

**ADVANCED GCE UNIT  
MATHEMATICS (MEI)**

Mechanics 2

**WEDNESDAY 10 JANUARY 2007**

**4762/01**

Afternoon  
Time: 1 hour 30 minutes

Additional materials:  
Answer booklet (8 pages)  
Graph paper  
MEI Examination Formulae and Tables (MF2)

**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

**ADVICE TO CANDIDATES**

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

1 A sledge and a child sitting on it have a combined mass of 29.5 kg. The sledge slides on horizontal ice with negligible resistance to its movement.

- (i) While at rest, the sledge is hit directly from behind by a ball of mass 0.5 kg travelling horizontally at  $10 \text{ m s}^{-1}$ . The coefficient of restitution in the collision is 0.8. After the impact the speeds of the sledge and the ball are  $V_1 \text{ m s}^{-1}$  and  $V_2 \text{ m s}^{-1}$  respectively.

Calculate  $V_1$  and  $V_2$  and state the direction in which the ball is travelling after the impact. [7]

- (ii) While at rest, the sledge is hit directly from behind by a snowball of mass 0.5 kg travelling horizontally at  $10 \text{ m s}^{-1}$ . The snowball sticks to the sledge.

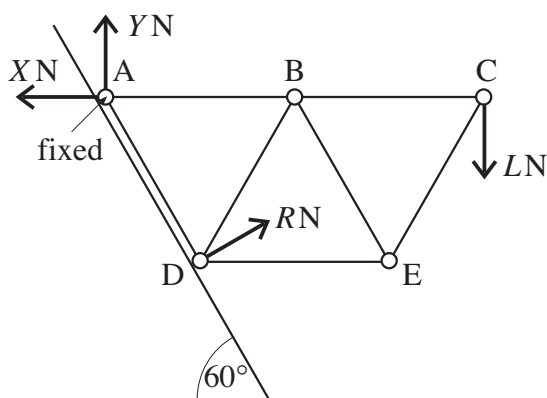
(A) Calculate the velocity with which the combined sledge and snowball start to move. [3]

(B) The child scoops up the 0.5 kg of snow and drops it over the back of the sledge. What happens to the velocity of the sledge? Give a reason for your answer. [2]

- (iii) In another situation, the sledge is travelling over the ice at  $2 \text{ m s}^{-1}$  with 10.5 kg of snow on it (giving a total mass of 40 kg). The child throws a snowball of mass 0.5 kg from the sledge, parallel to the ground and in the positive direction of the motion of the sledge. Immediately after the snowball is thrown, the sledge has a speed of  $V \text{ m s}^{-1}$  and the snowball and sledge are separating at a speed of  $10 \text{ m s}^{-1}$ .

Draw a diagram showing the velocities of the sledge and snowball before and after the snowball is thrown.

Calculate  $V$ . [5]



**Fig. 2**

Fig. 2 shows a framework in a vertical plane made from the equal, light, rigid rods AB, BC, AD, BD, BE, CE and DE. [The triangles ABD, BDE and BCE are all equilateral.]

The rods AB, BC and DE are horizontal.

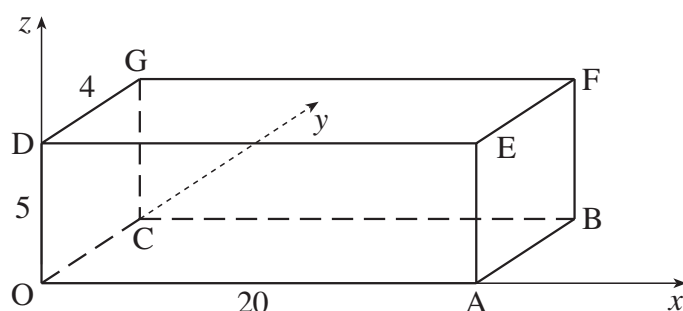
The rods are freely pin-jointed to each other at A, B, C, D and E.

The pin-joint at A is also fixed to an inclined plane. The plane is smooth and parallel to the rod AD. The pin-joint at D rests on this plane.

The following external forces act on the framework: a vertical load of  $LN$  at C; the normal reaction force  $RN$  of the plane on the framework at D; the horizontal and vertical forces  $XN$  and  $YN$ , respectively, acting at A.

- (i) Write down equations for the horizontal and vertical equilibrium of the framework. [3]
- (ii) By considering moments, find the relationship between  $R$  and  $L$ . Hence show that  $X = \sqrt{3}L$  and  $Y = 0$ . [4]
- (iii) Draw a diagram showing all the forces acting on the pin-joints, including the forces internal to the rods. [2]
- (iv) Show that the internal force in the rod AD is zero. [2]
- (v) Find the forces internal to AB, CE and BC in terms of  $L$  and state whether each is a tension or a thrust (compression). [You may leave your answers in surd form.] [7]
- (vi) Without calculating their values in terms of  $L$ , show that the forces internal to the rods BD and BE have equal magnitude but one is a tension and the other a thrust. [2]

- 3 A box is to be assembled in the shape of the cuboid shown in Fig. 3.1. The lengths are in centimetres. All the faces are made of the same uniform, rigid and thin material. All coordinates refer to the axes shown in this figure.



