GCE

Mathematics A

H240/03: Pure Mathematics and Mechanics

Advanced GCE

Mark Scheme for Autumn 2021

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Text Instructions

1. Annotations and abbreviations

Annotation in RM assessor	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	**Meaning**
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value **is given** in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value **is not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads “2 s.f”.

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

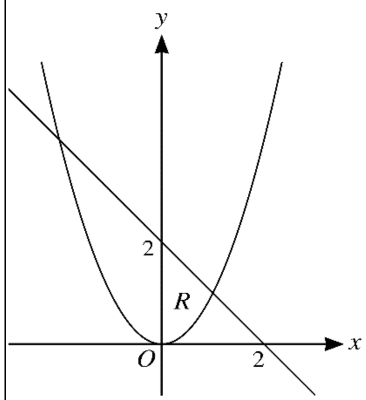
g Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

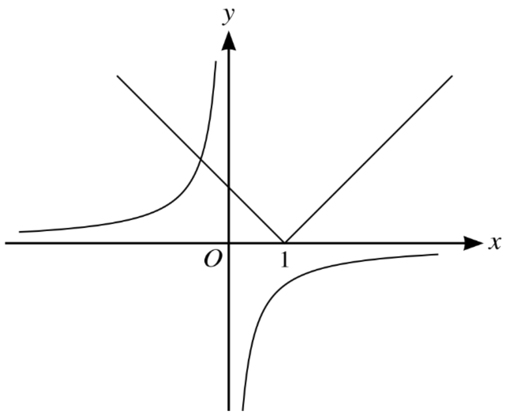
i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold “In this question you must show detailed reasoning”, or the command words “Show” or “Determine”. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1				B1	1.1	$y = x^2$ drawn correctly – must pass through the origin and (roughly) symmetrically about the *y*-axis	
				B1	1.1	$x + y = 2$ drawn correctly – must have positive *y*-intercept and negative gradient	Line need not intersect the *x*-axis and intercepts need not be labelled
				B1	2.2a	Correct identification of region (dependent on both previous **B** marks) – ignore shading if *R* correctly labelled	
				[3]			Condone dashed line/curve for full marks

Question			Answer	Marks	AO	Guidance	
2	(a)		$(p^2 =)(4+h)^2+(4-h)^2-2(4+h)(4-h)\cos 60$ $=(16+8h+h^2)+(16-8h+h^2)-(16-h^2)$	M1	1.1	Correct application of cosine rule	
			$p^2=16+3h^2$	A1	2.2a	**AG –** at least one line of intermediate working (must have $p^2 = \ldots$)	Any errors or missing brackets then **A0**
				[2]			
2	(b)		$(16+3h^2)^{\frac{1}{2}}=4(1+\ldots)^{\frac{1}{2}}$	B1	1.1	For reference: $4\left(1+\frac{3}{16}h^2\right)^{\frac{1}{2}}$	or for $16^{\frac{1}{2}}(1+\ldots)^{\frac{1}{2}}$
			$(1+kh^2)^{\frac{1}{2}}=1+\frac{1}{2}kh^2+\ldots$	M1	1.1	Correct first two terms for their *k*	$k\neq 1$
			$\ldots+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(kh^2)^2$	A1ft	1.1	Correct third term following through their *k*	
			$(p=)4+\frac{3}{8}h^2-\frac{9}{512}h^4+\ldots$	A1	1.1	$\lambda=\frac{3}{8},\mu=-\frac{9}{512}$ (oe)	**SC** if candidates assume that $p=4+\lambda h^2+\mu h^4$ and then substitute into $p^2=16+3h^2$ to find λ and μ then **B1** for correct λ and **B1** for correct μ (so 2/4 max.)
				[4]			

