General Certificate of Education
June 2008
Advanced Level Examination

MATHEMATICS **MS2B**
Unit Statistics 2B

Monday 2 June 2008 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MS2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.
- Unit Statistics 2B has a **written paper only**.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 It is thought that the incidence of asthma in children is associated with the volume of traffic in the area where they live.

Two surveys of children were conducted: one in an area where the volume of traffic was heavy and the other in an area where the volume of traffic was light.

For each area, the table shows the number of children in the survey who had asthma and the number who did not have asthma.

	Asthma	**No asthma**	**Total**
Heavy traffic	52	58	110
Light traffic	28	62	90
Total	80	120	200

(a) Use a χ^2 test, at the 5% level of significance, to determine whether the incidence of asthma in children is associated with the volume of traffic in the area where they live. *(8 marks)*

(b) Comment on the number of children in the survey who had asthma, given that they lived in an area where the volume of traffic was heavy. *(1 mark)*

2 (a) The number of telephone calls, X, received per hour for Dr Able may be modelled by a Poisson distribution with mean 6.

Determine $\mathrm{P}(X = 8)$. *(2 marks)*

(b) The number of telephone calls, Y, received per hour for Dr Bracken may be modelled by a Poisson distribution with mean λ and standard deviation 3.

(i) Write down the value of λ. *(1 mark)*

(ii) Determine $\mathrm{P}(Y > \lambda)$. *(2 marks)*

(c) (i) Assuming that X and Y are independent Poisson variables, write down the distribution of the **total** number of telephone calls received per hour for Dr Able and Dr Bracken. *(1 mark)*

(ii) Determine the probability that a total of at most 20 telephone calls will be received during any one-hour period. *(1 mark)*

(iii) The total number of telephone calls received during each of 6 one-hour periods is to be recorded. Calculate the probability that a total of at least 21 telephone calls will be received during exactly 4 of these one-hour periods. *(3 marks)*

3 Alan's company produces packets of crisps. The standard deviation of the weight of a packet of crisps is known to be 2.5 grams.

Alan believes that, due to the extra demand on the production line at a busy time of the year, the mean weight of packets of crisps is not equal to the target weight of 34.5 grams.

In an experiment set up to investigate Alan's belief, the weights of a random sample of 50 packets of crisps were recorded. The mean weight of this sample is 35.1 grams.

Investigate Alan's belief at the 5% level of significance. *(6 marks)*

4 The delay, in hours, of certain flights from Australia may be modelled by the continuous random variable T, with probability density function

$$f(t) = \begin{cases} \frac{2}{15}t & 0 \leqslant t \leqslant 3 \\ 1 - \frac{1}{5}t & 3 \leqslant t \leqslant 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the graph of f. *(3 marks)*

(b) Calculate:

(i) $P(T \leqslant 2)$; *(2 marks)*

(ii) $P(2 < T < 4)$. *(3 marks)*

(c) Determine $E(T)$. *(4 marks)*

5 The weight of fat in a digestive biscuit is known to be normally distributed.

Pat conducted an experiment in which she measured the weight of fat, x grams, in each of a random sample of 10 digestive biscuits, with the following results:

$$\sum x = 31.9 \qquad \text{and} \qquad \sum (x - \bar{x})^2 = 1.849$$

(a) (i) Construct a 99% confidence interval for the mean weight of fat in digestive biscuits. *(5 marks)*

(ii) Comment on a claim that the mean weight of fat in digestive biscuits is 3.5 grams. *(2 marks)*

(b) If 200 such 99% confidence intervals were constructed, how many would you expect **not** to contain the population mean? *(1 mark)*

Turn over ▶

6 The management of the Wellfit gym claims that the mean cholesterol level of those members who have held membership of the gym for more than one year is 3.8 .

A local doctor believes that the management's claim is too low and investigates by measuring the cholesterol levels of a random sample of 7 such members of the Wellfit gym, with the following results:

4.2 4.3 3.9 3.8 3.6 4.8 4.1

Is there evidence, at the 5% level of significance, to justify the doctor's belief that the mean cholesterol level is greater than the management's claim? State any assumption that you make. *(8 marks)*

7 (a) The number of text messages, N, sent by Peter each month on his mobile phone never exceeds 40.

When $0 \leqslant N \leqslant 10$, he is charged for 5 messages.
When $10 < N \leqslant 20$, he is charged for 15 messages.
When $20 < N \leqslant 30$, he is charged for 25 messages.
When $30 < N \leqslant 40$, he is charged for 35 messages.

The number of text messages, Y, that Peter is charged for each month has the following probability distribution:

y	5	15	25	35
$\mathbf{P}(Y = y)$	0.1	0.2	0.3	0.4

(i) Calculate the mean and the standard deviation of Y. *(4 marks)*

(ii) The Goodtime phone company makes a total charge for text messages, C pence, each month given by

$$C = 10Y + 5$$

Calculate $\mathrm{E}(C)$. *(1 mark)*

(b) The number of text messages, X, sent by Joanne each month on her mobile phone is such that

$$\mathrm{E}(X) = 8.35 \qquad \text{and} \qquad \mathrm{E}(X^2) = 75.25$$

The Newtime phone company makes a total charge for text messages, T pence, each month given by

$$T = 0.4X + 250$$

Calculate $\mathrm{Var}(T)$. *(4 marks)*

8 The continuous random variable X has cumulative distribution function

$$\mathrm{F}(x) = \begin{cases} 0 & x < -1 \\ \dfrac{x+1}{k+1} & -1 \leqslant x \leqslant k \\ 1 & x > k \end{cases}$$

where k is a positive constant.

(a) Find, in terms of k, an expression for $\mathrm{P}(X < 0)$. *(2 marks)*

(b) Determine an expression, in terms of k, for the lower quartile, q_1. *(3 marks)*

(c) Show that the probability density function of X is defined by

$$\mathrm{f}(x) = \begin{cases} \dfrac{1}{k+1} & -1 \leqslant x \leqslant k \\ 0 & \text{otherwise} \end{cases}$$ *(2 marks)*

(d) Given that $k = 11$:

(i) sketch the graph of f; *(2 marks)*

(ii) determine $\mathrm{E}(X)$ and $\mathrm{Var}(X)$; *(2 marks)*

(iii) show that $\mathrm{P}(q_1 < X < \mathrm{E}(X)) = 0.25$. *(2 marks)*

END OF QUESTIONS

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