OCR RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4754A

Applications of Advanced Mathematics (C4) Paper A

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Tuesday 13 January 2009
Morning

Duration: 1 hour 30 minutes

4754A

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

Section A (36 marks)

1 Express $\dfrac{3x+2}{x(x^2+1)}$ in partial fractions. **[6]**

2 Show that $(1+2x)^{\frac{1}{3}} = 1 + \frac{2}{3}x - \frac{4}{9}x^2 + \ldots$, and find the next term in the expansion.

State the set of values of x for which the expansion is valid. **[6]**

3 Vectors **a** and **b** are given by $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

Find constants λ and μ such that $\lambda\mathbf{a} + \mu\mathbf{b} = 4\mathbf{j} - 3\mathbf{k}$. **[5]**

4 Prove that $\cot\beta - \cot\alpha = \dfrac{\sin(\alpha-\beta)}{\sin\alpha\sin\beta}$. **[3]**

5 **(i)** Write down normal vectors to the planes $2x - y + z = 2$ and $x - z = 1$.

Hence find the acute angle between the planes. **[4]**

(ii) Write down a vector equation of the line through $(2, 0, 1)$ perpendicular to the plane $2x - y + z = 2$. Find the point of intersection of this line with the plane. **[4]**

6 **(i)** Express $\cos\theta + \sqrt{3}\sin\theta$ in the form $R\cos(\theta-\alpha)$, where $R > 0$ and α is acute, expressing α in terms of π. **[4]**

(ii) Write down the derivative of $\tan\theta$.

Hence show that $\displaystyle\int_0^{\frac{1}{3}\pi} \frac{1}{(\cos\theta + \sqrt{3}\sin\theta)^2}\,d\theta = \frac{\sqrt{3}}{4}$. **[4]**

Section B (36 marks)

7 Scientists can estimate the time elapsed since an animal died by measuring its body temperature.

(i) Assuming the temperature goes down at a constant rate of 1.5 degrees Fahrenheit per hour, estimate how long it will take for the temperature to drop

(A) from 98 °F to 89 °F,

(B) from 98 °F to 80 °F. **[2]**

In practice, rate of temperature loss is not likely to be constant. A better model is provided by Newton's law of cooling, which states that the temperature θ in degrees Fahrenheit t hours after death is given by the differential equation

$$\frac{d\theta}{dt} = -k(\theta - \theta_0),$$

where θ_0 °F is the air temperature and k is a constant.

(ii) Show by integration that the solution of this equation is $\theta = \theta_0 + Ae^{-kt}$, where A is a constant. **[5]**

The value of θ_0 is 50, and the initial value of θ is 98. The initial rate of temperature loss is 1.5 °F per hour.

(iii) Find A, and show that $k = 0.031\,25$. **[4]**

(iv) Use this model to calculate how long it will take for the temperature to drop

(A) from 98 °F to 89 °F,

(B) from 98 °F to 80 °F. **[5]**

(v) Comment on the results obtained in parts **(i)** and **(iv)**. **[1]**

[Question 8 is printed overleaf.]

8 Fig. 8 illustrates a hot air balloon on its side. The balloon is modelled by the volume of revolution about the x-axis of the curve with parametric equations

$$x = 2 + 2\sin\theta, \quad y = 2\cos\theta + \sin 2\theta, \quad (0 \leqslant \theta \leqslant 2\pi).$$

The curve crosses the x-axis at the point A (4, 0). B and C are maximum and minimum points on the curve. Units on the axes are metres.

