Oxford Cambridge and RSA

Wednesday 12 June 2019 – Morning

A Level Mathematics B (MEI)

H640/02 Pure Mathematics and Statistics

Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **12** pages.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient Rule $y = \dfrac{u}{v},\ \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h\to 0}\frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \tfrac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$ **or** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r p^r q^{n-r}$ where $q = 1 - p$

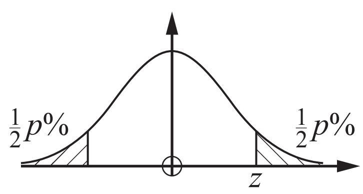
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line	Motion in two dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u + v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$
$v^2 = u^2 + 2as$	
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions

Section A (22 marks)

1 Fig. 1 shows the probability distribution of the discrete random variable X.

x	1	2	3	4	5
$\mathrm{P}(X=x)$	0.2	0.1	k	$2k$	$4k$

Fig. 1

(a) Find the value of k. **[2]**

(b) Find $\mathrm{P}(X \neq 4)$. **[2]**

2 Given that $y=(x^2+5)^{12}$,

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. **[2]**

(b) Hence find $\int 48x(x^2+5)^{11}\,\mathrm{d}x$. **[2]**

3 Fig. 3 shows the time Lorraine spent in hours, t, answering e-mails during the working day. The data were collected over a number of months.

Time in hours, t	$0 \leqslant t < 1$	$1 \leqslant t < 2$	$2 \leqslant t < 3$	$3 \leqslant t < 4$	$4 \leqslant t < 6$	$6 \leqslant t < 8$
Number of days	28	36	42	31	24	12

Fig. 3

(a) Calculate an estimate of the mean time per day that Lorraine spent answering e-mails over this period. **[1]**

(b) Explain why your answer to part **(a)** is an estimate. **[1]**

When Lorraine accepted her job, she was told that the mean time per day spent answering e-mails would not be more than 3 hours.

(c) Determine whether, according to the data in Fig. 3, it is possible that the mean time per day Lorraine spends answering e-mails is in fact more than 3 hours. **[1]**

4 Fig. 4 shows the graph of $y = \sqrt{1+x^3}$.

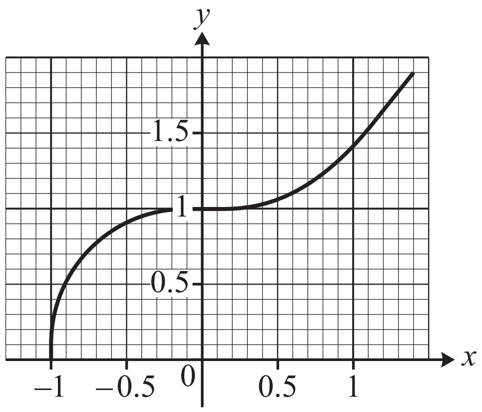


Fig. 4

(a) Use the trapezium rule with $h = 0.5$ to find an estimate of $\int_{-1}^{0} \sqrt{1+x^3}\,dx$, giving your answer correct to 6 decimal places. **[3]**

(b) State whether your answer to part **(a)** is an under-estimate or an over-estimate, justifying your answer. **[1]**

5 Fig. 5 shows the number of times that students at a sixth form college visited a recreational mathematics website during the first week of the summer term.

Number of visits to website	0	1	2	3	4	5
Number of students	24	38	17	12	4	2

Fig. 5

(a) State the value of the mid-range of the data. **[1]**

(b) Describe the shape of the distribution. **[1]**

(c) State the value of the mode. **[1]**

6 Find $\int \frac{32}{x^5} \ln x \, dx$. **[4]**

Answer **all** the questions

Section B (78 marks)

7 The area of a sector of a circle is $36.288\,\text{cm}^2$. The angle of the sector is θ radians and the radius of the circle is $r\,\text{cm}$.

(a) Find an expression for θ in terms of r. **[1]**

The perimeter of the sector is $24.48\,\text{cm}$.

(b) Show that $\theta = \dfrac{24.48}{r} - 2$. **[1]**

(c) Find the possible values of r. **[3]**

8 A team called "The Educated Guess" enter a weekly quiz. If they win the quiz in a particular week, the probability that they will win the following week is 0.4, but if they do not win, the probability that they will win the following week is 0.2.

In week 4 The Educated Guess won the quiz.

(a) Calculate the probability that The Educated Guess will win the quiz in week 6. **[3]**

Every week the same 20 quiz teams, each with 6 members, take part in a quiz. Every member of every team buys a raffle ticket. Five winning tickets are drawn randomly, without replacement. Alf, who is a member of one of the teams, takes part every week.

(b) Calculate the probability that, in a randomly chosen week, Alf wins a raffle prize. **[2]**

(c) Find the smallest number of weeks after which it will be 95% certain that Alf has won at least one raffle prize. **[4]**

9 You are given that

$\mathrm{f}(x) = 2x + 3 \quad$ for $x < 0$ and

$\mathrm{g}(x) = x^2 - 2x + 1 \quad$ for $x > 1$.

