

4753 (C3) Methods for Advanced Mathematics

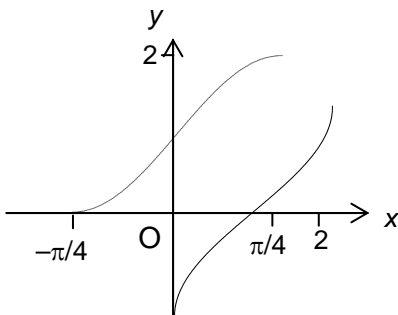
Section A

<p>1 $y = (1 + 6x^2)^{1/3}$ $\Rightarrow \frac{dy}{dx} = \frac{1}{3}(1 + 6x^2)^{-2/3} \cdot 12x$ $= 4x(1 + 6x^2)^{-2/3}$</p>	<p>M1 B1 A1 A1 [4]</p>	<p>chain rule used $\frac{1}{3}u^{-2/3}$ $\times 12x$ cao (must resolve $1/3 \times 12$) Mark final answer</p>
<p>2 (i) $fg(x) = f(x - 2)$ $= (x - 2)^2$ $gf(x) = g(x^2) = x^2 - 2.$</p>	<p>M1 A1 A1 [3]</p>	<p>forming a composite function mark final answer If fg and gf the wrong way round, M1A0A0</p>
<p>(ii) </p>	<p>B1ft B1ft [2]</p>	<p>fg – must have (2, 0) labelled (or inferable from scale). Condone no y-intercept, unless wrong gf – must have (0, -2) labelled (or inferable from scale) Condone no x-intercepts, unless wrong Allow ft only if fg and gf are correct but wrong way round.</p>
<p>3 (i) When $n = 1$, $10\,000 = A e^b$ when $n = 2$, $16\,000 = A e^{2b}$ $\Rightarrow \frac{16000}{10000} = \frac{Ae^{2b}}{Ae^b} = e^b$ $\Rightarrow e^b = 1.6$ $\Rightarrow b = \ln 1.6 = 0.470$ $A = 10000/1.6 = 6250.$</p>	<p>B1 B1 M1 E1 B1 B1 [6]</p>	<p>soi soi eliminating A (do not allow verification) SCB2 if initial 'B's are missing, and ratio of years = 1.6 $= e^b$ ln 1.6 or 0.47 or better (mark final answer) cao – allow recovery from inexact b's</p>
<p>(ii) When $n = 20$, $P = 6250xe^{0.470 \times 20}$ $= \text{£}75,550,000$</p>	<p>M1 A1 [2]</p>	<p>substituting $n = 20$ into their equation with their A and b Allow answers from £75 000 000 to £76 000 000.</p>
<p>4 (i) $5 = k/100 \Rightarrow k = 500^*$</p>	<p>E1 [1]</p>	<p>NB answer given</p>
<p>(ii) $\frac{dP}{dV} = -500V^{-2} = -\frac{500}{V^2}$</p>	<p>M1 A1 [2]</p>	<p>$(-1)V^{-2}$ o.e. – allow $-k/V^2$</p>
<p>(iii) $\frac{dP}{dt} = \frac{dP}{dV} \cdot \frac{dV}{dt}$ When $V = 100$, $dP/dV = -500/10000 = -0.05$ $dV/dt = 10$ $\Rightarrow dP/dt = -0.05 \times 10 = -0.5$ So P is decreasing at 0.5 Atm/s</p>	<p>M1 B1ft B1 A1 [4]</p>	<p>chain rule (any correct version) (soi) (soi) -0.5 cao</p>

<p>5(i) $p = 2, 2^p - 1 = 3$, prime $p = 3, 2^p - 1 = 7$, prime $p = 5, 2^p - 1 = 31$, prime $p = 7, 2^p - 1 = 127$, prime</p>	<p>M1 E1 [2]</p>	<p>Testing at least one prime testing all 4 primes (correctly) Must comment on answers being prime (allow ticks) Testing $p = 1$ is E0</p>
<p>(ii) $23 \times 89 = 2047 = 2^{11} - 1$ 11 is prime, 2047 is not So statement is false.</p>	<p>M1 E1 [2]</p>	<p>$2^{11} - 1$ must state or imply that 11 is prime ($p = 11$ is sufficient)</p>
<p>6 (i) $e^{2y} = x^2 + y$ $\Rightarrow 2e^{2y} \frac{dy}{dx} = 2x + \frac{dy}{dx}$ $\Rightarrow (2e^{2y} - 1) \frac{dy}{dx} = 2x$ $\Rightarrow \frac{dy}{dx} = \frac{2x}{2e^{2y} - 1} *$</p>	<p>M1 A1 M1 E1 [4]</p>	<p>Implicit differentiation – allow one slip (but with dy/dx both sides) collecting terms</p>
<p>(ii) Gradient is infinite when $2e^{2y} - 1 = 0$ $\Rightarrow e^{2y} = \frac{1}{2}$ $\Rightarrow 2y = \ln \frac{1}{2}$ $\Rightarrow y = \frac{1}{2} \ln \frac{1}{2} = -0.347$ (3 s.f.) $x^2 = e^{2y} - y = \frac{1}{2} - (-0.347)$ $= 0.8465$ $\Rightarrow x = 0.920$</p>	<p>M1 A1 M1 A1 [4]</p>	<p>must be to 3 s.f. substituting their y and solving for x cao – must be to 3 s.f., but penalise accuracy once only.</p>