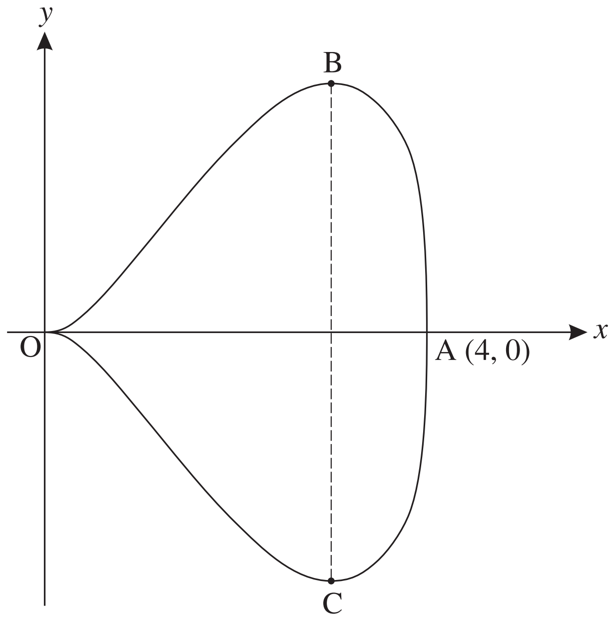

Fig. 8

(i) Find $\dfrac{dy}{dx}$ in terms of θ. **[4]**

(ii) Verify that $\dfrac{dy}{dx} = 0$ when $\theta = \frac{1}{6}\pi$, and find the exact coordinates of B.

Hence find the maximum width BC of the balloon. **[5]**

(iii) (A) Show that $y = x\cos\theta$.

(B) Find $\sin\theta$ in terms of x and show that $\cos^2\theta = x - \frac{1}{4}x^2$.

(C) Hence show that the cartesian equation of the curve is $y^2 = x^3 - \frac{1}{4}x^4$. **[7]**

(iv) Find the volume of the balloon. **[3]**

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RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4754B**

Applications of Advanced Mathematics (C4) Paper B: Comprehension

Candidates answer on the question paper

OCR Supplied Materials:
- Insert (inserted)
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Rough paper

Tuesday 13 January 2009
Morning

Duration: Up to 1 hour

Candidate Forename		Candidate Surname	
Centre Number		Candidate Number	

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the boxes above.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Write your answer to each question in the space provided, however additional paper may be used if necessary.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- The insert contains the text for use with the questions.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with your question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **18**.
- This document consists of **4** pages. Any blank pages are indicated.

Examiner's Use Only:	
1	
2	
3	
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5	
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Total	

For Examiner's Use

1 Show how the value $d = 8$ on line 32 is obtained. **[2]**

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2 Using the information given on lines 38 and 39, derive equation (1). **[3]**

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3 On lines 43 and 44 it is suggested that the volume of fuel in the tank in Figs. 2.1 and 2.2 could be calculated using the values of h and θ.

Calculate the volume of fuel in the case where $h = 5$ and $\theta = 30°$. **[3]**

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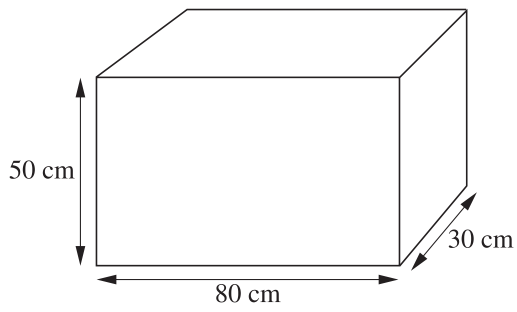
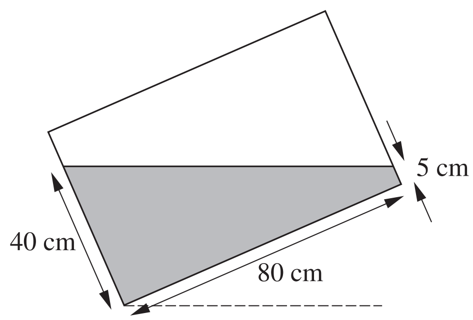
For Examiner's Use

4 A fuel tank in the shape of a cuboid is shown below.



It is partly filled with fuel and inclined at an angle to the horizontal. The side view is shown below.



