

Mathematics (MEI)

Advanced Subsidiary GCE 4751

Introduction to Advanced Mathematics (C1)

Mark Scheme for June 2010

SECTION A

1	$y = 3x + c$ or $y - y_1 = 3(x - x_1)$ $y - 5 = \text{their } m(x - 4)$ o.e. $y = 3x - 7$ or simplified equiv.	M1 M1 A1	allow M1 for 3 clearly stated/ used as gradient of required line or (4, 5) subst in their $y = mx + c$; allow M1 for $y - 5 = m(x - 4)$ o.e. condone $y = 3x + c$ and $c = -7$ or B3 www
2	(i) $250a^6b^7$ (ii) 16 cao (iii) 64	2 1 2	B1 for two elements correct; condone multiplication signs left in SC1 for eg $250 + a^6 + b^7$ condone ± 64 M1 for $[\pm]4^3$ or for $\sqrt{4096}$ or for only -64
3	$ac = \sqrt{y} - 5$ o.e. $ac + 5 = \sqrt{y}$ o.e. $[y =](ac + 5)^2$ o.e. isw	M1 M1 M1	M1 for each of 3 correct or ft correct steps s.o.i. leading to y as subject or some/all steps may be combined; allow B3 for $[y =](ac + 5)^2$ o.e. isw or B2 if one error
4 (i)	$2 - 2x > 6x + 5$ $-3 > 8x$ o.e. or ft $x < -3/8$ o.e. or ft isw	M1 M1 M1	or $1 - x > 3x + 2.5$ for collecting terms of their inequality correctly on opposite sides eg $-8x > 3$ allow B3 for correct inequality found after working with equation allow SC2 for $-3/8$ o.e. found with equation or wrong inequality
4 (ii)	$-4 < x < 1/2$ o.e.	2	accept as two inequalities M1 for one 'end' correct or for -4 and $1/2$
5 (i)	$7\sqrt{3}$	2	M1 for $\sqrt{48} = 4\sqrt{3}$ or $\sqrt{27} = 3\sqrt{3}$

5 (ii)	$\frac{10+15\sqrt{2}}{7}$ www isw	3	B1 for 7 [B0 for 7 wrongly obtained] and B2 for $10+15\sqrt{2}$ or B1 for one term of numerator correct; if B0 , then M1 for attempt to multiply num and denom by $3+\sqrt{2}$
6	$5 + 2k$ soi $k = 12$ attempt at $f(3)$ $27 + 36 + m = 59$ o.e. $m = -4$ cao	M1 A1 M1 A1 A1	allow M1 for expansion with $5x^3 + 2kx^3$ and no other x^3 terms or M1 for $(29 - 5) / 2$ soi must substitute 3 for x in cubic not product or long division as far as obtaining $x^2 + 3x$ in quotient or from division $m - (-63) = 59$ o.e. or for $27 + 3k + m = 59$ or ft their k
7	$1 + 2x + \frac{3}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{16}x^4$ oe (must be simplified) isw	4	B3 for 4 terms correct, or B2 for 3 terms correct or for all correct but unsimplified (may be at an earlier stage, but factorial or nC_r notation must be expanded/worked out) or B1 for 1, 4, 6, 4, 1 soi or for $1 + \dots + \frac{1}{16}x^4$ [must have at least one other term]
8	$5(x + 2)^2 - 14$	4	B1 for $a = 5$, and B1 for $b = 2$ and B2 for $c = -14$ or M1 for $c = 6$ - their ab^2 or M1 for [their a](6/their a - their b^2) [no ft for $a = 1$]
9	mention of -5 as a square root of 25 or $(-5)^2 = 25$ $-5 - 5 \neq 0$ o.e. or $x + 5 = 0$	M1 M1	condone $-5^2 = 25$ or, dep on first M1 being obtained, allow M1 for showing that 5 is the only soln of $x - 5 = 0$ allow M2 for $x^2 - 25 = 0$ $(x + 5)(x - 5) [= 0]$ so $x - 5 = 0$ or $x + 5 = 0$

