Centre Number			Candidate Number		
Surname					
Other Names					
Candidate Signature					



General Certificate of Education Advanced Level Examination June 2012

Mathematics

MPC3

Unit Pure Core 3

Thursday 31 May 2012 9.00 am to 10.30 am

For this paper you must have:

• the blue AQA booklet of formulae and statistical tables. You may use a graphics calculator.

Time allowed

• 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do not use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use						
Examine	r's Initials					
Question	Mark					
1						
2						
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5						
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7						
8						
9						
TOTAL						

Answer all questions.

Answer each question in the space provided for that question.

1	Use the mid-ordinate rule with four strips to find an estimate for	$\int_{0.4}^{1.2} \cot(x^2) \mathrm{d}x,$
	giving your answer to three decimal places.	(4 marks)

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- For $0 < x \le 2$, the curves with equations $y = 4 \ln x$ and $y = \sqrt{x}$ intersect at a single point where $x = \alpha$.
 - (a) Show that α lies between 0.5 and 1.5. (2 marks)
 - (b) Show that the equation $4 \ln x = \sqrt{x}$ can be rearranged into the form

$$x = e^{\left(\frac{\sqrt{x}}{4}\right)} \tag{1 mark}$$

(c) Use the iterative formula

$$x_{n+1} = e^{\left(\frac{\sqrt{x_n}}{4}\right)}$$

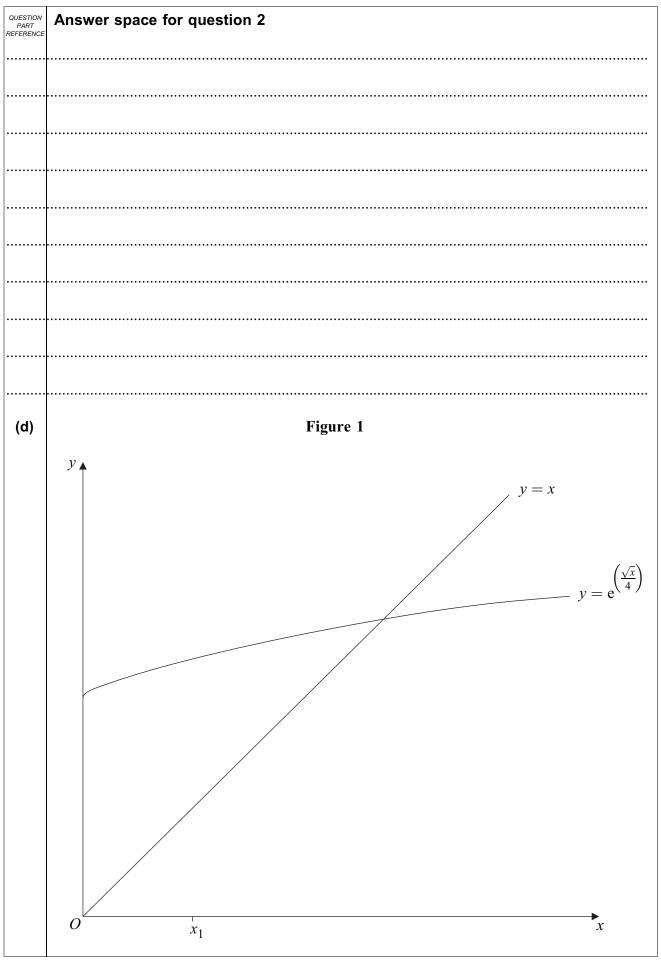
with $x_1 = 0.5$ to find the values of x_2 and x_3 , giving your answers to three decimal places. (2 marks)

(d) Figure 1, on the opposite page, shows a sketch of parts of the graphs of $y = e^{\left(\frac{\sqrt{x}}{4}\right)}$ and y = x, and the position of x_1 .

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis. (2 marks)

QUESTION PART REFERENCE	Answer space for question 2







3	A curve has equation $y = x^3 \ln x$.	
(a)	Find $\frac{dy}{dx}$.	(2 marks)

- (b) (i) Find an equation of the tangent to the curve $y = x^3 \ln x$ at the point on the curve where x = e.
 - (ii) This tangent intersects the x-axis at the point A. Find the exact value of the x-coordinate of the point A. (2 marks)

QUESTION PART REFERENCE	Answer space for question 3



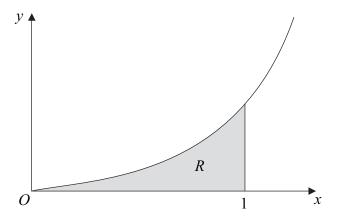
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4 (a) By using integration by parts, find $\int x e^{6x} dx$.

(4 marks)

(b) The diagram shows part of the curve with equation $y = \sqrt{x} e^{3x}$.



The shaded region R is bounded by the curve $y = \sqrt{x} e^{3x}$, the line x = 1 and the x-axis from x = 0 to x = 1.

Find the volume of the solid generated when the region R is rotated through 360° about the x-axis, giving your answer in the form $\pi(pe^6+q)$, where p and q are rational numbers. (3 marks)

QUESTION PART REFERENCE	Answer space for question 4



QUESTION PART REFERENCE	Answer space for question 4
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5	The	functions	f and	g are	defined	with	their	respective	domains	by

$$f(x) = \sqrt{2x - 5}, \quad \text{for } x \geqslant 2.5$$

$$g(x) = \frac{10}{x}$$
, for real values of x , $x \neq 0$

(a) State the range of f.

(2 marks)

(b) (i) Find fg(x).

(1 mark)

(ii) Solve the equation fg(x) = 5.

(2 marks)

- (c) The inverse of f is f^{-1} .
 - (i) Find $f^{-1}(x)$.

(3 marks)

(ii) Solve the equation $f^{-1}(x) = 7$.