Calculate the volume, in litres, of fuel in the tank. **[3]**

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For Examiner's Use

5 **(i)** Explain clearly how the equation on line 72 can be simplified to give the quadratic equation on line 74. **[1]**

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(ii) In line 76 only one root of the quadratic equation is given. Find the other root and explain why it is not relevant in the context of this problem. **[3]**

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6 On line 90 it is stated that if $H = h = 10$ then equation (4) gives a volume of 37.5 litres. Use equations (3) and (4) to show how this volume is derived. **[3]**

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OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4754B

Applications of Advanced Mathematics (C4) Paper B: Comprehension
INSERT

Tuesday 13 January 2009
Morning

Duration: Up to 1 hour

INSTRUCTIONS TO CANDIDATES

- This insert contains the text for use with the questions.

INFORMATION FOR CANDIDATES

- This document consists of **8** pages. Any blank pages are indicated.

Measuring the volume of fuel in a tank

Introduction

In many cars, aeroplanes and other vehicles, there is a display which provides information about various aspects of the engine system. This includes, for example, the temperature of the engine, the rate of fuel consumption and the volume of fuel remaining in the tank. This information is generated from measurements made by electronic sensors.

This article is concerned with the mathematics involved in calculating the volume of fuel in a fuel tank using measurements made by sensors in the tank.

When positioning sensors in a fuel tank, there are two major factors to consider. These are the shape of the tank and the possible orientations to the horizontal that the tank might experience during motion.

The shapes of fuel tanks in aircraft wings are determined by the shape of the wings. Consequently, aircraft fuel tanks have complex shapes and advanced mathematical techniques are required in order to calculate the volume of fuel in the tanks. In contrast, the fuel tanks in cars are not tightly constrained by the shape of the car and so can have relatively simple shapes.

Aircraft often fly at extreme angles to the horizontal. The sensors in the fuel tanks need to be positioned in such a way that they can always give meaningful measurements which can be used to calculate the volume of fuel in the tank.

Two shapes of tanks are considered in this article: a cylindrical tank, as an approximation to the fuel tanks used in some cars, and a trapezoidal tank, as an approximation to those used in some aircraft.

Cylindrical tanks

On level ground

Fig. 1 shows a vertical cylindrical tank with radius 20 cm and height 50 cm containing fuel to a depth of d cm.

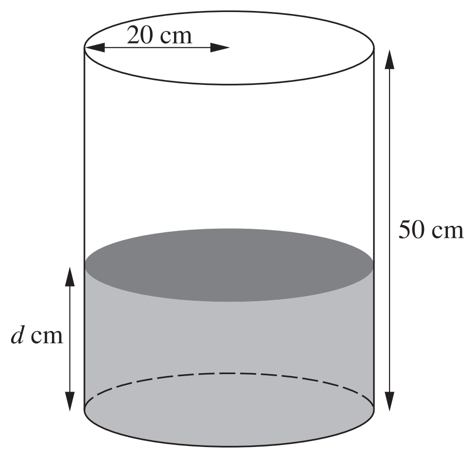

Fig. 1

The relationship between the depth, d cm, and volume, V litres, of the fuel is given by the formula

$$V = \frac{1}{1000} \times \pi \times 20^2 \times d.$$

For a cylindrical tank in this position, a sensor inside the tank would need to measure only the distance between the surface of the fuel and the top of the tank in order to calculate the volume of fuel.

The system could be set so that, if the volume of fuel drops below a certain amount, a warning light comes on. For example, if the critical volume is set as 10 litres, the warning light comes on when $d = 8$.

On a shallow incline

Figs. 2.1 and 2.2 (a vertical section) show the tank with its base inclined at an angle θ to the horizontal, where $\tan\theta = \dfrac{H - h}{40}$. PQ represents the surface of the fuel; PP′ and Q′Q are parallel to the base of the tank.