Question			Answer	Marks	AO	Guidance	
3			$2+2d=2r$	**B1**	1.1	Or for $a+2d=ar$	
			$2+12d=2r^2$	**B1**	1.1	Or for $a+12d=ar^2$	
			$1+6d=(1+d)^2$ or $2+12d=2(1+d)^2$	**M1***	1.1	Setting up an equation in *d* or *r* only – dependent on one **B** mark	$2+12(r-1)=2r^2$
			$d^2-4d=0 \Rightarrow d=...$	**M1dep***	1.1	Solving their two-term quadratic equation in *d* (or three-term quadratic in *r*)	$r^2-6r+5=0$ $(r-5)(r-1)=0$ $\Rightarrow r=...$
			$d=4$ and as the common difference is positive the progression is an increasing sequence	**A1**	2.4	Correct value for *d* and link to increasing sequence – must either say that *d* is positive (oe) or state at least the correct first four terms and comment that they are increasing	Condone no mention of $d \neq 0$
				[5]			
			Alternative method				
			$\left(\frac{u_3}{u_2}=\right)\frac{2+12d}{2+2d}$	**B1**		or for $\frac{u_3}{u_1}$	
			$\left(\frac{u_2}{u_1}=\right)\frac{2+2d}{2}$	**B1**			
			$\frac{2+12d}{2+2d}=\frac{2+2d}{2}$	**M1***		Setting up an equation in *d* only – dependent on one **B** mark	
			$d^2-4d=0 \Rightarrow d=...$	**M1dep***		Solving their two-term quadratic equation in *d*	
			$d=4$ and as the common difference is positive the progression is an increasing sequence	**A1**		As above	

Question			Answer	Marks	AO	Guidance	
4	(a)			B1 B1 [2]	1.1 1.1	$y=\|x-1\|$ drawn correctly – must touch (but not intersect) the positive *x*-axis $y=kx^{-1}$ drawn correctly – must not intersect axes	Intercepts with axes need not be labelled
4	(b)		The graphs in **(a)** intersect at only point (for any negative values of *k*) and therefore $\|x-1\|=\frac{k}{x}\Rightarrow x\|x-1\|=k$ has exactly one real root	B1 [1]	2.4	Dependent on both marks in **(a)** – must mention that the solution of the equation $x\|x-1\|=k$ corresponds to where the two graphs in **(a)** intersect (so just stating that the graphs in **(a)** intersect at only one point is **B0**)	
4	(c)		$x\|x-1\|=-6\Rightarrow x(1-x)=-6$ $x^2-x-6=0$ $x=-2$	M1 A1 [2]	3.1a 2.2a	Uses graph and sets up quadratic (oe) – allow if $x^2-x+6=0$ stated as well (but **M0** if this is the only quadratic (oe) considered) **BC** $x=-2$ only (www) **SC** If no marks awarded, then **B1** for $x=-2$ only and then **B1** for explicitly showing that $-2\|-2-1\|=-2(3)=-6$	Or setting up a four-term quartic from $(x-1)^2=36x^{-2}$

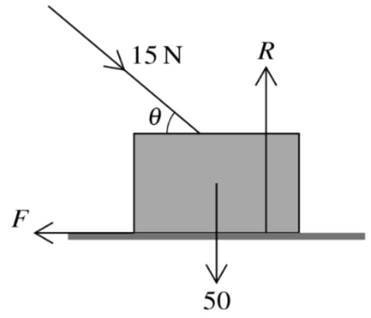
Question			Answer	Marks	AO	Guidance	
5	(a)		$R = 13$	B1	1.1	**B0** for $\sqrt{169}$ or for ± 13	
			$\left.\begin{matrix} R\cos\alpha = 12 \\ R\sin\alpha = 5 \end{matrix}\right\} \tan\alpha = \frac{5}{12}$	M1	1.1	**M1** for $\tan\alpha = k$ where $k = \pm\frac{5}{12}, \pm\frac{12}{5}$	or for $\cos\alpha = \pm\frac{12}{R}$, $\sin\alpha = \pm\frac{5}{R}$
			$\alpha = 0.3948$	A1	1.1	**A0** if in degrees (must be stated to exactly **4** significant figures at some point)	For reference: 0.3947911197…
				[3]			
5	(b)		$13\cos(t - 0.3948) = \pm 3 \Rightarrow t = 0.3948 + \arccos\left(\frac{\pm 3}{13}\right)$	M1	1.1	Sets $R\cos(t - \alpha)$ (with their *R* and α) equal to either 3 or -3, and attempt to solve (with correct order of operations)	**M1** only if in degrees
			1.73	A1	1.1	awrt 1.73	1.7327192…
			2.20	A1	1.1	awrt 2.20 (condone 2.2) – ignore other values that are greater than 2.20	2.1984556…
				[3]			

