OCR
Oxford Cambridge and RSA

Wednesday 17 May 2017 – Morning

A2 GCE MATHEMATICS (MEI)

4762/01 Mechanics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4762/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Fig. 1.1 shows the masses and speeds of two small uniform circular discs, A and B, sliding towards one another on a smooth horizontal surface. Each of the discs is moving with its centre on the line shown in the figure; there is a barrier which is perpendicular to this line. The discs collide and separate.

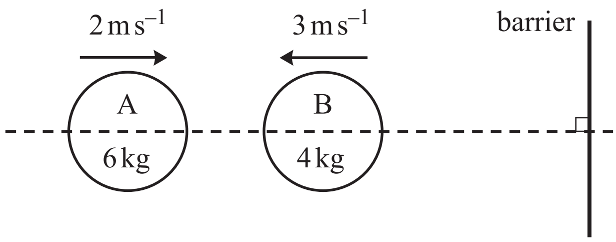

Fig. 1.1

(i) By considering only linear momentum, explain how you know that the direction of motion of each disc must be reversed in the collision. **[2]**

You are now given that the coefficient of restitution is e.

(ii) Show that after the collision A has speed $2e$. Find an expression in terms of e for the speed of B after the collision. **[5]**

(iii) Find an expression in terms of e for the magnitude of the impulse in the collision. **[1]**

Three seconds after its collision with A, disc B has a perfectly elastic direct collision with the barrier shown in Fig. 1.1 and later collides again with A.

(iv) What time will elapse between the two collisions of the discs? **[3]**

In a different situation, B has an oblique impact with a smooth barrier. This barrier is inclined at an angle α to the direction of motion of B. The coefficient of restitution in this collision is $\frac{1}{3}$ and the direction of motion of B is turned through 90° as a result of the collision, as shown in Fig. 1.2.

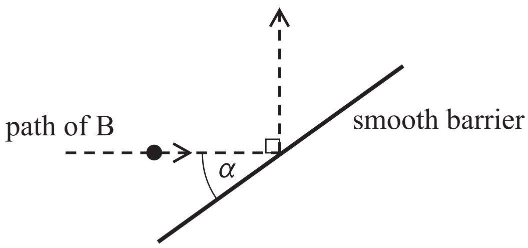

Fig. 1.2

(v) Calculate α, giving your answer in degrees. **[5]**

2 In this question take $g = 10$.

Fig. 2 shows a small object Q, of mass 5 kg, which slides in a straight line up a rough ramp. The point D is at the top of the ramp, where the ramp is joined to a horizontal platform. The point C on the ramp is a vertical distance of 2 m below D. CD is inclined at an angle α to the horizontal.

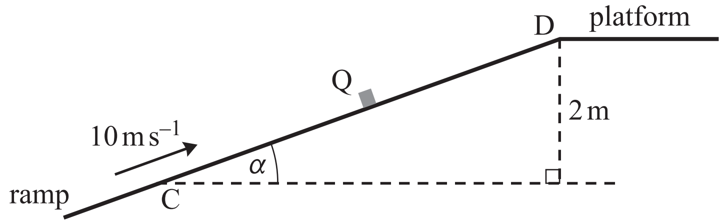

Fig. 2

Q is struck so that it moves up a line of greatest slope of the ramp. Q passes through C with a speed of 10 m s^{-1} and comes to rest at D.

The motion of Q is first modelled by assuming that the only resistance to its motion is friction with the plane.

(i) Without assuming that the ramp is uniformly rough, calculate the work done by the frictional force as Q travels from C to D. **[3]**

Now assume that the coefficient of friction between the object and the plane has the constant value $\frac{5}{8}$.

(ii) Show that the work done by the frictional force acting on Q as it travels from C to D may be expressed as $\dfrac{125}{2\tan\alpha}$.

Calculate $\tan\alpha$. **[7]**

A new ramp is built that is inclined at an angle greater than α to the horizontal. This also ends at D. Q is now struck so that it moves up a line of greatest slope of the new ramp. As in the previous situation, it passes through a point at a vertical level 2 m below D at a speed of 10 m s^{-1}. The coefficient of friction between Q and the new ramp is the same as that between Q and the old ramp.

(iii) Does Q now come to rest below D or still come to rest at D or is Q still moving at D?
You should explain your answer but you need not produce detailed calculations. **[2]**

Subsequently Q is moving on the horizontal platform and is made to travel in a straight line by a force which has a constant power of 50 W. The resistance to the motion of Q is F N, where F is constant. The velocity and acceleration of Q at time t s are v m s^{-1} and a m s^{-2} in the direction of its motion.

(iv) Write down the equation of motion of Q in terms of F, v and a. **[3]**

(v) Given that Q is travelling at a constant speed of 4 m s^{-1}, calculate F. **[1]**

(vi) Show that a can only be constant if it is zero.

Q changes speed from U m s^{-1} to V m s^{-1} in T s. Explain why it would not be appropriate to calculate the distance travelled in this time as $\frac{1}{2}T(U+V)$. **[3]**

3 **(a)** Fig. 3.1 shows a framework JKL in a vertical plane. The framework is made from three light rigid rods JK, KL and LJ which are freely pin-jointed to each other at J, K and L. The pin-joint at J is attached to a fixed vertical wall; the pin-joint at L is in contact with a fixed smooth horizontal beam. JK is 5 m, KL is 3 m and LJ is 4 m. Angle KLJ is 90°.

