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Mark Scheme (Final)
Summer 2007

GCE

GCE Mathematics (6684/01)

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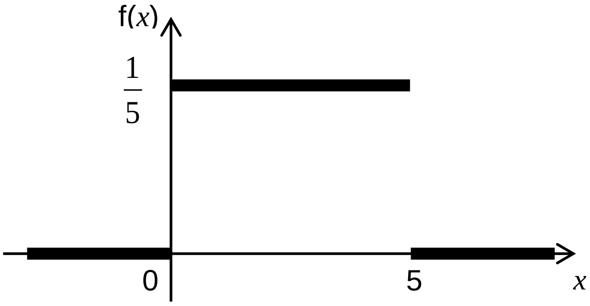
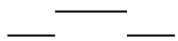
June 2007
6684 Statistics S2
Mark Scheme

Question Number	Scheme	Marks
1(a)	<u>Continuous</u> <u>uniform</u> distribution *or* <u>rectangular</u> distribution.	B1
	 1/5,(0),5 — 0 may be implied by start at y axis	B1
		B1
		(3)
(b)	$E(X) = 2.5$ ft from their a and b, must be a number	B1ft
	$\text{Var}(X) = \frac{1}{12}(5-0)^2$ or attempt to use $\int_0^5 f(x)x^2 dx - \mu^2$ use their f(x)	M1
	$= \frac{25}{12}$ or 2.08 o.e awrt 2.08	A1
		(3)
(c)	$P(X > 3) = \frac{2}{5} = 0.4$ 2 times their 1/5 from diagram	B1ft
		(1)
(d)	$P(X = 3) = 0$	B1
		(1)
		(Total 8)

Question Number	Scheme	Marks
2	One tail test Method 1 $H_0 : \lambda = 5$ ($\lambda = 2.5$) may use λ or μ $H_1 : \lambda > 5$ ($\lambda > 2.5$) $X \sim Po(2.5)$ may be implied $P(X \geq 7) = 1 - P(X \leq 6)$ $= 1 - 0.9858$ $= 0.0142$ awrt 0.0142 $0.0142 < 0.05$ att $P(X \geq 7)$ | $P(X \geq 6)$: [$P(X \geq 5) = 1 - 0.8912 = 0.1088$] $P(X \geq 6) = 1 - 0.9580 = 0.0420$ CR $X \geq 6$ $7 \geq 6$ or 7 is in critical region or 7 is significant (Reject H_0.) There is significant evidence at the 5% significance level that the factory is polluting the river with bacteria. **or** The scientists claim is justified	B1 B1 M1 M1 A1 M1 B1 (7) Total 7
	Method 2 $H_0 : \lambda = 5$ ($\lambda = 2.5$) may use λ or μ $H_1 : \lambda > 5$ ($\lambda > 2.5$) $X \sim Po(2.5)$ may be implied $P(X < 7)$ $= 0.9858$ $0.9858 > 0.95$ att $P(X < 7)$ | $P(X < 6)$: [$P(X < 5) = 0.8912$] $P(X < 6) = 0.9580$ CR $X \geq 6$ wrt 0.986 $7 \geq 6$ or 7 is in critical region or 7 is significant (Reject H_0.) There is significant evidence at the 5% significance level that the factory is polluting the river with bacteria. **or** The scientists claim is justified	B1 B1 M1 M1 A1 M1 B1 (7)

Two tail test

Method 1

$H_0 : \lambda = 5$ ($\lambda = 2.5$) — may use λ or μ	B1	
$H_1 : \lambda \neq 5$ ($\lambda \neq 2.5$)	B0	
$X \sim \text{Po}(2.5)$	M1	
$P(X \geq 7) = 1 - P(X \leq 6)$ $= 1 - 0.9858$ | [$P(X \geq 6) = 1 - 0.9580 = 0.0420$] $P(X \geq 7) = 1 - 0.9858 = 0.0142$ — att $P(X \geq 7)$ | $P(X \geq 7)$	M1	
$= 0.0142$ | CR $X \geq 7$ — awrt 0.0142	A1	
$0.0142 < 0.025$ | $7 \geq 7$ or 7 is in critical region or 7 is significant	M1	
(Reject H_0.) There is significant evidence at the 5% significance level that the factory is polluting the river with bacteria. **or** The scientists claim is justified	B1	(7)

Method 2

$H_0 : \lambda = 5$ ($\lambda = 2.5$) — may use λ or μ	B1	
$H_1 : \lambda \neq 5$ ($\lambda \neq 2.5$)	B0	
$X \sim \text{Po}(2.5)$	M1	
$P(X < 7)$ | [$P(X < 6) = 0.9580$] $P(X < 7) = 0.9858$ — att $P(X < 7)$ | $P(X < 7)$		
$= 0.9858$ | CR $X \geq 7$ — awrt 0.986	M1A1	
$0.9858 > 0.975$ | $7 \geq 7$ or 7 is in critical region or 7 is significant	M1	
(Reject H_0.) There is significant evidence at the 5% significance level that the factory is polluting the river with bacteria. **or** The scientists claim is justified	B1	(7)