SECTION B

10 (i)	$(2x - 3)(x + 1)$ $x = 3/2$ and -1 obtained	M2 B1	M1 for factors with one sign error or giving two terms correct allow M1 for $2(x - 1.5)(x + 1)$ with no better factors seen or ft their factors
10 (ii)	graph of quadratic the correct way up and crossing both axes crossing x -axis only at $3/2$ and -1 or ft from their roots in (i), or their factors if roots not given crossing y -axis at -3	B1 B1 B1	for $x = 3/2$ condone 1 and 2 marked on axis and crossing roughly halfway between; intns must be shown labelled or worked out nearby
10 (iii)	use of $b^2 - 4ac$ with numbers subst (condone one error in substitution) (may be in quadratic formula) $25 - 40 < 0$ or -15 obtained	M1 A1	may be in formula or $(x - 2.5)^2 = 6.25 - 10$ or $(x - 2.5)^2 + 3.75 = 0$ oe (condone one error) or $\sqrt{-15}$ seen in formula or $(x - 2.5)^2 = -3.75$ oe or $x = 2.5 \pm \sqrt{-3.75}$ oe
10 (iv)	$2x^2 - x - 3 = x^2 - 5x + 10$ o.e. $x^2 + 4x - 13 [= 0]$ use of quad. formula on resulting eqn (do not allow for original quadratics used) $-2 \pm \sqrt{17}$ cao	M1 M1 M1 A1	attempt at eliminating y by subst or subtraction or $(x + 2)^2 = 17$; for rearranging to form $ax^2 + bx + c [= 0]$ or to completing square form condone one error for each of 2 nd and 3 rd M1s or $x + 2 = \pm\sqrt{17}$ o.e. 2 nd and 3 rd M1s may be earned for good attempt at completing square as far as roots obtained

<p>11 (i)</p>	<p>grad AB = $\frac{1-3}{5-(-1)}$ [= -1/3] $y - 3 =$ their grad $(x - (-1))$ or $y - 1 =$ their grad $(x - 5)$</p> <p>$y = -1/3x + 8/3$ or $3y = -x + 8$ o.e isw</p>	<p>M1</p> <p>M1</p> <p>A1</p>	<p>or use of $y =$ their gradient $x + c$ with coords of A or B or M2 for $\frac{y-3}{1-3} = \frac{x-(-1)}{5-(-1)}$ o.e.</p> <p>o.e. eg $x + 3y - 8 = 0$ or $6y = 16 - 2x$ allow B3 for correct eqn www</p>
<p>11 (ii)</p>	<p>when $y = 0, x = 8$; when $x = 0,$ $y = 8/3$ or ft their (i)</p> <p>[Area =] $\frac{1}{2} \times 8/3 \times 8$ o.e. cao isw</p>	<p>M1</p> <p>M1</p>	<p>allow $y = 8/3$ used without explanation if already seen in eqn in (i)</p> <p>M1 NB answer $32/3$ given; allow $4 \times 8/3$ if first M1 earned; or M1 for $\int_0^8 \left[\frac{1}{3}(8-x) \right] dx = \left[\frac{1}{3} \left(8x - \frac{1}{2}x^2 \right) \right]_0^8$ and M1 dep for $\frac{1}{3}(64 - 32[-0])$</p>
<p>11 (iii)</p>	<p>grad perp = $-1/\text{grad AB}$ stated, or used after their grad AB stated in this part</p> <p>midpoint [of AB] = (2, 2)</p> <p>$y - 2 =$ their grad perp $(x - 2)$ or ft their midpoint</p> <p><u>alt method working back from ans:</u></p> <p>grad perp = $-1/\text{grad AB}$ and showing/stating same as given line</p> <p>finding intn of their $y = -1/3x - 8/3$ and $y = 3x - 4$ is (2, 2)</p> <p>showing midpt of AB is (2, 2)</p>	<p>M1</p> <p>M1</p> <p>M1</p> <p>or</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>M1</p>	<p>or showing $3 \times -1/3 = -1$ if (i) is wrong, allow the first M1 here ft, provided the answer is correct ft</p> <p>must state ‘midpoint’ or show working</p> <p>for M3 this must be correct, starting from grad AB = $-1/3$, and also needs correct completion to given ans $y = 3x - 4$</p> <p>mark one method or the other, to benefit of candidate, not a mixture</p> <p>eg stating $-1/3 \times 3 = -1$</p> <p>or showing that (2, 2) is on $y = 3x - 4,$ having found (2, 2) first</p> <p>[for both methods: for M3 must be fully correct]</p>