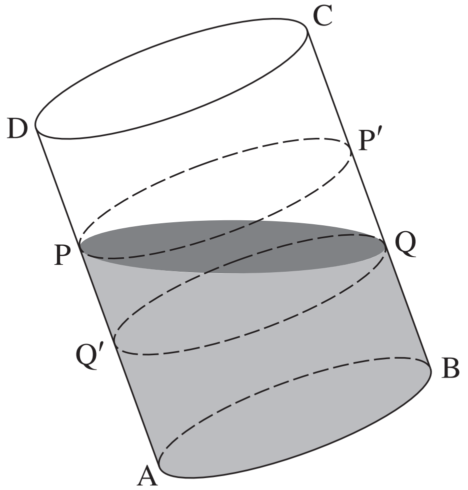

Fig. 2.1

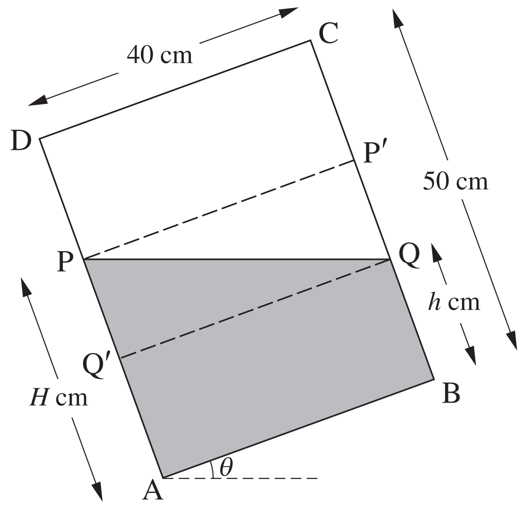

Fig. 2.2

The part of the tank between PP′ and Q′Q is divided into two congruent halves by the surface of the fuel. Therefore the volume of the fuel in the tank can be calculated by adding the volume of the cylinder ABQQ′ to half of the volume of the cylinder Q′QP′P. It follows that the volume, V litres, is given by

$$V = \frac{1}{5}\pi(H + h). \qquad (1)$$

Two sensors could measure the distances DP and CQ. These measurements could then be used to calculate the volume of fuel in the tank. Alternatively, one of these sensors could be used together with a different sensor which measures θ to calculate this volume.

Table 3 shows some possible values of h, H and θ for which this tank is three-quarters full. Notice that when $\theta = 32.0°$, the surface of the fuel touches the top of the tank at D.

h	37.5	35	32.5	30	27.5	25
H	37.5	40	42.5	45	47.5	50
θ	0°	7.1°	14.0°	20.6°	26.6°	32.0°

Table 3

In this case, and for any other volume of fuel in the tank, there is an angle of inclination beyond which equation (1) will no longer apply.

On a steep incline

Fig. 4 shows a vertical section of the tank inclined at a steep angle to the horizontal.

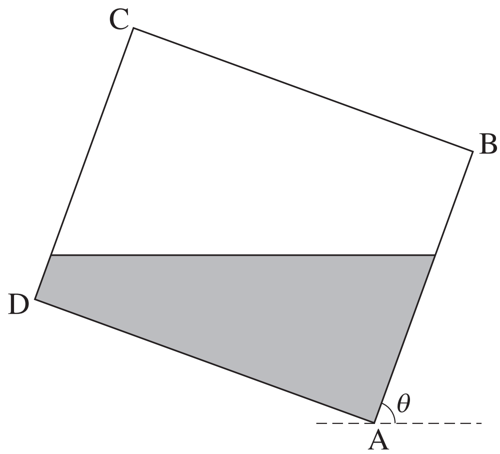

Fig. 4

Sensors designed to measure the distance of the surface of the fuel from D along DA and from C along CB would no longer provide useful information.

This is not a serious issue for cars with such tanks since the steepest roads have an angle of inclination of only about 15°.

Fuel tanks in aircraft

In aircraft, the shapes of fuel tanks are determined by the shape of the wing. Calculating the volume of fuel in such tanks requires advanced modelling and computational techniques.

A trapezoidal tank is considered below as an approximation to an aircraft fuel tank. The calculations give an indication of the way in which the volumes of such shapes are calculated.