Question Number	Scheme		Marks
3(a)	$X \sim \text{Po}(1.5)$	need Po and 1.5	B1 (1)
(b)	<u>Faulty</u> components occur at a constant rate. <u>Faulty</u> components occur independently or randomly. <u>Faulty</u> components occur singly.	any two of the 3 only need faulty once	B1 B1 (2)
(c)	$P(X = 2) = \quad P(X \le 2) - P(X \le 1) \qquad \text{or} \qquad \dfrac{e^{-1.5}(1.5)^2}{2}$		M1
	$= 0.8088 - 0.5578$		
	$= 0.251$	awrt 0.251	A1 (2)
(d)	$X \sim \text{Po}(4.5)$	4.5 may be implied	B1
	$P(X \ge 1) = \quad 1 - P(X = 0)$ $= \quad 1 - e^{-4.5}$ $= \quad 1 - 0.0111$		M1
	$= \quad 0.9889$	awrt 0.989	A1 (3)
			Total 8

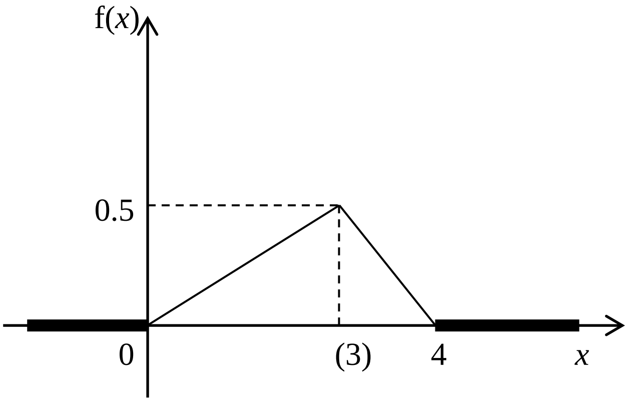
Question Number	Scheme	Marks
4	Attempt to write down combinations — at least one seen	M1
	(5,5,5), (5,5,10) any order (10,10,5) any order, (10,10,10)	A1
	(5,10,5), (10,5,5), (10,5,10), (5,10,10), — all 8 cases considered. May be implied by 3 * (10,5,10) and 3 * (5,5,10)	A1
	median 5 and 10	B1
	Median = 5 P(M = m) = $\left(\frac{1}{4}\right)^3 + 3\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right) = \frac{10}{64}$ = 0.15625 — add at least two prob using ¼ and ¾. identified by having same median of 5 or 10 Allow no 3 for M	M1 A1
	Median = 10 P(M = m) = $\left(\frac{3}{4}\right)^3 + 3\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right) = \frac{54}{64}$ = 0.84375	A1
		(7) Total 7

Question Number	Scheme		Marks
5(a)	If $X \sim B(n,p)$ and		
	n is large, $n > 50$		B1
	p is small, $p < 0.2$		B1
	then X can be approximated by $Po(np)$		
			(2)
(b)	P(2 consecutive calls) $= 0.01^2$		M1
	$= 0.0001$		A1
			(2)
(c)	$X \sim B(5, 0.01)$	may be implied	B1
	$P(X > 1) = 1 - P(X = 1) - P(X = 0)$		M1
	$= 1 - 5(0.01)(0.99)^4 - (0.99)^5$		
	$= 1 - 0.0480298\ldots - 0.95099\ldots$		
	$= 0.00098$	awrt 0.00098	A1
			(3)
(d)	$X \sim B(1000, 0.01)$	may be implied by correct mean and variance	B1
	Mean $= np = 10$		B1
	Variance $= np(1-p) = 9.9$		B1
			(3)
(e)	$X \sim Po(10)$		
	$P(X > 6) = 1 - P(X \le 6)$		M1
	$= 1 - 0.1301$		
	$= 0.8699$	awrt 0.870	A1
			(2)
			Total 12

Question Number	Scheme	Marks
6	<u>One tail test</u> <u>Method 1</u> $H_0 : p = 0.2$ $H_1 : p > 0.2$ $X \sim B(5, 0.2)$ may be implied $P(X \geq 3) = 1 - P(X \leq 2)$ $= 1 - 0.9421$ | $[P(X \geq 3) = 1 - 0.9421 = 0.0579]$ att $P(X \geq 3)$ | $P(X \geq 4)$ $P(X \geq 4) = 1 - 0.9933 = 0.0067$ $= 0.0579$ | CR $X \geq 4$ awrt 0.0579 $0.0579 > 0.05$ | $3 \leq 4$ or 3 is not in critical region or 3 is not significant (Do not reject H_0.) There is insufficient evidence at the 5% significance level that there is an increase in the number of times <u>the taxi/driver is late.</u> **Or** Linda’s claim is not justified	 B1 B1 M1 M1 A1 M1 B1 (7) Total 7
	<u>Method 2</u> $H_0 : p = 0.2$ $H_1 : p > 0.2$ $X \sim B(5, 0.2)$ may be implied $P(X < 3) =$ | $[P(X < 3) = 0.9421]$ att $P(X < 3)$ | $P(X < 4)$ $P(X < 4) = 0.9933$ 0.9421 | CR $X \geq 4$ awrt 0.942 $0.9421 < 0.95$ | $3 \leq 4$ or 3 is not in critical region or 3 is not significant (Do not reject H_0.) There is insufficient evidence at the 5% significance level that there is an increase in the number of times the <u>taxi/driver is late.</u> **Or** Linda’s claim is not justified	 B1 B1 M1 M1A1 M1 B1 (7)

