

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

4753/1

Methods for Advanced Mathematics (C3)

Wednesday

25 MAY 2005

Afternoon

1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer all the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 72.

Section A (36 marks)

1 Solve the equation |3x + 2| = 1. [3]

2 Given that
$$\arcsin x = \frac{1}{6}\pi$$
, find x. Find $\arccos x$ in terms of π . [3]

3 The functions f(x) and g(x) are defined for the domain x > 0 as follows:

$$f(x) = \ln x, \quad g(x) = x^3.$$

Express the composite function fg(x) in terms of $\ln x$.

State the transformation which maps the curve
$$y = f(x)$$
 onto the curve $y = fg(x)$. [3]

4 The temperature $T^{\circ}C$ of a liquid at time t minutes is given by the equation

$$T = 30 + 20e^{-0.05t}$$
, for $t \ge 0$.

Write down the initial temperature of the liquid, and find the initial rate of change of temperature.

5 Using the substitution
$$u = 2x + 1$$
, show that
$$\int_0^1 \frac{x}{2x + 1} dx = \frac{1}{4} (2 - \ln 3).$$
 [6]

6 A curve has equation $y = \frac{x}{2+3 \ln x}$. Find $\frac{dy}{dx}$. Hence find the exact coordinates of the stationary point of the curve. [7]

7 Fig. 7 shows the curve defined implicitly by the equation

$$y^2 + y = x^3 + 2x,$$

together with the line x = 2.

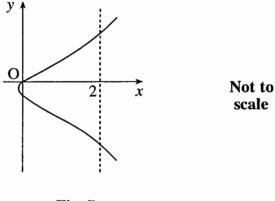


Fig. 7

Find the coordinates of the points of intersection of the line and the curve.

Find $\frac{dy}{dx}$ in terms of x and y. Hence find the gradient of the curve at each of these two points.

Section B (36 marks)

8 Fig. 8 shows part of the curve $y = x \sin 3x$. It crosses the x-axis at P. The point on the curve with x-coordinate $\frac{1}{6}\pi$ is Q.

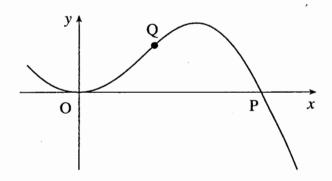


Fig. 8

(i) Find the x-coordinate of P. [3]

(ii) Show that Q lies on the line y = x. [1]

(iii) Differentiate $x \sin 3x$. Hence prove that the line y = x touches the curve at Q. [6]

(iv) Show that the area of the region bounded by the curve and the line y = x is $\frac{1}{72}(\pi^2 - 8)$. [7]

9 The function $f(x) = \ln(1 + x^2)$ has domain $-3 \le x \le 3$.

Fig. 9 shows the graph of y = f(x).

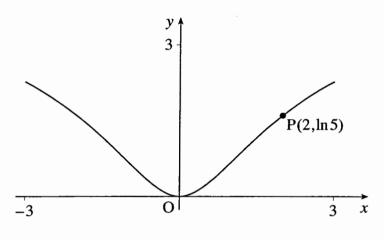


Fig. 9

(i) Show algebraically that the function is even. State how this property relates to the shape of the curve. [3]

(iii) Explain why the function does not have an inverse for the domain $-3 \le x \le 3$.

The domain of f(x) is now restricted to $0 \le x \le 3$. The inverse of f(x) is the function g(x).

(iv) Sketch the curves y = f(x) and y = g(x) on the same axes.

State the domain of the function g(x).

Show that
$$g(x) = \sqrt{e^x - 1}$$
. [6]

(v) Differentiate g(x). Hence verify that $g'(\ln 5) = 1\frac{1}{4}$. Explain the connection between this result and your answer to part (ii). [5]