OCR

Monday 16 June 2014 – Morning

A2 GCE MATHEMATICS

4723/01 Core Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4723/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Given that $y = 4x^2 \ln x$, find the value of $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ when $x = \mathrm{e}^2$. **[5]**

2 By first using appropriate identities, solve the equation

$$5\cos 2\theta \operatorname{cosec}\theta = 2$$

for $0° < \theta < 180°$. **[6]**

3 **(i)** Use Simpson's rule with four strips to find an approximation to

$$\int_0^2 \mathrm{e}^{\sqrt{x}}\,\mathrm{d}x\ ,$$

giving your answer correct to 3 significant figures. **[4]**

(ii) Deduce an approximation to $\int_0^2 (1 + 10\mathrm{e}^{\sqrt{x}})\mathrm{d}x$. **[2]**

4 The functions f and g are defined for all real values of x by

$$\mathrm{f}(x) = 2x^3 + 4 \quad \text{and} \quad \mathrm{g}(x) = \sqrt[3]{x - 10}\ .$$

(i) Evaluate $\mathrm{f}^{-1}(-50)$. **[2]**

(ii) Show that $\mathrm{fg}(x) = 2x - 16$. **[2]**

(iii) Differentiate $\mathrm{gf}(x)$ with respect to x. **[3]**

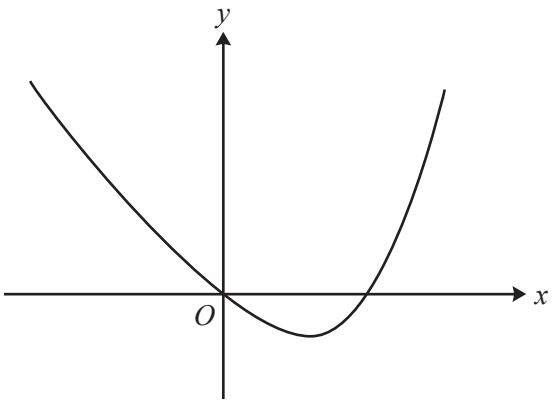
5 **(a)** The mass, M grams, of a substance at time t years is given by

$$M = 58\mathrm{e}^{-0.33t}\ .$$

Find the rate at which the mass is decreasing at the instant when $t = 4$. Give your answer correct to 2 significant figures. **[3]**

(b) The mass of a second substance is increasing exponentially. The initial mass is 42.0 grams and, 6 years later, the mass is 51.8 grams. Find the mass at a time 24 years after the initial value. **[4]**

6



The diagram shows the curve $y = x^4 - 8x$.

(i) By sketching a second curve on the copy of the diagram, show that the equation

$$x^4 + x^2 - 8x - 9 = 0$$

has two real roots. State the equation of the second curve. **[2]**

(ii) The larger root of the equation $x^4 + x^2 - 8x - 9 = 0$ is denoted by α.

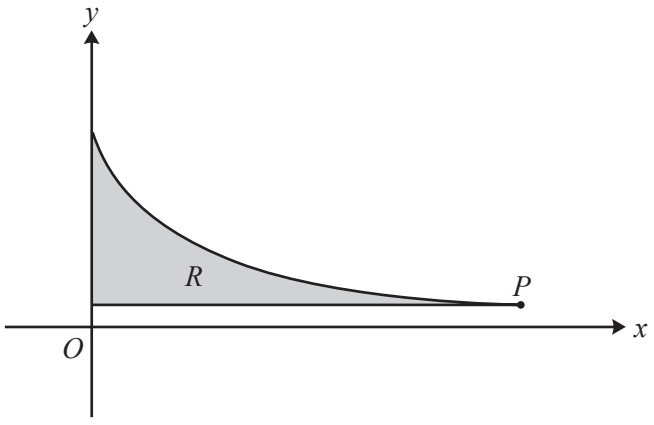
(a) Show by calculation that $2.1 < \alpha < 2.2$. **[2]**

(b) Use an iterative process based on the equation

$$x = \sqrt[4]{9 + 8x - x^2},$$

with a suitable starting value, to find α correct to 3 decimal places. Give the result of each step of the iterative process. **[4]**

7

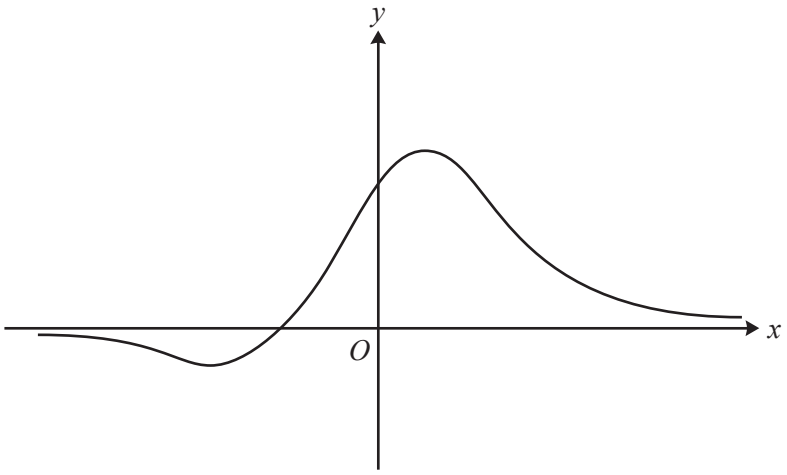


The diagram shows the curve $y = \sqrt{\dfrac{3}{4x+1}}$ for $0 \leqslant x \leqslant 20$. The point P on the curve has coordinates $\left(20, \frac{1}{9}\sqrt{3}\right)$. The shaded region R is enclosed by the curve and the lines $x = 0$ and $y = \frac{1}{9}\sqrt{3}$.

(i) Find the exact area of R. **[4]**

(ii) Find the exact volume of the solid obtained when R is rotated completely about the x-axis. **[6]**

8



The diagram shows the curve $y = \dfrac{2x+4}{x^2+5}$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and hence find the coordinates of the two stationary points. **[6]**

(ii) The function g is defined for all real values of x by

$$\mathrm{g}(x) = \left|\frac{2x+4}{x^2+5}\right|.$$

(a) Sketch the curve $y = \mathrm{g}(x)$ and state the range of g. **[3]**

(b) It is given that the equation $\mathrm{g}(x) = k$, where k is a constant, has exactly two distinct real roots. Write down the set of possible values of k. **[2]**

9 **(i)** Express $5\cos(\theta - 60°) + 3\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. **[4]**

(ii) Hence

(a) give details of the transformations needed to transform the curve $y = 5\cos(\theta - 60°) + 3\cos\theta$ to the curve $y = \sin\theta$, **[3]**

(b) find the smallest positive value of β satisfying the equation

$$5\cos\left(\tfrac{1}{3}\beta - 40°\right) + 3\cos\left(\tfrac{1}{3}\beta + 20°\right) = 3. \quad \textbf{[5]}$$

END OF QUESTION PAPER