(a) Find $\mathrm{gf}(x)$, stating the domain. **[3]**

(b) State the range of $\mathrm{gf}(x)$. **[1]**

(c) Find $(\mathrm{gf})^{-1}(x)$. **[5]**

10 Club 65–80 Holidays fly jets between Liverpool and Magaluf. Over a long period of time records show that half of the flights from Liverpool to Magaluf take less than 153 minutes and 5% of the flights take more than 183 minutes.

An operations manager believes that flight times from Liverpool to Magaluf may be modelled by the Normal distribution.

(a) Use the information above to write down the mean time the operations manager will use in his Normal model for flight times from Liverpool to Magaluf. **[1]**

(b) Use the information above to find the standard deviation the operations manager will use in his Normal model for flight times from Liverpool to Magaluf, giving your answer correct to 1 decimal place. **[3]**

(c) Data is available for 452 flights. A flight time of under 2 hours was recorded in 16 of these flights. Use your answers to parts **(a)** and **(b)** to determine whether the model is consistent with this data. **[3]**

The operations manager suspects that the mean time for the journey from Magaluf to Liverpool is less than from Liverpool to Magaluf. He collects a random sample of 24 flight times from Magaluf to Liverpool. He finds that the mean flight time is 143.6 minutes.

(d) Use the Normal model used in part **(c)** to conduct a hypothesis test to determine whether there is evidence at the 1% level to suggest that the mean flight time from Magaluf to Liverpool is less than the mean flight time from Liverpool to Magaluf. **[7]**

(e) Identify two ways in which the Normal model for flight times from Liverpool to Magaluf might be adapted to provide a better model for the flight times from Magaluf to Liverpool. **[2]**

11 Fig. 11 shows the graph of $y = x^2 - 4x + x\ln x$.

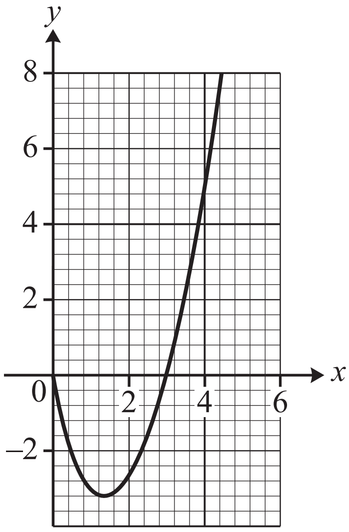


Fig. 11

(a) Show that the x-coordinate of the stationary point on the curve may be found from the equation $2x - 3 + \ln x = 0$. **[4]**

(b) Use an iterative method to find the x-coordinate of the stationary point on the curve $y = x^2 - 4x + x\ln x$, giving your answer correct to 4 decimal places. **[4]**

12 The jaguar is a species of big cat native to South America. Records show that 6% of jaguars are born with black coats. Jaguars with black coats are known as black panthers. Due to deforestation a population of jaguars has become isolated in part of the Amazon basin. Researchers believe that the percentage of black panthers may not be 6% in this population.

(a) Find the minimum sample size needed to conduct a two-tailed test to determine whether there is any evidence at the 5% level to suggest that the percentage of black panthers is not 6%. **[3]**

A research team identifies 70 possible sites for monitoring the jaguars remotely. 30 of these sites are randomly selected and cameras are installed. 83 different jaguars are filmed during the evidence gathering period. The team finds that 10 of the jaguars are black panthers.

(b) Conduct a hypothesis test to determine whether the information gathered by the research team provides any evidence at the 5% level to suggest that the percentage of black panthers in this population is not 6%. **[7]**

13 The population of Melchester is 185 207. During a nationwide flu epidemic the number of new cases in Melchester are recorded each day. The results from the first three days are shown in Fig. 13.

Day	1	2	3
Number of new cases	8	24	72

Fig. 13

A doctor notices that the numbers of new cases on successive days are in geometric progression.

(a) Find the common ratio for this geometric progression. **[1]**

The doctor uses this geometric progression to model the number of new cases of flu in Melchester.

(b) According to the model, how many new cases will there be on day 5? **[1]**

(c) Find a formula for the **total** number of cases from day 1 to day n inclusive according to this model, simplifying your answer. **[1]**

(d) Determine the maximum number of days for which the model could be viable in Melchester. **[3]**

(e) State, with a reason, whether it is likely that the model will be viable for the number of days found in part **(d)**. **[1]**

14 The pre-release material includes data concerning crude death rates in different countries of the world. Fig. 14.1 shows some information concerning crude death rates in countries in Europe and in Africa.

	Europe	Africa
n	48	56
minimum	6.28	3.58
lower quartile	8.50	7.31
median	9.53	8.71
upper quartile	11.41	11.93
maximum	14.46	14.89

Fig. 14.1

(a) Use your knowledge of the large data set to suggest a reason why the statistics in Fig. 14.1 refer to only 48 of the 51 European countries. **[1]**

(b) Use the information in Fig. 14.1 to show that there are no outliers in either data set. **[3]**

The crude death rate in Libya is recorded as 3.58 and the population of Libya is recorded as 6 411 776.