Section B

<p>7(i) $y = 2x \ln(1+x)$ $\Rightarrow \frac{dy}{dx} = \frac{2x}{1+x} + 2 \ln(1+x)$ When $x = 0$, $dy/dx = 0 + 2 \ln 1 = 0$ \Rightarrow origin is a stationary point.</p>	M1 B1 A1 E1 [4]	product rule $d/dx(\ln(1+x)) = 1/(1+x)$ soi www (i.e. from correct derivative)
<p>(ii) $\frac{d^2y}{dx^2} = \frac{(1+x).2 - 2x.1}{(1+x)^2} + \frac{2}{1+x}$ $= \frac{2}{(1+x)^2} + \frac{2}{1+x}$ When $x = 0$, $d^2y/dx^2 = 2 + 2 = 4 > 0$ $\Rightarrow (0, 0)$ is a min point</p>	M1 A1ft A1 M1 E1 [5]	Quotient or product rule on their $2x/(1+x)$ correctly applied to their $2x/(1+x)$ o.e., e.g. $\frac{4+2x}{(1+x)^2}$ cao substituting $x = 0$ into their d^2y/dx^2 www – dep previous A1
<p>(iii) Let $u = 1+x \Rightarrow du = dx$ $\Rightarrow \int \frac{x^2}{1+x} dx = \int \frac{(u-1)^2}{u} du$ $= \int \frac{(u^2 - 2u + 1)}{u} du$ $= \int (u - 2 + \frac{1}{u}) du$ * $\Rightarrow \int_0^1 \frac{x^2}{1+x} dx = \int_1^2 (u - 2 + \frac{1}{u}) du$ $= \left[\frac{1}{2}u^2 - 2u + \ln u \right]_1^2$ $= 2 - 4 + \ln 2 - (\frac{1}{2} - 2 + \ln 1)$ $= \ln 2 - \frac{1}{2}$</p>	M1 E1 B1 B1 M1 A1 [6]	$\frac{(u-1)^2}{u}$ www (but condone du omitted except in final answer) changing limits (or substituting back for x and using 0 and 1) $\left[\frac{1}{2}u^2 - 2u + \ln u \right]$ substituting limits (consistent with u or x) cao
<p>(iv) $A = \int_0^1 2x \ln(1+x) dx$ Parts: $u = \ln(1+x)$, $du/dx = 1/(1+x)$ $dv/dx = 2x \Rightarrow v = x^2$ $= \left[x^2 \ln(1+x) \right]_0^1 - \int_0^1 \frac{x^2}{1+x} dx$ $= \ln 2 - \ln 2 + \frac{1}{2} = \frac{1}{2}$</p>	M1 A1 M1 A1 [4]	soi substituting their $\ln 2 - \frac{1}{2}$ for $\int_0^1 \frac{x^2}{1+x} dx$ cao

<p>8 (i) Stretch in x-direction s.f. $\frac{1}{2}$ translation in y-direction 1 unit up</p>	<p>M1 A1 M1 A1 [4]</p>	<p>(in either order) – allow ‘contraction’ dep ‘stretch’ allow ‘move’, ‘shift’, etc – direction can be inferred from $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ dep ‘translation’. $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ alone is M1 A0</p>
<p>(ii) $A = \int_{-\pi/4}^{\pi/4} (1 + \sin 2x) dx$ $= \left[x - \frac{1}{2} \cos 2x \right]_{-\pi/4}^{\pi/4}$ $= \pi/4 - \frac{1}{2} \cos \pi/2 + \pi/4 + \frac{1}{2} \cos (-\pi/2)$ $= \pi/2$</p>	<p>M1 B1 M1 A1 [4]</p>	<p>correct integral and limits. Condone dx missing; limits may be implied from subsequent working. substituting their limits (if zero lower limit used, must show evidence of substitution) or 1.57 or better – cao (www)</p>
<p>(iii) $y = 1 + \sin 2x$ $\Rightarrow dy/dx = 2\cos 2x$ When $x = 0$, $dy/dx = 2$ So gradient at $(0, 1)$ on $f(x)$ is 2 \Rightarrow gradient at $(1, 0)$ on $f^{-1}(x) = \frac{1}{2}$</p>	<p>M1 A1 A1ft B1ft [4]</p>	<p>differentiating – allow 1 error (but not $x + 2\cos 2x$) If 1, then must show evidence of using reciprocal, e.g. $1/1$</p>
<p>(iv) Domain is $0 \leq x \leq 2$.</p> 	<p>B1 M1 A1 [3]</p>	<p>Allow 0 to 2, but not $0 < x < 2$ or y instead of x clear attempt to reflect in $y = x$ correct domain indicated (0 to 2), and reasonable shape</p>
<p>(v) $y = 1 + \sin 2x \quad x \leftrightarrow y$ $x = 1 + \sin 2y$ $\Rightarrow \sin 2y = x - 1$ $\Rightarrow 2y = \arcsin(x - 1)$ $\Rightarrow y = \frac{1}{2} \arcsin(x - 1)$</p>	<p>M1 A1 [2]</p>	<p>or $\sin 2x = y - 1$ cao</p>