Question			Answer	Marks	AO	Guidance	
6	(a)		Considers both $f(0.5)$ and $f(0.6)$ where $f(x) = \pm\{6\arcsin(2x-1) - x^2\}$	M1	1.1	With at least one correct value – values should be given to at least 2 sf (rot)	Allow degrees for **M1** only: $f(0.6) = 68.8617...$
			$f(0.5) = -0.25 < 0,\ f(0.6) = 0.8481... > 0$ change of sign indicates that the root lies between 0.5 and 0.6	A1	2.4	Correct values together with explanation in words (change of sign) and conclusion	
				[2]			
6	(b)		$6\arcsin(2x-1) - x^2 = 0 \Rightarrow \arcsin(2x-1) = \frac{1}{6}x^2$				
			So $2x - 1 = \sin\left(\frac{1}{6}x^2\right)$	M1	1.1	Correct order of operations to get $2x-1=\sin\left(kx^2\right)$	$k \neq 0$
			$x = \frac{1}{2} + \frac{1}{2}\sin\left(\frac{1}{6}x^2\right)$	A1	2.2a	$p = \frac{1}{2}$, $q = \frac{1}{2}$ and $r = \frac{1}{6}$ (oe)	
				[2]			
6	(c)		$(x_0 = 0.5)$ $(x_1 =)0.5208273057...$ $(x_2 =)0.5225973903...$ $(x_3 =)0.5227511445....$ $(x_4 =)0.5227645245...$	M1	1.1	Uses their iterative formula with correct starting value to produce terms up to at least x_2 to at least 4 significant figures	Allow degrees for **M1** only: For reference: $x_1 = 0.5003636...$ $x_2 = 0.5003641...$ $x_3 = 0.5003641...$
			0.5228	A1	1.1	Must be stated to exactly **4** significant figures	
				[2]			

Question			Answer	Marks	AO	Guidance	
7	(a)		$\frac{dy}{dx} = 3\left(\frac{y-2}{x-3}\right)$	M1 A1 [2]	2.5 1.1	Correct left-hand side (including equals sign) and right-hand side must be of the form $\frac{f(y)}{g(x)}$ or $\frac{g(x)}{f(y)}$ cao (oe)	
7	(b)		$\int \frac{1}{y-2}\,dy = 3\int \frac{1}{x-3}\,dx$ $\ln(y-2) = 3\ln(x-3)(+c)$ $(4,3) \Rightarrow c = 0,\ y-2 = (x-3)^3$ $y = (x-3)^3 + 2$	M1* A1ft M1dep* A1 [4]	1.1a 1.1 1.1 2.2a	Separation of variables – dependent on the **M** mark in **(a)** Follow through their differential eq. from **(a)** Attempt to find *c* and eliminate logs oe (but must be of the form $y = f(x)$)	With an indication of integration Condone no constant
7	(c)		Translation $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$	B1 B1ft [2]	1.1 1.1	**B0** if another type of transformation stated or if shift/move etc. used Follow through their $y = (x-p)^3 + q$ - **B0** if this transformation is given as a stretch/rotation/reflection/enlargement etc. (but condone no transformation stated or shift/move etc.) – need not be given as a vector	$p, q \neq 0$

Question			Answer	Marks	AO	Guidance	
8	(a)			M1*	2.1	Differentiates y with respect to x – answer of the form $\pm e^{-2x} \pm \lambda x e^{-2x}$	$\lambda \neq 0$
			$y' = e^{-2x}(1-2x)$	A1	1.1		
			$y'' = e^{-2x}(-4+4x)$	A1ft	1.1	Follow through their first derivative	
				M1dep*	3.1a	Solves $y''=0$ (or attempts to verify $y''=0$ by substituting $x = 1$) **or** considers sign change either side of y''	
			$y''=0$ at $x = 1$ and $y''(0.5) = -2e^{-1} < 0$, $y''(1.5) = 2e^{-3} > 0$ (so change of sign indicates a point of inflection at $x = 1$)	A1 [5]	2.2a	Conclusion not required for this mark	
8	(b)		$\int xe^{-2x}dx = -\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}dx$	M1*	2.1	Integration by parts – of the form $\pm\alpha xe^{-2x} \pm \beta\int e^{-2x}dx$	Where $\alpha, \beta = 2, \frac{1}{2}$
			$\int xe^{-2x}dx = -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$	A1	1.1		
			$\int_0^1 xe^{-2x}dx = \left[-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}\right]_0^1 = \left(-\frac{1}{2}e^{-2} - \frac{1}{4}e^{-2}\right) - \left(0 - \frac{1}{4}\right)$	M1dep*	1.1	Use of correct limits in their fully integrated expression – need not be simplified (or equivalent)	
			$\frac{1}{4} - \frac{3}{4}e^{-2}$	A1	1.1	Allow unsimplified	
			Area of triangle below $OP = \frac{1}{2}e^{-2}$	B1	1.1	Or by correctly evaluating $\int_0^1 e^{-2}x\,dx$	Allow unsimplified
			$= \frac{1}{4}(1-5e^{-2})$	A1 [6]	2.2a	$a = 1$, $b = -5$ (must be in this form)	