The framework is held in equilibrium with JK horizontal by means of a single applied force of 80 N acting at K parallel to JL. Fig. 3.1 shows this force and the angle α between JL and JK. Fig. 3.1 also shows the horizontal component, X N, of the force on the framework due to J being attached to the wall. Note that the diagram does not show the vertical component of the force acting at J nor the force on the framework at L due to contact with the beam.

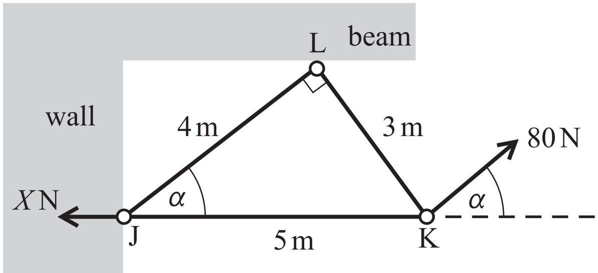

Fig. 3.1

Show that $X = 64$.

By first considering the equilibrium of the pin-joint at K, or otherwise, calculate the forces internal to the rods JK, KL and LJ, stating whether each rod is in tension or thrust (compression). **[8]**

(b) Fig. 3.2 shows a uniform heavy ladder AB, of weight W N, standing on rough horizontal ground and resting on a smooth peg at C. The ladder has length 5 m. C is 3 m above the ground and is a horizontal distance of 1.25 m from A. The ladder is in equilibrium.

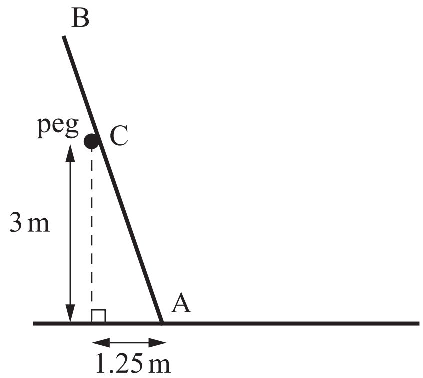

Fig. 3.2

Show that the force exerted on the ladder by the peg at C is $\frac{50W}{169}$.

Calculate the range of possible values of the coefficient of friction between the ladder and the ground. **[12]**

4 **(a)** In this part question, all coordinates refer to the axes shown in Fig. 4.1.

Fig. 4.1 shows a uniform rectangular lamina OABC with OA and OC on the x- and y-coordinate axes. The units of the axes are metres.

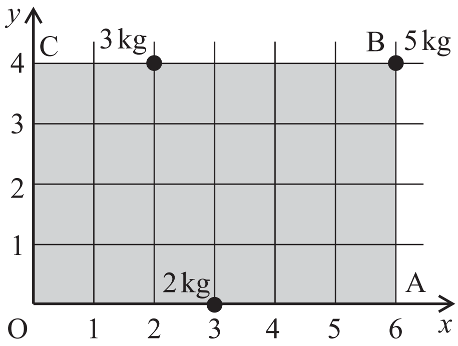

Fig. 4.1

The mass of the lamina is 2 kg and 3 heavy particles are fixed to it. The masses of these particles are 2 kg, 3 kg and 5 kg and they are at points with coordinates (3, 0), (2, 4) and (6, 4), respectively.

(i) Calculate the coordinates of the combined centre of mass of the lamina and the 3 particles. **[3]**

A particle of mass m kg is now fixed to the lamina at a point $(X, 0)$, with $0 < X \leqslant 6$, so that the combined centre of mass of the original 3 particles, the lamina and the mass m kg lies on OB (part of the line with equation $y = \frac{2}{3}x$).

(ii) Establish that $mX = 6$. **[4]**

(b) Fig. 4.2 shows a thin heavy uniform wire ABC bent into the shape of a semi-circle with centre O and radius r. More of the same wire, DE, has its centre at O and lies on the straight line AOC. The distance DE is kr, where $k \geqslant 2$ is a constant. The two wires are joined at A and C to form object P.

You may use without proof the information that the centre of mass of the curved semi-circular section of the wire, ABC, lies on the line OB at a distance $\frac{2r}{\pi}$ from O.

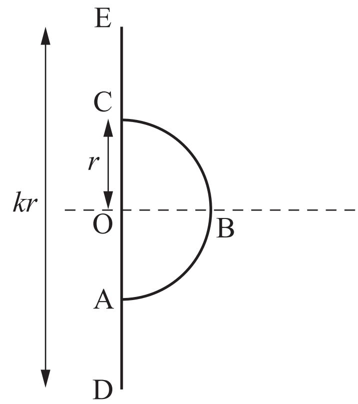

Fig. 4.2

(i) Show that the centre of mass of P lies on OB at a distance $\frac{2r}{\pi + k}$ from O. **[4]**

Two light inelastic strings are attached to P, one at E and the other at B. Both strings are vertical and P is in equilibrium with DE vertical. The tension in the string attached at E is T_E and the tension in the string attached at B is T_B.

(ii) Find the value of k for which $T_E = 2T_B$. **[6]**

END OF QUESTION PAPER

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