OCR
Oxford Cambridge and RSA

…day June 20XX – Morning/Afternoon

A Level Further Mathematics B (MEI)

Y434 Numerical Methods

SAMPLE MARK SCHEME

Duration: 1 hour 15 minutes

MAXIMUM MARK 60

This document consists of 12 pages

Text Instructions

1. **Annotations and abbreviations**

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for A Level Further Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.). When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for g. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some papers. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

k Anything in the mark scheme which is in square brackets […] is not required for the mark to be earned on this occasion, but shows what a complete solution might look like.

Question			Answer	Marks	AOs	Guidance	
1	(i)		x = 101 y = −100	**B1** **B1** **[2]**	**1.1** **1.1**		
1	(ii)		Substitution of 0.985 and 0.995 to obtain −200 < y < − 66.67 67.67 < x < 201 E.g. calculation involves subtraction of nearly equal numbers	 **B1** **B1** **E1** **[3]**	 **1.1** **1.1** **2.4**	 For either condition For either condition E.g. equations represent lines which are almost parallel, so a small change in gradient affects the point of intersection dramatically	 Or maximum possible error is 100% Or full argument based on determinants or gradients Do not allow general statements such as "the problem is ill-conditioned"
2	(i)		To determine whether 1 is replaced by 1.5	**B1** **[1]**	**2.4**	Accept explanation in terms of method	
2	(ii)		E2, C2	**B1** **[1]**	**2.2a**	In correct order	The resulting formula must be fully correct, so that it could be copied to produce the results in column C.
2	(iii)		$0.5 \times 0.5^n < 5 \times 10^{-7}$ soi Row 22	**M1** **A1** **[2]**	**3.1a** **1.1**	So n > 19.93 i.e. 20 cells further down	NB $\ln 10^{-6} \div \ln 0.5$ If **M0**, **B2** for correct answer www

Question			Answer	Marks	AOs	Guidance	
3	(i)		$\frac{1-\sinh 1}{1}$ soi $-0.175.., -6.7128\ldots, -61.442\ldots$ so fails	M1 A1 [2]	1.1 2.2a	At least 3 iterates BC	Alternative Method $\alpha \approx 1$ and substitution in $g'(x) = \frac{-x\cosh x - 1 + \sinh x}{x^2}$ $\lvert g'(x)\rvert \approx 1.368 > 1$ so iteration fails
3	(ii)		0.706199, 0.612353, 0.601606, 0.601403, 0.601402,.. 0.601402 cao	M1 A1 [2]	1.1 1.1	Must see at least 3 iterates BC	Answer only does not score
4	(i)		Evidence of correct use of formula h | 0.2 | 0.1 | 0.05 $\frac{dy}{dx}$ | 0.03594.. | 0.06447.. | 0.07058	M1 A1 A1 A1 [4]	1.1 1.1 1.1 1.1	Answers to 3 or more significant figures	
4	(ii)		0.07 is reasonable Accept any correct answer based on differences	B1 E1 [2]	1.1 2.4	E.g. The difference between successive approximations has reduced by roughly one fifth, so further reduction in h is unlikely to generate an answer larger than 0.075	

Question			Answer	Marks	AOs	Guidance	
5	(i)		[net] force is a linear function [of time]	B1 [1]	2.4		
5	(ii)		Uses values at 5, 10, 15 First differences: 5.875, 7.375 Second difference: 1.5 $5.125+\frac{(t-5)}{5}\times 5.875+\frac{(t-5)(t-10)}{5^2\times 2!}\times 1.5$ $v=0.03t^2+0.725t+0.75$	M1 M1 M1 A1 [4]	3.3 1.2 1.1 3.3	soi FT differences	
5	(iii)		f(12) = 13.77 Quite close to 14.000, so the model may be a reasonable approximation	B1 E1 [2]	1.1 3.5a		
5	(iv)		$\int_5^{15} v\,dt=\left[\frac{0.03t^3}{3}+\frac{0.725t^2}{2}+0.75t\right]_5^{15}$ 112.5	M1 A1 [2]	3.4 1.1	Attempt at integration must be seen.	
5	(v)		Lagrange's form of the interpolating polynomial	B1 [1]	3.5c	accept $\frac{(x-5)(x-10)(x-15)}{(12-5)(12-10)(12-15)}\times 14.000$	

Question			Answer	Marks	AOs	Guidance	
6	(i)		$\alpha = \frac{-1\times-1-0\times-1.38347}{-1--1.38347}$ $= 2.601636$ cos2.601636 = − 0.8577… and ln(− 0.8577…) is undefined so the method breaks down	M1 A1 A1 [3]	1.1 1.1 2.4		
6	(ii)		The values are approaching zero	B1 [1]	2.3		"getting smaller" is insufficient
6	(iii)		=(A2*D2-C2*B2)/(D2-B2) oe	B2 [2]	3.1a 1.1	**B1** if * omitted	must show application of secant method
6	(iv)	(A)	= E2 – E1 finds the differences between successive estimates of the root = F2/F1 finds the ratio of differences of these estimates = F2/F1^2 finds the ratio of differences of each estimate to the square of the previous estimate	B1 B1 B1 [3]	1.2 2.5 2.4		
6	(iv)	(B)	The values in column G suggest that convergence [of secant method] is faster than 1st order The values in column H suggest that the convergence [of secant method] is slower than 2nd order	B1 B1 [2]	2.2b 2.2b	 If **B0B0, SC1** for "neither 1st order nor 2nd order convergence"	
6	(v)		The values in E6, E7 and E8 are stored to greater accuracy than is displayed oe	B1 [1]	2.4		

Question			Answer	Marks	AOs	Guidance	
7	(i)		$1.9353662285 \leq I < 1.946821025$ The curve is concave down over this interval, so the Trapezium Rule generates an underestimate, and the Midpoint Rule generates an over-estimate	**B1** **E1** **[2]**	**2.2a** **2.4**		
7	(ii)	(A)	diffs ratio 0.12361679 0.359815 0.04447923 0.359412 0.01598637 $r \approx 0.36$ Generally r should be close to 0.25, as the Trapezium Rule is a second order method	**M1** **A1** **E1** **[3]**	**1.1** **1.1** **2.4**	Differences and attempt to find ratio	Condone minor arithmetic slips
7	(ii)	(B)	$1.93536623 + 0.01598637 \times \frac{0.36}{0.64} = 1.9443(58563)$ 1.94 This agrees with T_8 to two decimal places	**M1** **A1** **A1** **E1** **[4]**	**1.1** **1.1** **1.1** **3.2b**	**A0** for incomplete extrapolation	
7	(iii)		It is not appropriate. Extrapolation is an approximate method (in this case it has led to the values "swapping places".)	**B1** **B1** **[2]**	**2.3** **2.4**		
7	(iv)		$\frac{2 \times 1.95135259 + 1.91937986}{3} = 1.940695$	**M1** **A1** **[2]**	**1.1** **1.1**		

Question			Answer	Marks	AOs	Guidance	
7	(v)		$1.94424906 + 0.00003623 \times \frac{0.354}{1-0.354}$	M1	3.1a	Extrapolation using most accurate Simpson value	
				M1	1.1	Use of r = 0.354 or 0.35	
				M1	2.1	Extrapolation to infinity	
			= 1.944268919 to 1.944268568	A1	1.1		
			1.9443 looks secure, 1.94427 or 1.944269 is possible.	A1	3.2b		
			Sensible reasoning consistent with their answer.	E1	2.4	Accept argument based on comparison with 1.94424906 for 4 sf or on further improvement due to satisfactory demonstration of convergence of r for 5 or 6 sf	
				[6]			