Version 1.0

# General Certificate of Education
# June 2010

## Mathematics MPC4

## Pure Core 4

# Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available to download from the AQA Website: www.aqa.org.uk



Set and published by the Assessment and Qualifications Alliance.

**Key to mark scheme and abbreviations used in marking**

| | | | |
|---|---|---|---|
| M | mark is for method | | |
| m or dM | mark is dependent on one or more M marks and is for method | | |
| A | mark is dependent on M or m marks and is for accuracy | | |
| B | mark is independent of M or m marks and is for method and accuracy | | |
| E | mark is for explanation | | |
| | | | |
| √ or ft or F | follow through from previous incorrect result | MC | mis-copy |
| CAO | correct answer only | MR | mis-read |
| CSO | correct solution only | RA | required accuracy |
| AWFW | anything which falls within | FW | further work |
| AWRT | anything which rounds to | ISW | ignore subsequent work |
| ACF | any correct form | FIW | from incorrect work |
| AG | answer given | BOD | given benefit of doubt |
| SC | special case | WR | work replaced by candidate |
| OE | or equivalent | FB | formulae book |
| A2,1 | 2 or 1 (or 0) accuracy marks | NOS | not on scheme |
| $-x$ EE | deduct $x$ marks for each error | G | graph |
| NMS | no method shown | c | candidate |
| PI | possibly implied | sf | significant figure(s) |
| SCA | substantially correct approach | dp | decimal place(s) |

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

**MPC4**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **1(a)** | $\mathrm{f}\left(\frac{1}{4}\right)=8\times\frac{1}{64}+6\times\frac{1}{16}-14\times\frac{1}{4}-1$ | M1 | | Use $x=\frac{1}{4}$ in evaluation |
| | $=-4$ | A1 | 2 | NMS 2/2; no ISW |
| **(b)(i)** | $\mathrm{g}\left(\frac{1}{4}\right)=$ number(s) $+\,d=0$ | M1 | | Use factor theorem to find $d$<br>See some processing |
| | $d=3$ | A1 | 2 | NMS 2/2 |
| **(ii)** | $\mathrm{g}(x)=(4x-1)\left(2x^2+bx-3\right)$ | B1F | | $a=2\quad c=-3$; F on $d$ $(c=-d)$ |
| | $x^2\quad 6=4b-2$ or $x\quad -14=-b-12$ | M1 | | Any appropriate method; PI |
| | $b=2$ | A1 | 3 | NMS 2/2 |
| | **Total** | | **7** | |
| | **Alternatives:** | | | |
| **(a)** | $$\begin{array}{r} 2x^2+2x-3 \\ 4x-1\overline{\big)8x^3+6x^2-14x-1} \\ \underline{8x^3-2x^2}\phantom{-14x-1} \\ 8x^2-14x\phantom{-1} \\ \underline{8x^2-2x}\phantom{-1} \\ -12x-1 \\ \underline{-12x+3} \\ -4 \end{array}$$ | (M1) | | Complete division with integer remainder |
| | | (A1) | (2) | Remainder = –4 stated |
| **(b)(i)** | Division as for (a) $\Rightarrow d-3$ last line | (M1) | | Candidate's $-3$ |
| | $d=3$ | (A1) | (2) | |
| **2(a)** | $\frac{\mathrm{d}x}{\mathrm{d}t}=-3\qquad\frac{\mathrm{d}y}{\mathrm{d}t}=6t^2$ | B1 | | Both derivatives correct; PI |
| | $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{6t^2}{3}$ | M1 | | Correct use of chain rule |
| | $=-2t^2$ | A1 | 3 | CSO |
| **(b)** | $t=1\quad m_{\mathrm{T}}=-2\quad m_{\mathrm{N}}=\frac{1}{2}$ | M1 | | Substitute $t=1\quad m_N=-\frac{1}{m_T}$ |
| | | A1F | | F on gradient; $m_{\mathrm{T}}\neq\pm1$ |
| | Attempt at equation of normal using $(x,\,y)=(-2,\,3)$ | M1 | | Condone one error |
| | Normal has equation $y-3=\frac{1}{2}(x+2)$ | A1 | 4 | CSO; ACF |
| **(c)** | $t=\frac{1-x}{3}$ or $t=\sqrt[3]{\frac{y-1}{2}}$ | M1 | | Correct expression for $t$ in terms of $x$ or $y$ |
| | $y=1+2\left(\frac{1-x}{3}\right)^3$ | A1 | 2 | ACF |
| | **Total** | | **9** | |