**Fig. 3.1**

- (i) The four vertical faces  $OAED$ ,  $ABFE$ ,  $FGCB$  and  $CODG$  are assembled first to make an open box without a base or a top. Write down the coordinates of the centre of mass of this open box. [1]

The base  $OABC$  is added to the vertical faces.

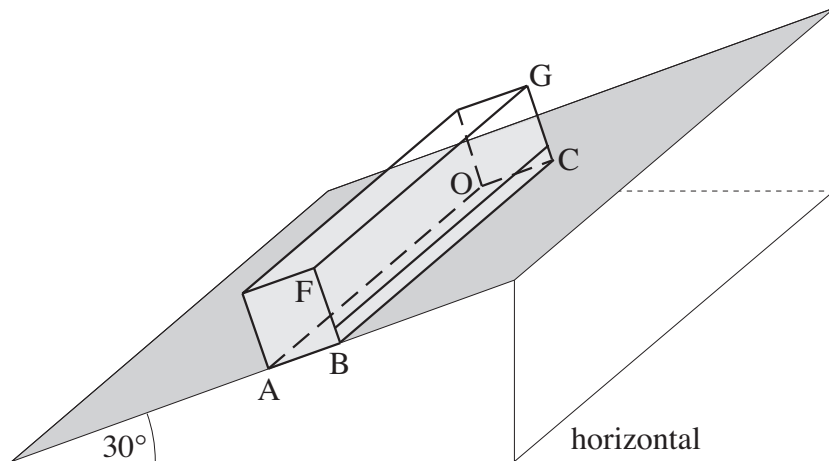
- (ii) Write down the  $x$ - and  $y$ -coordinates of the centre of mass of the box now. Show that the  $z$ -coordinate is now 1.875. [5]

The top face  $FGDE$  is now added. This is a lid hinged to the rest of the box along the line  $FG$ . The lid is open so that it hangs in a vertical plane touching the face  $FGCB$ .

- (iii) Show that the coordinates of the centre of mass of the box in this situation are  $(10, 2.4, 2.1)$ . [6]

**[This question is continued on the facing page.]**

The box, with the lid still touching face FGCB, is now put on a sloping plane with the edge OA horizontal and the base inclined at  $30^\circ$  to the horizontal, as shown in Fig. 3.2.



**Fig. 3.2**

The weight of the box is 40 N. A force  $P$  N acts parallel to the plane and is applied to the mid-point of FG at  $90^\circ$  to FG. This force tends to push the box down the plane. The box does not slip and is on the point of toppling about the edge AO.

- (iv) Show that the clockwise moment of the weight of the box about the edge AO is about 0.411 Nm. [4]
- (v) Calculate the value of  $P$ . [2]

**[Question 4 is printed overleaf.]**

- 4 Some tiles on a roof are being replaced. Each tile has a mass of 2 kg and the coefficient of friction between it and the existing roof is 0.75. The roof is at  $30^\circ$  to the horizontal and the bottom of the roof is 6 m above horizontal ground, as shown in Fig. 4.

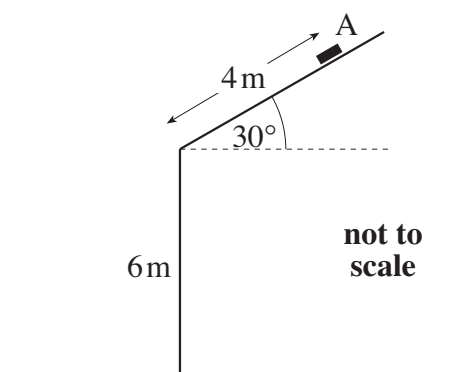


Fig. 4

- (i) Calculate the limiting frictional force between a tile and the roof.

A tile is placed on the roof. Does it slide? (Your answer should be supported by a calculation.) [5]

- (ii) The tiles are raised 6 m from the ground, the only work done being against gravity. They are then slid 4 m up the roof and placed at the point A shown in Fig. 4.

(A) Show that each tile gains 156.8 J of gravitational potential energy. [3]

(B) Calculate the work done against friction per tile. [2]

(C) What average power is required to raise 10 tiles per minute from the ground to A? [2]

- (iii) A tile is kicked from A directly down the roof. When the tile is at B,  $x$  m from the edge of the roof, its speed is  $4 \text{ m s}^{-1}$ . It subsequently hits the ground travelling at  $9 \text{ m s}^{-1}$ . In the motion of the tile from B to the ground, the work done against sliding and other resistances is 90 J.

Use an energy method to find  $x$ . [5]