(2 marks)

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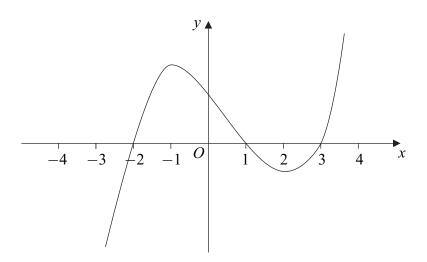
6	Use the substitution $u = x^4 + 2$ to find the value of $\int_0^1 \frac{x^7}{(x^4 + 2)^2} dx$, giving your answer in the form $p \ln q + r$, where p , q and r are rational numbers. (6 marks)
	answer in the form $p \ln q + r$, where p , q and r are rational numbers. (6 marks)
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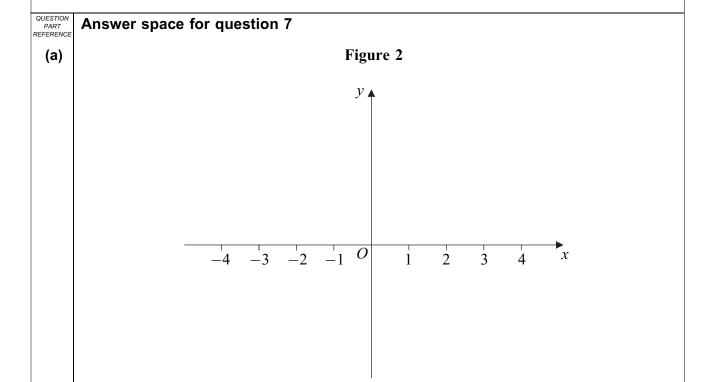
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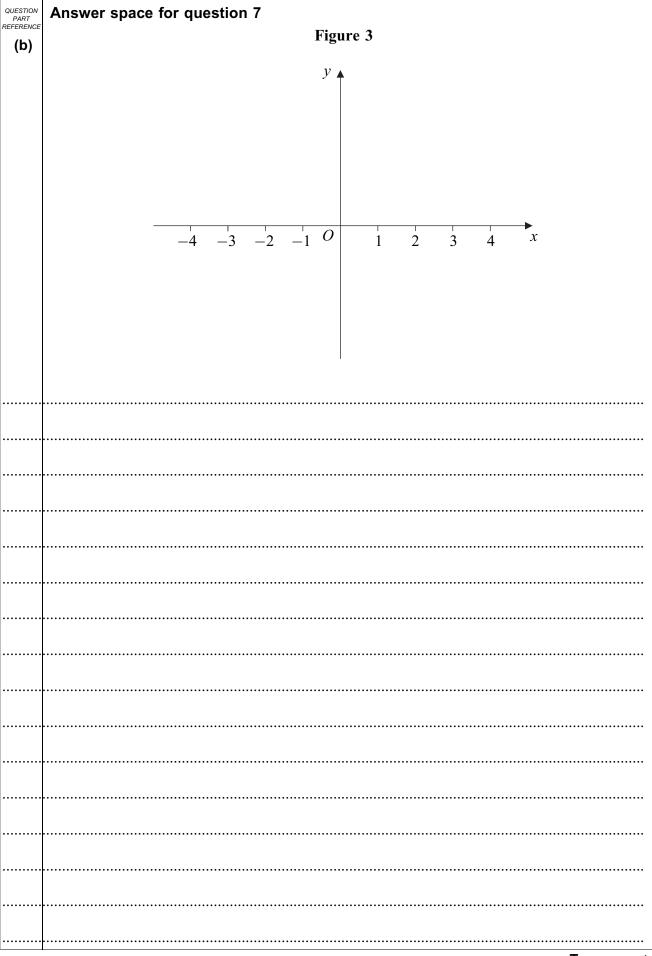


7 The sketch shows part of the curve with equation y = f(x).



- (a) On Figure 2 below, sketch the curve with equation y = |f(x)|. (3 marks)
- (b) On Figure 3 opposite, sketch the curve with equation y = f(|x|). (2 marks)
- (c) Describe a sequence of two geometrical transformations that maps the graph of y = f(x) onto the graph of $y = \frac{1}{2}f(x+1)$. (4 marks)
- (d) The maximum point of the curve with equation y = f(x) has coordinates (-1, 10). Find the coordinates of the maximum point of the curve with equation $y = \frac{1}{2}f(x+1)$.







8 (a) Show that the equation

$$\frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta} = 32$$

can be written in the form

$$\csc^2 \theta = 16$$
 (4 marks)

(b) Hence, or otherwise, solve the equation

$$\frac{1}{1 + \cos(2x - 0.6)} + \frac{1}{1 - \cos(2x - 0.6)} = 32$$

giving all values of x in radians to two decimal places in the interval $0 < x < \pi$.

(5 marks)

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9 (a) Given that $x = \frac{\sin y}{\cos y}$, use the quotient rule to show that

$$\frac{\mathrm{d}x}{\mathrm{d}y} = \sec^2 y \tag{3 marks}$$

(b) Given that $\tan y = x - 1$, use a trigonometrical identity to show that

$$\sec^2 y = x^2 - 2x + 2 \tag{2 marks}$$

(c) Show that, if $y = \tan^{-1}(x - 1)$, then

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x^2 - 2x + 2} \tag{1 mark}$$

- (d) A curve has equation $y = \tan^{-1}(x-1) \ln x$.
 - (i) Find the value of the x-coordinate of each of the stationary points of the curve.

(4 marks)

(ii) Find
$$\frac{d^2y}{dx^2}$$
. (2 marks)

(iii) Hence show that the curve has a minimum point which lies on the x-axis. (2 marks)

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	END OF QUESTIONS
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