OCR
RECOGNISING ACHIEVEMENT

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**Advanced Subsidiary General Certificate of Education**
**Advanced General Certificate of Education**

# MATHEMATICS 4723

Core Mathematics 3

Wednesday **18 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

**TIME** 1 hour 30 minutes

## INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

**This question paper consists of 4 printed pages.**



[Turn over

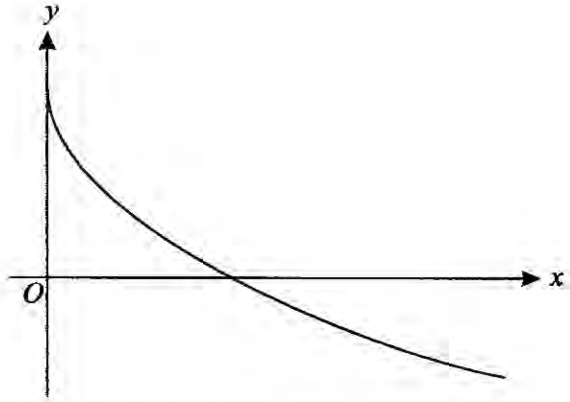
**1** Show that $\displaystyle\int_2^8 \frac{3}{x}\,dx = \ln 64$. [4]

**2** Solve, for $0° < \theta < 360°$, the equation $\sec^2\theta = 4\tan\theta - 2$. [5]

**3** **(a)** Differentiate $x^2(x+1)^6$ with respect to $x$. [3]

**(b)** Find the gradient of the curve $y = \dfrac{x^2+3}{x^2-3}$ at the point where $x = 1$. [3]

**4**



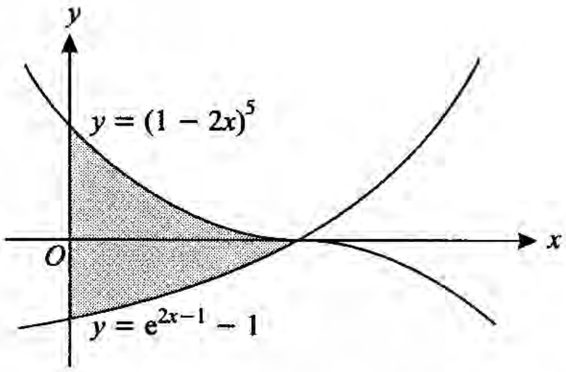

The function f is defined by $f(x) = 2 - \sqrt{x}$ for $x \geqslant 0$. The graph of $y = f(x)$ is shown above.

**(i)** State the range of f. [1]

**(ii)** Find the value of $ff(4)$. [2]

**(iii)** Given that the equation $|f(x)| = k$ has two distinct roots, determine the possible values of the constant $k$. [2]

**5**




The diagram shows the curves $y = (1-2x)^5$ and $y = e^{2x-1} - 1$. The curves meet at the point $(\frac{1}{2}, 0)$. Find the exact area of the region (shaded in the diagram) bounded by the $y$-axis and by part of each curve. [8]

**6** **(a)**

| $t$ | 0 | 10 | 20 |
|---|---|---|---|
| $X$ | 275 | 440 | |

The quantity $X$ is increasing exponentially with respect to time $t$. The table above shows values of $X$ for different values of $t$. Find the value of $X$ when $t = 20$. [3]

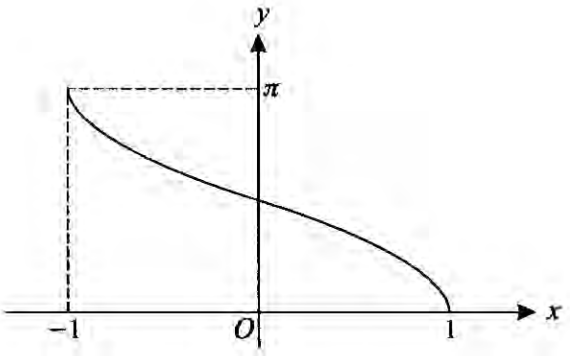
**(b)** The quantity $Y$ is decreasing exponentially with respect to time $t$ where

$$Y = 80e^{-0.02t}.$$

**(i)** Find the value of $t$ for which $Y = 20$, giving your answer correct to 2 significant figures. [3]

**(ii)** Find by differentiation the rate at which $Y$ is decreasing when $t = 30$, giving your answer correct to 2 significant figures. [3]