Question			Answer	Marks	AO	Guidance	
9	(a)		$18 = 14 + 20a \Rightarrow a = 0.2 \left(\text{m s}^{-2}\right)$	**B1** **[1]**	1.1		
9	(b)		$14^2 = u^2 + 2(0.2)(330)$ $u = 8\left(\text{m s}^{-1}\right)$	**M1** **A1** **[2]**	3.3 1.1	Use of SUVAT equation to determine *u* with their *a* or other complete method to find *u*	**M0** if using 18 for *v*

Question			Answer	Marks	AO	Guidance	
10	(a)			B1 [1]	1.2	All three forces added correctly – need not be labelled	Extra forces is **B0**
10	(b)		$F = 15\cos\theta$	B1	3.3	Not for just seeing the expression $15\cos\theta$	
				M1	3.3	**M1** for resolving vertically – allow sign errors and sin/cos confusion (three terms)	Condone inclusion of g with the 50 for the **M** mark only
			$R = 15\sin\theta + 50$	A1	1.1		
			$15\cos\theta \le \frac{1}{5}(15\sin\theta + 50)$	M1	3.4	Use of $F \le \frac{1}{5}R$ with their F and $R \ne 50$	Allow $F = \frac{1}{5}R$ for the **M** mark only
			$15\cos\theta - 3\sin\theta \le 10$	A1 [5]	2.2a	**AG**	

Question			Answer	Marks	AO	Guidance	
11	(a)			M1	3.3	Use of $s = ut + \frac{1}{2}at^2$ with $a = \pm g$ and $s = \pm 4$	Allow sin/cos confusion
			$-4 = (25\sin 15)t - \frac{1}{2}(10)t^2$	A1	1.1		
			$t = 1.75$ (s)	A1	2.2a	**BC** (1.750981765…) 1.75 only	For reference: 1.779296952… (if g = 9.8 used)
				[3]			**Penalise g = 9.8 only once in the question**
			Alternative method				
			$0 = (25\sin 15)^2 + 2(-10)s_1$ and $0 = 25\sin 15 + (-10)t_1$	M1*		Finding the maximum height $s_1 (= 2.093353...)$ above A **and** corresponding time $t_1 (= 0.647047...)$	Using v = 0 and $a = \pm 10$
			$4 + s_1 = \frac{1}{2}\ 10\ t_2^2$ and $t = t_1 + t_2$	M1dep*		Complete correct method to find t	Using u = 0 and where $t_2\ (= 1.1039341...)$ is the time from the maximum height to the ground
			$t = 0.6470476... + 1.1039341... = 1.75$ (s)	A1			
11	(b)		$(25\cos 15)t$	M1	3.4	Use of $s = ut$ with their t from **(a)**	Allow sin/cos confusion
			42.3 (m)	A1FT	1.1	42.2829627… - ft their positive value of t from **(a)** but must be using $(25\cos 15)t$	For reference: 42.96672196… (if $g = 9.8$ used)
				[2]			

Question			Answer	Marks	AO	Guidance	
11	(c)		$v_h = 25\cos 15$	B1	1.2	Correct expression for horizontal velocity component (soi)	24.14814…
			$v_v = 25\sin 15 - 10(1.5)$	B1	3.3	Correct expression for vertical velocity component at $t = 1.5$ (condone positive value)	− 8.529523...
			$\tan\theta = \frac{v_v}{v_h}$	M1	3.1b	Use of tan to find angle (allow reciprocal) – dependent on one B mark earned	**M0** if using expressions for displacements
			19.5° below the horizontal	A1 [4]	3.2a	oe (e.g., 70.5° to the downward vertical)	For reference: 18.8° (if g = 9.8 used)
11	(d)		e.g., a less accurate value of *g* was used e.g., no consideration of the wind e.g., no consideration of (back)spin on the ball (but not topspin)	B1 [1]	3.5a	Any valid reason (do not accept mention of resistance e.g., air/wind <u>resistance</u>)	