**MPC4 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **3(a)(i)** | $7x-3=A(3x-2)+B(x+1)$ | M1 | | |
| | $x=-1 \qquad x=\frac{2}{3}$ | m1 | | Substitute two values of *x* and solve for *A* and *B* |
| | $A=2 \qquad B=1$ | A1 | 3 | Or solve $\left.\begin{matrix}7=3A+B\\-3=-2A+B\end{matrix}\right\}$ condone one error |
| **(ii)** | $\int \frac{7x-3}{(x+1)(3x-2)}\,dx=$ | | | |
| | $p\ln(x+1)+q\ln(3x-2)$ | M1 | | Condone missing brackets |
| | $=2\ln(x+1)+\frac{1}{3}\ln(3x-2)\ (+c)$ | A1F | 2 | F on *A* and *B*; constant not required |
| **(b)** | $\frac{6x^2+x+2}{2x^2-x+1}=\frac{6x^2-3x+3+4x-1}{2x^2-x+1}$ | M1 | | |
| | $=3+\frac{4x-1}{2x^2-x+1}$ | B1<br>A1 | 3 | $P=3$<br>$Q=4$ and $R=-1$ |
| | **Total** | | **8** | |
| | **Alternatives**: | | | |
| **(a)(i)** | By cover up rule | | | |
| | $x=-1 \quad A=\frac{-7-3}{-5}$ | | | |
| | $x=\frac{2}{3} \quad B=\frac{\frac{14}{3}-3}{\frac{5}{3}}$ | (M1) | | $x=-1$ and $x=\frac{2}{3}$<br>and attempt to find *A* and *B* |
| | $A=2 \quad B=1$ | (A1,A1) | (3) | SC NMS *A* and *B* both correct 3/3<br>One of *A* or *B* correct 1/3 |
| **(b)** | $\begin{array}{r} 3 \\ 2x^2-x+1\overline{)6x^2+\ \ x+2} \\ \underline{6x^2-3x+3} \\ 4x-1 \end{array}$ | (M1)<br>(B1)<br>(A1) | (3) | Complete division, with $ax+b$ remainder<br>$P=3$ stated<br>$Q=4$ and $R=-1$ stated or written as expression |
| | **or**<br>$6x^2+x+2=P(2x^2-x+1)+Qx+R$ | | | |
| | $=2Px^2+(Q-P)x+P+R$ | (M1) | | Multiply across and equate coefficients or use numerical values of *x* |
| | $P=3$<br>$Q-P=1$<br>$P+R=2$ | (B1) | | $P=3$ stated |
| | $Q=4 \quad \text{and} \quad R=-1$ | (A1) | (3) | $Q=4$ and $R=-1$ stated or written as expression |