	<u>Two tail test</u> <u>Method 1</u> $H_o : p = 0.2$ $H_1 : p \neq 0.2$ $X \sim X \sim B(5, 0.2)$ may be implied $P(X \geq 3) = 1 - P(X \leq 2)$ | $[P(X \geq 3) = 1 - 0.9421 = 0.0579]$ att $P(X \geq 3)$ | $P(X \geq 4)$ $= 1 - 0.9421$ | $P(X \geq 4) = 1 - 0.9933 = 0.0067$ $= 0.0579$ | CR $X \geq 4$ awrt 0.0579 $0.0579 > 0.025$ | $3 \leq 4$ or 3 is not in critical region or 3 is not significant (Do not reject H_0.) There is insufficient evidence at the 5% significance level that there is an increase in the number of times the <u>taxi/driver is late.</u> **Or** Linda's claim is not justified	B1 B0 M1 M1 A1 M1 B1	 (7)
	<u>Method 2</u> $H_o : p = 0.2$ $H_1 : p \neq 0.2$ $X \sim X \sim B(5, 0.2)$ may be implied $P(X < 3) =$ | $[P(X < 3) = 0.9421]$ att $P(X < 3)$ | $P(X < 4)$ | $P(X < 4) = 0.9933$ 0.9421 | CR $X \geq 4$ awrt 0.942 $0.9421 < 0.975$ | $3 \leq 4$ or 3 is not in critical region or 3 is not significant Do not reject H_0. There is insufficient evidence at the 5% significance level that there is an increase in the number of times <u>the taxi/driver is late.</u> **Or** Linda's claim is not justified	B1 B0 M1 M1A1 M1 B1	 (7)
	<u>Special Case</u> If they use a probability of $\frac{1}{7}$ throughout the question they may gain B1 B1 M0 M1 A0 M1 B1. NB they must attempt to work out the probabilities using $\frac{1}{7}$		

Question Number	Scheme	Marks
7(a) i	If $X \sim B(n,p)$ and n is large or $n > 10$ **or** $np > 5$ **or** $nq > 5$ p is close to 0.5 **or** $nq > 5$ <u>and</u> $np > 5$ then X can be approximated by $N(np, np(1-p))$	B1 B1 (2)
ii	mean = np variance = $np(1-p)$ — must be in terms of p	B1 B1 (2)
(b)	$X \sim N(60, 58.2)$ or $X \sim N(60, 7.63^2)$ — 60, 58.2 $P(X \geq 40) = P(X > 39.5)$ — using 39.5 or 40.5 $= 1 - P\left(z < \pm\left(\frac{39.5-60}{\sqrt{58.2}}\right)\right)$ — standardising 39.5 or 40 or 40.5 and their μ and σ $= 1 - P(z < -2.68715\ldots)$ $= 0.9965$ — allow answers in range 0.996 – 0.997	B1, B1 M1 M1 A1dep on both M (5)
(c)	$E(X) = 60$ — may be implied or ft from part (b) Expected profit $= (2000 - 60) \times 11 - 2000 \times 0.70$ $= £19\,940.$	B1ft M1 A1 (3) Total 12

Question Number	Scheme	Marks
8(a)	 (0), 4, 0.5 — 0 may be implied by start at y axis both patio must be straight	B1 B1 B1 (3)
(b)	Mode is $x = 3$	B1 (1)
(c)	$F(x) = \int_0^x \frac{1}{6}t \, dt$ (for $0 \le x \le 3$) — ignore limits for M	M1
	$= \frac{1}{12}x^2$ — must use limit of 0	A1
	$F(x) = \int_3^x 2 - \frac{1}{2}t \, dt; + \int_0^3 \frac{1}{6}t \, dt$ (for $3 < x \le 4$) — need limit of 3 and variable upper limit; need limit 0 and 3	M1; M1
	$= 2x - \frac{1}{4}x^2 - 3$	A1
	$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{12}x^2 & 0 \le x \le 3 \\ 2x - \frac{1}{4}x^2 - 3 & 3 < x \le 4 \\ 1 & x > 4 \end{cases}$ — middle pair; ends	B1 ft B1 (7)
(d)	$F(m) = 0.5$ — either eq	M1
	$\frac{1}{12}x^2 = 0.5$ — eq for their $0 \le x \le 3$	A1ft
	$x = \sqrt{6} = 2.45$ — $\sqrt{6}$ or awrt 2.45	A1 (3) Total 14