(c) Calculate an estimate of the number of deaths in Libya in a year. **[1]**

The median age in Germany is 46.5 and the crude death rate is 11.42. The median age in Cyprus is 36.1 and the crude death rate is 6.62.

(d) Explain why a country like Germany, with a higher median age than Cyprus, might also be expected to have a higher crude death rate than Cyprus. **[1]**

Fig. 14.2 shows a scatter diagram of median age against crude death rate for countries in Africa and Fig. 14.3 shows a scatter diagram of median age against crude death rate for countries in Europe.

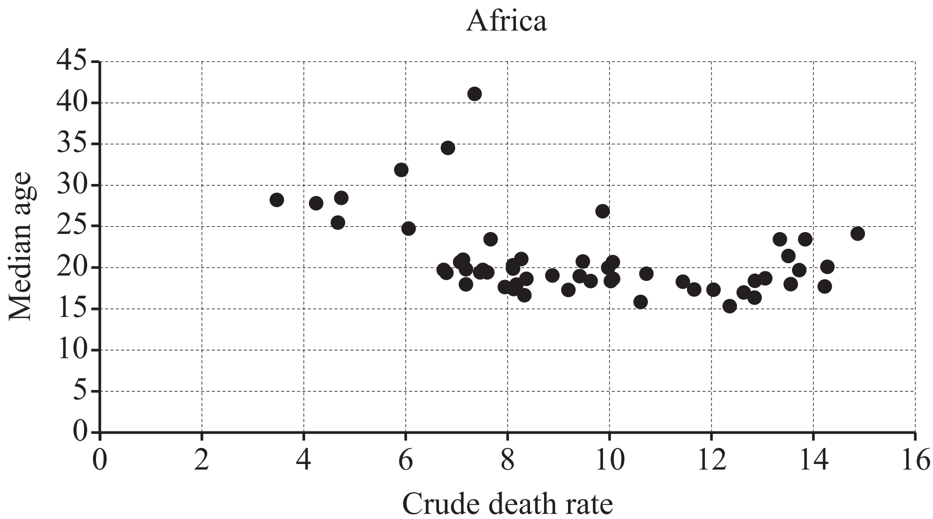


Fig. 14.2

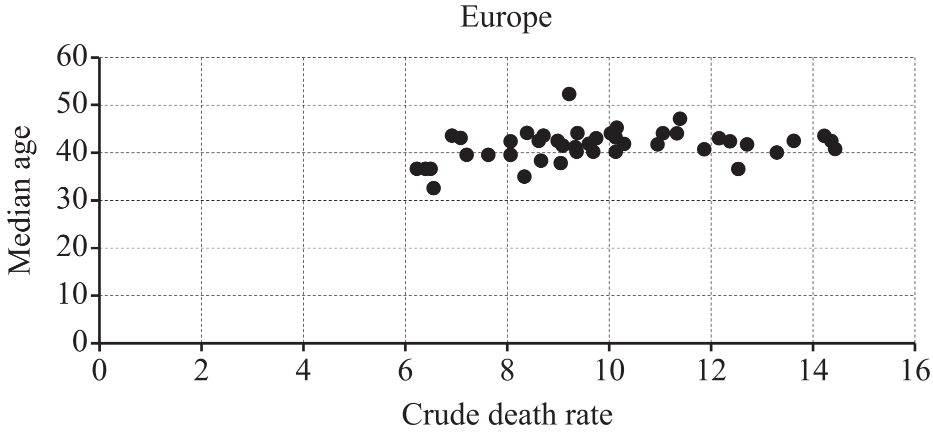


Fig. 14.3

The rank correlation coefficient for the data shown in Fig. 14.2 is $-0.281\,206$.

The rank correlation coefficient for the data shown in Fig. 14.3 is $0.335\,215$.

(e) Compare and contrast what may be inferred about the relationship between median age and crude death rate in countries in Africa and in countries in Europe. **[2]**

15 **You must show detailed reasoning in this question.**

The screenshot in Fig. 15 shows the probability distribution for the continuous random variable X, where $X \sim \mathrm{N}(\mu, \sigma^2)$.

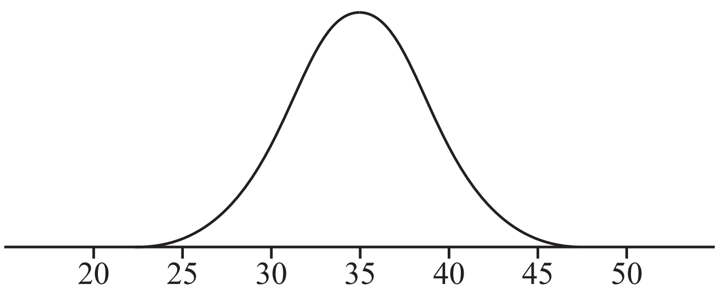


Fig. 15

The distribution is symmetrical about the line $x = 35$ and there is a point of inflection at $x = 31$.

Fifty independent readings of X are made. Show that the probability that at least 45 of these readings are between 30 and 40 is less than 0.05. **[6]**

END OF QUESTION PAPER