Question			Answer	Marks	AO	Guidance	
12	(a)			M1*	3.3	Moments (with correct number of terms) about *A*, *C*, *D*, *B* or com	$\bar{x}$ is the centre of mass of the rod from *A* Allow *g* to be absent
			$0.5T_C + (4-0.7)T_D = 20g\bar{x}$ (moments about *A*) $(\bar{x}-0.5)(20g) = (4-1.2)T_D$ (moments about *C*) $(4-1.2)T_C = 20g(4-0.7-\bar{x})$ (moments about *D*) $0.7T_D + (4-0.5)T_C = 20g(4-\bar{x})$ (moments about *B*) $(\bar{x}-0.5)T_C = (4-0.7-\bar{x})T_D$ (moments about com)	A1FT	1.1	Follow though their T_C and T_D (allow just *T*) and allow *W* for 20*g*	
			$T_C = 3T_D$ **or** $T_C = 3T$ and $T_D = T$	B1	3.3	Correct relationship(s) for the tensions at *C* and *D* (soi)	
			$T_C = 147 (= 15g)$ and/or $T_D = 49 (= 5g)$ (from $T_C + T_D = 20g$) used in relevant moment equation(s)	M1dep*	1.1	Equation in $\bar{x}$ only, e.g., • W = 4T and any one mom. eq. • 4T = 20g and any one mom. eq. • W = 20g and any two mom. eq.	**M0** if tension used are the same in both ropes
			$\bar{x} = 1.2$ (m)	A1 **[5]**	2.2a	Must be 1.2 as question asks for com from *A*	Alternative: **B1** as main scheme (soi), then **M1** for splitting 2.8 in the ratio 1: 3 or 3:1 then **A1** for correct 1:3 then **B2** for 0.5 + 0.25(2.8) = 1.2(m)
12	(b)		$0.7mg = 20g(4-\bar{x}-0.7)$ (moments about *D*) $20g(\bar{x}-0.5)+3.5mg = (4-0.5-0.7)(20g+mg)$ (moments about *C*)	M1	3.1b	Moments about *D* (oe) – correct number of terms – if taking moments about another point then must set $T_C = 0$ and $T_D = 20g + mg$	Allow *g* to be absent
			$m = 60$	A1 **[2]**	2.2a		

Question			Answer	Marks	AO	Guidance	
13	(a)		$\mathbf{F} = (4t-8)\mathbf{i} + 6\left((2t-1)^2 - 9\right)\mathbf{j}$	M1	3.1b	Combines given forces and considers either component equal to zero – allow **M1** for either $4t-8=0$ **or** for $6(2t-1)^2 - 54 = 0$	
			When $t = 2$, the forces are in equilibrium	A1	1.1	$t = 2$ only – need only consider **i** or **j** component but any contradictory working/answers scores **A0**	
				[2]			
13	(b)		$m = 2 \Rightarrow \mathbf{a} = (2t-4)\mathbf{i} + 3\left((2t-1)^2 - 9\right)\mathbf{j}$	B1	3.3	Using **F** = 2**a** correctly	Allow 2**a = …**
				M1*	3.1b	Attempt to integrate **a** (or **F**) wrt t – two of their terms integrated correctly	**M0** if only considering one force or one component for **a** or **F**
			$\mathbf{v} = (t^2 - 4t)\mathbf{i} + 3\left(\frac{1}{6}(2t-1)^3 - 9t\right)\mathbf{j}(+\mathbf{c})$	A1	1.1	Condone no +**c** for this mark $\mathbf{v} = (t^2 - 4t)\mathbf{i} + (4t^3 - 6t^2 - 24t)\mathbf{j}$ (oe)	Allow 2**v** =…
			$t = 0, \mathbf{v} = \mathbf{0} \Rightarrow \mathbf{c} = \frac{1}{2}\mathbf{j}$	M1dep*	3.4	Uses correct initial conditions to find **c** (or **c** = **0** if expanded version used)	
			Moving parallel to $\mathbf{j} \Rightarrow \mathbf{i} = \mathbf{0}$ therefore $t(t-4) = 0$	M1	3.1a	Sets **i**-component of **v** equal to 0 to obtain a quadratic equation in t	Dependent on first **M** mark
			$t = 4 \Rightarrow \lvert\mathbf{v}\rvert = 64$ (ms^{-1})	A1	2.2a	Must have found +**c** for this mark	
				[6]			