Trapezoidal tanks

On level ground

Fig. 5 shows a trapezoidal tank; it is a prism with an isosceles trapezium cross-section.

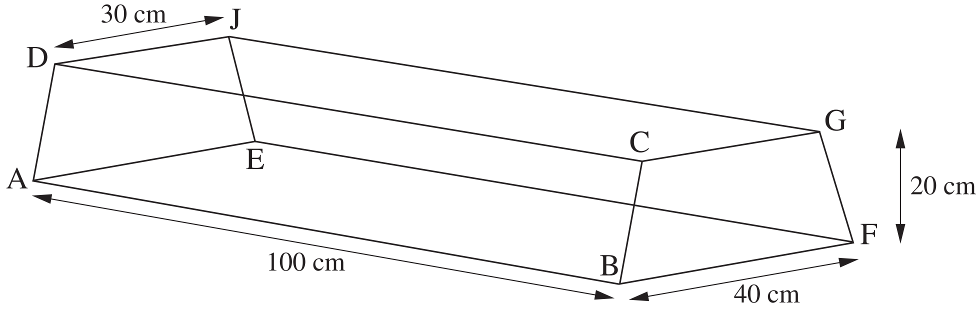

Fig. 5

The capacity of this tank is

$$\frac{1}{2} \times 20 \times (30 + 40) \times 100\,\text{cm}^3 = 70\,\text{litres}.$$

Fig. 6 shows the cross-section of the tank when it contains fuel to a depth of y cm.

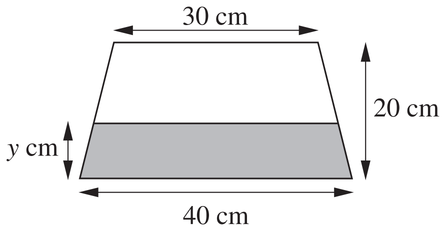

Fig. 6

The shaded area, $A\,\text{cm}^2$, in Fig. 6 is given by

$$A = \frac{1}{2} \times y \times \left(80 - \frac{y}{2}\right).$$

The volume, V litres, of fuel in the tank is given by

$$V = \frac{1}{2} \times y \times \left(80 - \frac{y}{2}\right) \times \frac{1}{10}. \qquad (2)$$

A sensor in this tank could be set so that, if the volume drops to 10 litres, a warning light comes on. This happens when y satisfies the equation

$$\frac{1}{2} \times y \times \left(80 - \frac{y}{2}\right) \times \frac{1}{10} = 10.$$

This equation simplifies to

$$y^2 - 160y + 400 = 0.$$

By solving this equation, it can be shown that the warning light comes on when the depth of the fuel drops to 2.54 cm.

On a shallow incline

The tank in Fig. 5, containing fuel, is now tilted about the edge AE. Fig. 7 shows a vertical section of the tank.

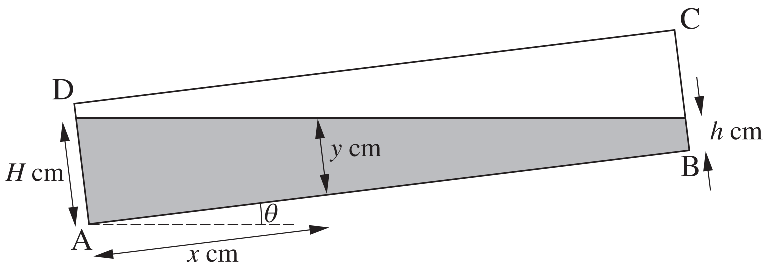

Fig. 7

The two ends of the tank and a cross-section parallel to them are shown in Fig. 8. The distance of the cross-section from the end AEJD is x cm, where $0 \leqslant x \leqslant 100$.