**7**




The diagram shows the curve with equation $y = \cos^{-1} x$.

**(i)** Sketch the curve with equation $y = 3\cos^{-1}(x - 1)$, showing the coordinates of the points where the curve meets the axes. [3]

**(ii)** By drawing an appropriate straight line on your sketch in part **(i)**, show that the equation $3\cos^{-1}(x - 1) = x$ has exactly one root. [1]

**(iii)** Show by calculation that the root of the equation $3\cos^{-1}(x - 1) = x$ lies between 1.8 and 1.9. [2]

**(iv)** The sequence defined by

$$x_1 = 2, \qquad x_{n+1} = 1 + \cos\left(\tfrac{1}{3}x_n\right)$$

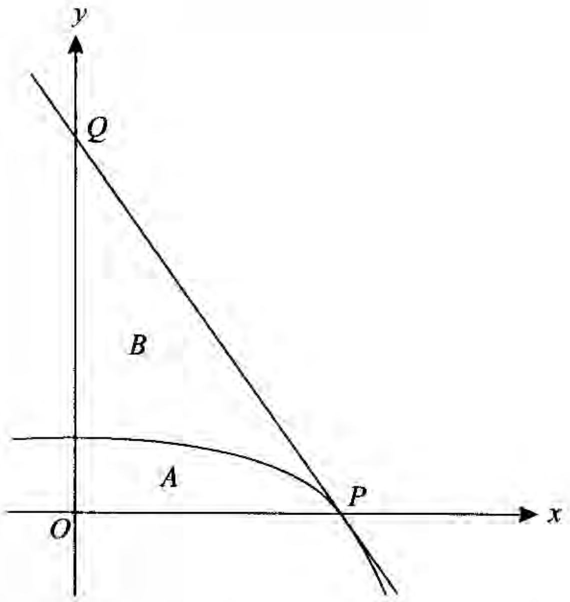
converges to a number $\alpha$. Find the value of $\alpha$ correct to 2 decimal places and explain why $\alpha$ is the root of the equation $3\cos^{-1}(x - 1) = x$. [5]

**[Questions 8 and 9 are printed overleaf.]**



[Turn over

**8**




The diagram shows part of the curve $y = \ln(5 - x^2)$ which meets the $x$-axis at the point $P$ with coordinates $(2, 0)$. The tangent to the curve at $P$ meets the $y$-axis at the point $Q$. The region $A$ is bounded by the curve and the lines $x = 0$ and $y = 0$. The region $B$ is bounded by the curve and the lines $PQ$ and $x = 0$.

**(i)** Find the equation of the tangent to the curve at $P$. [5]

**(ii)** Use Simpson's Rule with four strips to find an approximation to the area of the region $A$, giving your answer correct to 3 significant figures. [4]

**(iii)** Deduce an approximation to the area of the region $B$. [2]

**9** **(i)** By first writing $\sin 3\theta$ as $\sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta. \quad [4]$$

**(ii)** Determine the greatest possible value of

$$9\sin\left(\tfrac{10}{3}\alpha\right) - 12\sin^3\left(\tfrac{10}{3}\alpha\right),$$

and find the smallest positive value of $\alpha$ (in degrees) for which that greatest value occurs. [3]

**(iii)** Solve, for $0° < \beta < 90°$, the equation $3\sin 6\beta \operatorname{cosec} 2\beta = 4$. [6]