Question			Answer	Marks	AO	Guidance	
13	(c)			M1*	1.1	Attempt to integrate **v** wrt *t* – two of their terms integrated correctly – dependent on first M mark in **(b)**	no vector constant of integration required in **(c)**
			$\mathbf{r}=\left(\frac{1}{3}t^3-2t^2\right)\mathbf{i}+3\left(\frac{1}{48}(2t-1)^4-\frac{9}{2}t^2+\frac{1}{6}t\right)\mathbf{j}$	A1	1.1	$\mathbf{r}=(\frac{1}{3}t^3-2t^2)\mathbf{i}+(t^4-2t^3-12t^2)\mathbf{j}$	
			$t=0\Rightarrow\mathbf{r}=\frac{1}{16}\mathbf{j}$, $t=3\Rightarrow\mathbf{r}=-9\mathbf{i}-\frac{1295}{16}\mathbf{j}$	M1dep*	1.1	Attempt to find **r** at both *t* = 0 and *t* = 3	
			Dist. = $\sqrt{(-9)^2+\left(-\frac{1295}{16}-\frac{1}{16}\right)^2}$	M1	1.1	Correct expression for the distance between given times (dependent on both previous **M** marks)	
			81.5 (m)	A1	2.2a	(For reference: 81.49846624…)	$\sqrt{6642}=9\sqrt{82}$
				[5]			

Question			Answer	Marks	AO	Guidance	
14	(a)		$T_{AB} - 2g\sin 30 = 2a$	M1*	3.3	N2L parallel to plane for *A* – correct number of terms, allow cos/sin confusion	Dimensionally consistent equations for **M** marks
			$4g\sin 60 - T_{BC} = 4a$	M1*	3.3	N2L parallel to plane for *C* – correct number of terms, allow cos/sin confusion	**M1M0M0** if *T* used in both equations
			$T_{BC} - T_{AB} - F_B = 3a$	M1*	3.3	N2L parallel to plane for *B*	
			$4g\sin 60 - 4a - 2g\sin 30 - 2a - F_B = 3a$	M1dep*	2.1	Eliminates both tensions	Allow in terms of F_B
			$9a = g(4\sin 60 - 2\sin 30 - 3\mu)$	A1	3.3	Use of $F_B = \mu(3g)$ to get a correct equation in *a* and μ	$9a = g(2\sqrt{3} - 1 - 3\mu)$
			$(\mu =)\frac{1}{3}\left(2\sqrt{3} - 1 - 9\frac{a}{g}\right) > 0$	M1	3.1b	Explicitly uses $\mu > 0$ to get a strict inequality in *a* and *g* only. If $a = \frac{1}{9}g(2\sqrt{3} - 1 - 3\mu)$ $\Rightarrow a < \frac{1}{9}g(2\sqrt{3} - 1)$ without justification is **M0**	**Dependent on all previous M marks**
			$a < \frac{1}{9}g(2\sqrt{3} - 1)$	A1 [7]	2.2a	**AG –** must follow from a correct equation involving μ, a and g	**SC** considering whole system with no friction **B2** only for deriving $9a = g(2\sqrt{3} - 1)$
14	(b)		$a = \frac{1}{9}g \Rightarrow \mu = \frac{2}{3}(\sqrt{3} - 1)$	B1	1.1	Correct value of μ (oe) using given *a* (soi)	$\mu = 0.488033...$
			$F_B = 2(\sqrt{3} - 1)g$	B1	3.4	Correct value for F_B	$F_B = 14.34819...$
			$\sqrt{(3g)^2 + (2(\sqrt{3} - 1)g)^2}$	M1	3.1a	$\sqrt{(3g)^2 + F_B^2}$ allow any value for F_B or even just the expression F_B	
			32.7 (N)	A1 [4]	2.2a	(For reference: 32.71438099…)	

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