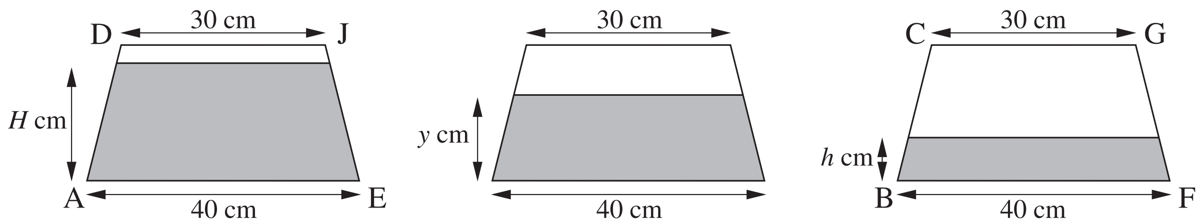

Fig. 8

The relationship between y, as shown in Figs. 7 and 8, and x is given by

$$y = H - \frac{H-h}{100}x. \qquad (3)$$

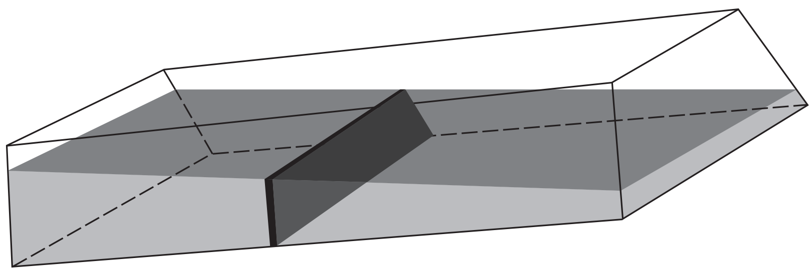

Fig. 9

To calculate the volume of fuel in the tank in Fig. 7, you can think of the region occupied by the fuel as being made up of a large number of thin trapezoidal prisms like the one shown in Fig. 9. It can then be shown that the volume, V litres, of fuel is given by

$$V = \frac{1}{1000}\int_0^{100} \frac{y}{2}\left(80 - \frac{y}{2}\right) \mathrm{d}x. \qquad (4)$$

For example, if sensors indicate that $H = 20$ and $h = 0$, then equation (3) gives $y = 20 - \frac{1}{5}x$. Equation (4) then gives a volume of $36\frac{2}{3}$ litres.

Similarly, if $H = h = 10$, then equation (4) gives a volume of 37.5 litres. Since in this case the tank is on level ground, the volume could have been found using equation (2).

On a steep incline

Fig. 10 shows a vertical section of the tank at a steeper angle of inclination to the horizontal.

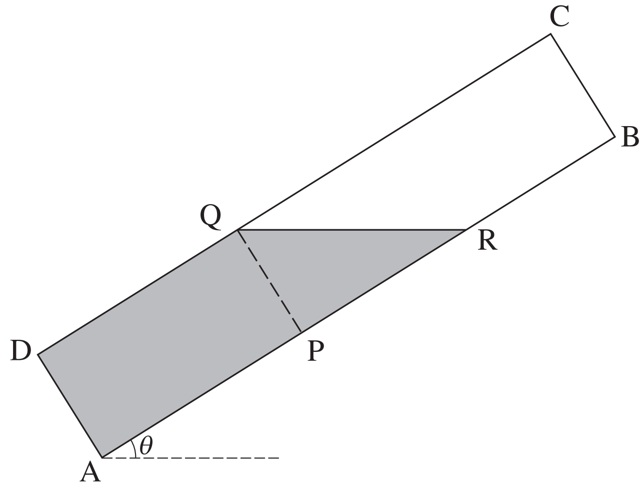

Fig. 10

The techniques required to calculate the volume indicated in Fig. 10 are similar to those used in deriving equation (4). Sensors measure the distances BR and CQ which are needed in order to calculate this volume.

In conclusion

Sensors in aircraft do much more than measure the volume of fuel in the tank. When designing and testing tanks for use in aircraft, sensors provide information about the movement of fuel in the tank. It is crucially important to ensure that, at all times, fuel is being sucked from the tanks into the engines. Sensors allow the designers and engineers to ensure that the shapes of the tanks meet these crucial requirements.