**MPC4 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 4(a)(i) | $(1+x)^{\frac{3}{2}} = 1+\frac{3}{2}x+kx^2$ | M1 | | |
| | $= 1+\frac{3}{2}x+\frac{3}{8}x^2$ | A1 | 2 | |
| (ii) | $(16+9x)^{\frac{3}{2}} = 16^{\frac{3}{2}}\left(1+\frac{9}{16}x\right)^{\frac{3}{2}}$ | B1 | | |
| | $= k\left(1+\frac{3}{2}\times\frac{9}{16}x+\frac{3}{8}\left(\frac{9}{16}x\right)^2\right)$ | M1 | | $x$ replaced by $\frac{9}{16}x$ or start binomial again<br>Condone missing brackets |
| | $= 64+54x+\frac{243}{32}x^2$ | A1 | 3 | Accept $7.59375x^2$ |
| (b) | $x=-\frac{1}{3}$ | M1 | | Use $x=-\frac{1}{3}$ |
| | $13^{\frac{3}{2}} \approx 46+\frac{27}{32}$ | A1 | 2 | 46 seen with $a$ = 27 $b$ =32, or $\left(\frac{k\times 27}{k\times 32}\right)$ |
| | **Total** | | **7** | |
| | **Alternative:** | | | |
| (a)(ii) | $(16+9x)^{\frac{3}{2}} =$<br>$16^{\frac{3}{2}}+\frac{3}{2}\times16^{\frac{1}{2}}\times9x+\frac{3}{2}\times\frac{1}{2}\times\frac{1}{2}\times16^{-\frac{1}{2}}\times(9x)^2$ | (M1) | | Use $(a+bx)^n$ from FB. Allow one error.<br>Condone missing brackets. |
| | $= 64+54x+\frac{243}{32}x^2$ | (A2) | (3) | Accept $7.59375x^2$ |
| 5(a)(i) | $\cos 2x = 1-2\sin^2 x$ | B1 | | ACF in terms of sin (PI later) |
| | $3\left(1-2\sin^2 x\right)+2\sin x+1=0$ | M1 | | Substitute candidate's cos2$x$ in terms of sin $x$ (at least 2 terms) |
| | $-6\sin^2 x+2\sin x+4=0$<br>$3\sin^2 x-\sin x-2=0$ | A1 | 3 | AG |
| (ii) | $(3\sin x+2)(\sin x-1)=0$ | M1 | | Factorise correctly or use formula correctly |
| | $\sin x=-\frac{2}{3}$ $\quad\sin x=1$ | A1 | 2 | Both; condone $-0.67$ or $-0.66$ or better |
| (b)(i) | $R=\sqrt{13}$ | B1 | | Accept 3.6 or better |
| | $\tan\alpha=\frac{2}{3}$ $\quad\alpha=33.7$ | M1A1 | 3 | OE; accept $\alpha$ = 33.69(0) |
| (ii) | $2x-\alpha=\cos^{-1}\left(\frac{-1}{R}\right)$ | M1 | | Candidate's $R$. Or $\cos(2x-\alpha)=\frac{-1}{R}$ |
| | $2x-\alpha=106.1°,\ 253.9°$ | | | |
| | $x=69.9°,\ 143.8°$ | A1 | | One correct answer |
| | | A1 | 3 | Both correct, no extras in range |
| | **Total** | | **11** | |

**MPC4 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **6(a)** | $x^3 + \cos\pi = 7 \Rightarrow x^3 - 1 = 7$ | M1 | | Or $x = \sqrt[3]{7 - \cos\pi}$ |
| | $x = 2$ | A1 | 2 | CSO |
| **(b)** | $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^3 y\right) = 3x^2 y + x^3 \frac{\mathrm{d}y}{\mathrm{d}x}$ | M1 | | 2 terms added, one with $\frac{\mathrm{d}y}{\mathrm{d}x}$ |
| | | A1 | | |
| | $\frac{\mathrm{d}}{\mathrm{d}x}(\cos\pi y) = -\pi\sin(\pi y)\frac{\mathrm{d}y}{\mathrm{d}x}$ | B1 | | |
| | At $(2,1)$ $\quad 3\times 4 + 8\frac{\mathrm{d}y}{\mathrm{d}x} - \pi\sin\pi\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ | M1 | | Substitute candidate's $x$ from (a) and $y = 1$ with 0 on RHS and both derivatives attempted and no extra derivatives |
| | $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{3}{2}$ | A1 | 5 | CSO; OE |
| | **Total** | | **7** | |

**MPC4 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **7(a)** | $\overrightarrow{OB}=\begin{bmatrix}1\\-1\\2\end{bmatrix}$ | B1 | | PI |
| | $\overrightarrow{AB}=\begin{bmatrix}1\\-1\\2\end{bmatrix}-\begin{bmatrix}4\\-3\\2\end{bmatrix}=\begin{bmatrix}-3\\2\\0\end{bmatrix}$ | M1<br>A1 | 3 | Use $\pm\left(\overrightarrow{OB}-\overrightarrow{OA}\right)$ |
| **(b)(i)** | $4+2\lambda=-1+\mu$<br>$-3\quad=3-2\mu$<br>$2+\lambda\quad=4-\mu$ | M1 | | $\begin{bmatrix}4+2\lambda\\-3\\2+\lambda\end{bmatrix}=\begin{bmatrix}1+\mu\\3-2\mu\\4-\mu\end{bmatrix}$<br>or set up 3 equations |
| | $-6=-2\mu\qquad\mu=3$ | m1 | | Solve for $\lambda$ and $\mu$ |
| | $\lambda=4-3-2\qquad\lambda=-1$ | A1 | | Both correct |
| | $4+2\lambda=4-2=2$<br>$-1+\mu=-1+3=2$ | A1 | 4 | Independent check with conclusion: minimum "intersect" |
| **(ii)** | $P$ is $(2,-3,1)$ | B1 | 1 | |
| **(c)** | $\overrightarrow{OC}=\overrightarrow{OA}+\overrightarrow{AC}$<br>$=\overrightarrow{OA}+\overrightarrow{PB}$ | | | Or $\overrightarrow{OC}=\overrightarrow{OB}+\overrightarrow{BC}$<br>$=\overrightarrow{OB}+\overrightarrow{PA}$ |
| | $\overrightarrow{OC}=\begin{bmatrix}4\\-3\\2\end{bmatrix}+\begin{bmatrix}1-2\\-1--3\\2-1\end{bmatrix}$ | M1 | | $\overrightarrow{OA}+\overrightarrow{PB}$ in components |
| | $C$ is $(3,-1,3)$ | A1 | | |
| | or $\overrightarrow{OC}=\overrightarrow{OB}+\overrightarrow{BC}$<br>$=\overrightarrow{OB}+\overrightarrow{AP}$ | | | |
| | $\overrightarrow{OC}=\begin{bmatrix}1\\-1\\2\end{bmatrix}+\begin{bmatrix}2-4\\-3--3\\1-2\end{bmatrix}$ | M1 | | $\overrightarrow{OB}+\overrightarrow{AP}$ in components |
| | $C$ is $(-1,-1,1)$ | A1 | 4 | |
| | **Total** | | **12** | |

