

General Certificate of Education
June 2008
Advanced Level Examination



MATHEMATICS
Unit Mechanics 2B

MM2B

Friday 6 June 2008 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
 - the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.
- Unit Mechanics 2B has a **written paper only**.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 A particle moves in a straight line and at time t seconds has velocity $v \text{ m s}^{-1}$, where

$$v = 6t^2 + 4t - 7, \quad t \geq 0$$

- (a) Find an expression for the acceleration of the particle at time t . (2 marks)

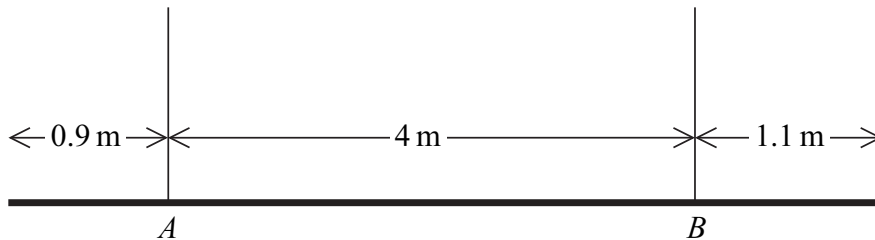
- (b) The mass of the particle is 3 kg.

Find the resultant force on the particle when $t = 4$. (2 marks)

- (c) When $t = 0$, the displacement of the particle from the origin is 5 metres.

Find an expression for the displacement of the particle from the origin at time t . (4 marks)

- 2 A uniform plank, of length 6 metres, has mass 40 kg. The plank is held in equilibrium in a horizontal position by two vertical ropes attached to the plank at A and B , as shown in the diagram.



- (a) Draw a diagram to show the forces acting on the plank. (1 mark)

- (b) Show that the tension in the rope attached to the plank at B is $21g \text{ N}$. (3 marks)

- (c) Find the tension in the rope that is attached to the plank at A . (2 marks)

- (d) State where in your solution you have used the fact that the plank is uniform. (1 mark)

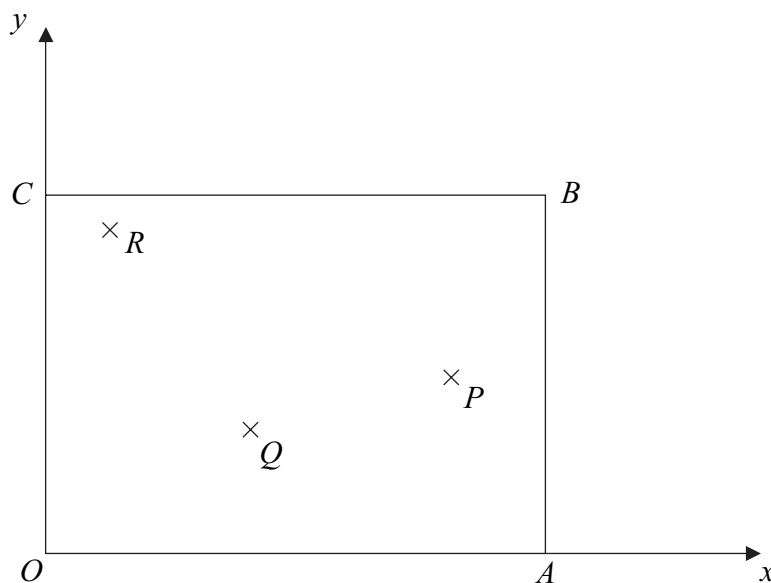
- 3 Three particles are attached to a light rectangular lamina $OABC$, which is fixed in a horizontal plane.

Take OA and OC as the x - and y -axes, as shown.

Particle P has mass 1 kg and is attached at the point $(25, 10)$.

Particle Q has mass 4 kg and is attached at the point $(12, 7)$.

Particle R has mass 5 kg and is attached at the point $(4, 18)$.



Find the coordinates of the centre of mass of the three particles.

(4 marks)

- 4 A van, of mass 1500 kg, has a maximum speed of 50 m s^{-1} on a straight horizontal road. When the van travels at a speed of $v \text{ m s}^{-1}$, it experiences a resistance force of magnitude $40v$ newtons.

