac{13.7 - 13.1}{\sqrt{10.24/64}} = 1.5 \text{ or } p = 0.0668$ $1.5 < 1.645 \text{ or } 0.0668 > 0.05$ <p>Do not reject H_0. Insufficient evidence that time taken on average is greater than 13.1 min</p>	B1 M1 M1 A1 B2 M1 A1 B1 M1 A1 11	13.7 stated Correct formula for biased estimate $\times \frac{64}{63}$ used, or equivalent, can come in later Variance or SD 10.24 or 10.2 Both correct. [SR: One error, B1, but x or t or \bar{x} or \bar{t} , 0] Standardise, or find CV, with $\sqrt{64}$ or 64 $z =$ a.r.t. 1.50, or $p = 0.0668$, or CV 13.758 [$\sqrt{\quad}$ on z] Compare z & 1.645, or p & 0.05 (must be correct tail), or $z = 1.645$ & 13 with CV Correct comparison & conclusion, needs 64, <i>not</i> $\mu = 13.7$ Contextualised, some acknowledgement of uncertainty [13.1 – 13.7: (6), M1 A0 B1 M0]
6 (ii)	Yes, not told that dist is normal	B1 1	Equivalent statement, <i>not</i> “ n is large”, don't need “yes”
8 (i)	$N(14.7, 4.41)$ Valid because $np = 14.7 > 5; nq = 6.3 > 5$ $1 - \Phi\left(\frac{15.5 - 14.7}{\sqrt{4.41}}\right) = 1 - \Phi(0.381)$ $= 1 - 0.6484$ $= \mathbf{0.3516}$	M1 A1 B1 B1 M1 A1 A1 7	Normal, attempt at np Both parameters correct Check $np > 5$; } If both asserted but not both nq or $npq > 5$ } 14.7 and 6.3 seen: B1 only [Allow “ n large, p close to $\frac{1}{2}$ ”] Standardise, answer < 0.5 , no \sqrt{n} z , a.r.t. 0.381 Answer in range [0.351, 0.352] [Exact: M0]
8 (ii)	$\bar{K} \sim N(14.7, 4.41/36)$ $[= N(14.7, 0.35^2)]$ Valid by Central Limit Theorem as 36 is large $\Phi\left(\frac{14.0 + \frac{1}{2} - 14.7}{\sqrt{4.41/36}}\right) = \Phi(-1.96)$ $= \mathbf{0.025}$	M1 A1√ B1 M1 A1 A1 A1 7	Normal, their np from (i) Their variance/36 Refer to CLT or large n ($= 36$, <i>not</i> 21), or “ $K \sim N$ so $\bar{K} \sim N$ ”, <i>not</i> same as (i), <i>not</i> $np > 5$, $nq > 5$ for \bar{K} Standardise 14.0 with 36 or $\sqrt{36}$ cc included, allow 0.5 here, e.g. 14.5 – 14.7 $z = -1.96$ or -2.00 or -2.04 , allow $+$ if answer < 0.5 0.025 or 0.0228 [0.284 loses last 2] [Po(25.2) etc: probably 0]
OR:	$B(756, 0.7) \approx N(529.2, 158.76)$ $\Phi\left(\frac{504.5 - 529.2}{\sqrt{158.76}}\right) = \Phi(-1.96)$ $= \mathbf{0.025}$	M1M1A1 B1 M1 A1 A1	$\times 36$; $N(529.6, \dots)$; 158.76 CLT as above, or $np > 5$, $nq > 5$, can be asserted here Standardise 14×36 cc correct and \sqrt{npq} 0.025 or 0.0228