**MPC4 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| | **Alternative:** | | | |
| **7(c)** | $\overrightarrow{AP} = \overrightarrow{BC}$ | | | |
| | $\left\lvert \overrightarrow{AP} \right\rvert = \left\lvert \overrightarrow{BC} \right\rvert =$ $\sqrt{(2-4)^2 + (-3--3)^2 + (1-2)^2}$ $=\sqrt{5}$ | (M1) | | |
| | $\overrightarrow{BC} = k\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ $\quad \left\lvert \overrightarrow{BC} \right\rvert = \sqrt{k}\sqrt{5}$ | | | |
| | so $k = \pm 1$ | (A1*) | | For $k = 1$ and $k = -1$ |
| | $\overrightarrow{OC} = \overrightarrow{OB} + k\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ | | | |
| | $= \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ | (M1) | | Either |
| | $= \begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix}$ or $\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$ | (A1) | (4) | Both |
| | *If $k = 1$ or $k = -1$ (ie only one $k$), one correct point gets 2/4 | | | |

**MPC4 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **8(a)** | $\int \frac{\mathrm{d}x}{\sqrt{x+1}} = \int -\frac{1}{5}\,\mathrm{d}t$ | B1 | | Correct separation; or $\frac{\mathrm{d}t}{\mathrm{d}x} = -5(x+1)^{-\frac{1}{2}}$<br>Condone missing integral signs |
| | $2\sqrt{x+1} = -\frac{1}{5}t \quad (+C)$ | B1B1 | | Correct integrals; condone $\frac{\sqrt{x+1}}{\frac{1}{2}}$ |
| | $x = 80 \quad t = 0 \quad C = 2\sqrt{81}$ | M1 | | Use $(0, 80)$ to find a constant $C$ |
| | $= 18$ | A1F | | F on integrals if in form $\sqrt{x+1} = qt + c$ |
| | $x = \left(9 - \frac{1}{10}t\right)^2 - 1$ | A1 | 6 | OE; CSO; $x$ = correct expression in $t$ |
| **(b)** | $t = 60 \quad x = \mathrm{f}(60)$ | M1 | | Evaluate $\mathrm{f}(60)$, ie $x = \ldots$ ($C$ not required) |
| | $= 8$ | A1 | 2 | CSO |
| **(c)(i)** | $\frac{\mathrm{d}A}{\mathrm{d}t} = kA(9 - A)$ | M1 | | $\frac{\mathrm{d}A}{\mathrm{d}t}$ = product involving $A$; $k$ required<br>Condone terms in $t$ |
| | | A1 | 2 | |
| **(ii)** | $4.5 = \frac{9}{1 + 4\mathrm{e}^{-0.09t}}$ | M1 | | Condone one slip in denominator |
| | $\mathrm{e}^{-0.09t} = \frac{1}{4}$ | A1 | | |
| | $-0.09t = \ln\left(\frac{1}{4}\right)$ | m1 | | Take ln correctly |
| | $t = \frac{\ln\left(\frac{1}{4}\right)}{-0.09}$ | | | |
| | $= 15.4$ (hours) | A1 | 4 | CAO; condone more than 3sf if correct<br>15.40327068 Allow 15h 24m |
| | **Total** | | **14** | |
| | **TOTAL** | | **75** | |