11 (iv)	subst $x = 3$ into $y = 3x - 4$ and obtaining centre = (3, 5) $r^2 = (5 - 3)^2 + (1 - 5)^2$ o.e. $r = \sqrt{20}$ o.e. cao eqn is $(x - 3)^2 + (y - 5)^2 = 20$ or ft their r and y -coord of centre	M1 M1 A1 B1	or using $(-1 - 3)^2 + (3 - b)^2 = (5 - 3)^2 + (1 - b)^2$ and finding (3, 5) or $(-1 - 3)^2 + (3 - 5)^2$ or ft their centre using A or B condone $(x - 3)^2 + (y - b)^2 = r^2$ o.e. or $(x - 3)^2 + (y - \text{their } 5)^2 = r^2$ o.e. (may be seen earlier)
12 (i)	trials of at calculating $f(x)$ for at least one factor of 30 details of calculation for $f(2)$ or $f(-3)$ or $f(-5)$ attempt at division by $(x - 2)$ as far as $x^3 - 2x^2$ in working correctly obtaining $x^2 + 8x + 15$ factorising a correct quadratic factor $(x - 2)(x + 3)(x + 5)$	M1 A1 M1 A1 M1 A1	M0 for division or inspection used or equiv for $(x + 3)$ or $(x + 5)$; or inspection with at least two terms of quadratic factor correct or B2 for another factor found by factor theorem for factors giving two terms of quadratic correct; M0 for formula without factors found condone omission of first factor found; ignore '= 0' seen allow last four marks for $(x - 2)(x + 3)(x + 5)$ obtained; for all 6 marks must see factor theorem use first
12 (ii)	sketch of cubic right way up, with two turning points values of intns on x axis shown, correct $(-5, -3, \text{ and } 2)$ or ft from their factors/ roots in (i) y -axis intersection at -30	B1 B1 B1	0 if stops at x -axis on graph or nearby in this part mark intent for intersections with both axes or $x = 0, y = -30$ seen in this part if consistent with graph drawn

12	(iii)	<p>$(x - 1)$ substituted for x in either form of eqn for $y = f(x)$</p>	M1	<p>correct or ft their (i) or (ii) for factorised form; condone one error; allow for new roots stated as $-4, -2$ and 3 or ft</p>
		<p>$(x - 1)^3$ expanded correctly (need not be simplified) or two of their factors multiplied correctly</p>	M1 dep	<p>or M1 for correct or correct ft multiplying out of all 3 brackets at once, condoning one error [$x^3 - 3x^2 + 4x^2 + 2x^2 + 8x - 6x - 12x - 24$]</p>
		<p>correct completion to given answer [condone omission of 'y =']</p>	M1	<p>unless all 3 brackets already expanded, must show at least one further interim step allow SC1 for $(x + 1)$ subst <u>and</u> correct exp of $(x + 1)^3$ or two of their factors ft</p> <p><u>or</u>, for those using given answer: M1 for roots stated or used as $-4, -2$ and 3 or ft A1 for showing all 3 roots satisfy given eqn B1 for comment re coefft of x^3 or product of roots to show that eqn of translated graph is not a multiple of RHS of given eqn</p>

Section B Total: 36