(a) Show that the maximum power of the van is 100 000 watts. (2 marks)

(b) The van is travelling along a straight horizontal road.

Find the maximum possible acceleration of the van when its speed is 25 m s^{-1} .

(3 marks)

(c) The van starts to climb a hill which is inclined at 6° to the horizontal. Find the maximum possible constant speed of the van as it travels in a straight line up the hill.

(6 marks)

Turn over ►

- 5 A particle moves on a horizontal plane in which the unit vectors \mathbf{i} and \mathbf{j} are directed east and north respectively.

At time t seconds, the particle's position vector, \mathbf{r} metres, is given by

$$\mathbf{r} = 8\left(\cos\frac{1}{4}t\right)\mathbf{i} - 8\left(\sin\frac{1}{4}t\right)\mathbf{j}$$

- (a) Find an expression for the velocity of the particle at time t . (2 marks)
- (b) Show that the speed of the particle is a constant. (3 marks)
- (c) Prove that the particle is moving in a circle. (2 marks)
- (d) Find the angular speed of the particle. (2 marks)
- (e) Find an expression for the acceleration of the particle at time t . (2 marks)
- (f) State the magnitude of the acceleration of the particle. (1 mark)
- 6 A car, of mass m , is moving along a straight smooth horizontal road. At time t , the car has speed v . As the car moves, it experiences a resistance force of magnitude $0.05mv$. No other horizontal force acts on the car.

- (a) Show that

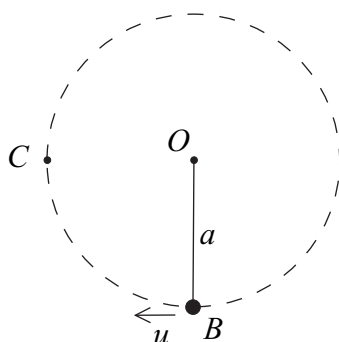
$$\frac{dv}{dt} = -0.05v \quad (1 \text{ mark})$$

- (b) When $t = 0$, the speed of the car is 20 m s^{-1} .

Show that $v = 20e^{-0.05t}$. (4 marks)

- (c) Find the time taken for the speed of the car to reduce to 10 m s^{-1} . (3 marks)

- 7 A small bead, of mass m , is suspended from a fixed point O by a light inextensible string, of length a . The bead is then set into circular motion with the string taut at B , where B is vertically below O , with a horizontal speed u .



- (a) Given that the string does not become slack, show that the least value of u required for the bead to make complete revolutions about O is $\sqrt{5ag}$. *(5 marks)*
- (b) In the case where $u = \sqrt{5ag}$, find, in terms of g and m , the tension in the string when the bead is at the point C , which is at the same horizontal level as O , as shown in the diagram. *(3 marks)*
- (c) State one modelling assumption that you have made in your solution. *(1 mark)*

Turn over for the next question

Turn over ►

- 8 (a) Hooke's law states that the tension in a stretched string of natural length l and modulus of elasticity λ is $\frac{\lambda x}{l}$ when its extension is x .

Using this formula, prove that the work done in stretching a string from an unstretched position to a position in which its extension is e is $\frac{\lambda e^2}{2l}$. (3 marks)

- (b) A particle, of mass 5 kg, is attached to one end of a light elastic string of natural length 0.6 metres and modulus of elasticity 150 N. The other end of the string is fixed to a point O .

- (i) Find the extension of the elastic string when the particle hangs in equilibrium directly below O . (2 marks)
- (ii) The particle is pulled down and held at the point P , which is 0.9 metres vertically below O .

Show that the elastic potential energy of the string when the particle is in this position is 11.25 J. (2 marks)

- (iii) The particle is released from rest at the point P . In the subsequent motion, the particle has speed $v \text{ m s}^{-1}$ when it is x metres **above** P .

Show that, while the string is taut,

$$v^2 = 10.4x - 50x^2 \quad (7 \text{ marks})$$

- (iv) Find the value of x when the particle comes to rest for the first time after being released, given that the string is still taut. (2 marks)

END OF QUESTIONS

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