General Certificate of Education
June 2005
Advanced Subsidiary Examination

MATHEMATICS **MPC1**
Unit Pure Core 1

Tuesday 7 June 2005 Afternoon Session

In addition to this paper you will require:
- an 8-page answer book;
- the **blue** AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The point A has coordinates (6, 5) and the point B has coordinates (2, -1).

(a) Find the coordinates of the midpoint of AB. *(2 marks)*

(b) Show that AB has length $k\sqrt{13}$, where k is an integer. *(3 marks)*

(c) (i) Find the gradient of the line AB. *(2 marks)*

(ii) Hence, or otherwise, show that the line AB has equation $3x - 2y = 8$. *(2 marks)*

(d) The line AB intersects the line with equation $2x + y = 10$ at the point C. Find the coordinates of C. *(3 marks)*

2 (a) Express $x^2 - 6x + 16$ in the form $(x - p)^2 + q$. *(2 marks)*

(b) A curve has equation $y = x^2 - 6x + 16$.

Using your answer from part (a), or otherwise:

(i) find the coordinates of the vertex (minimum point) of the curve; *(2 marks)*

(ii) sketch the curve, indicating the value where the curve crosses the y-axis; *(2 marks)*

(iii) state the equation of the line of symmetry of the curve. *(1 mark)*

(c) Describe geometrically the transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 - 6x + 16$. *(3 marks)*

3 A circle has centre $C(2, -1)$ and radius 5. The point P has coordinates $(6, 2)$.

(a) Write down an equation of the circle. *(3 marks)*

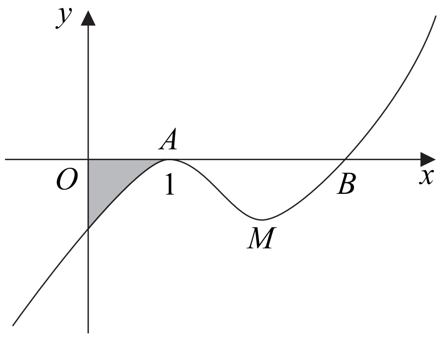
(b) Verify that the point P lies on the circle. *(2 marks)*

(c) Find the gradient of the line CP. *(2 marks)*

(d) (i) Find the gradient of a line which is perpendicular to CP. *(2 marks)*

(ii) Hence find an equation for the tangent to the circle at the point P. *(1 mark)*

4 The curve with equation $y = x^3 - 5x^2 + 7x - 3$ is sketched below.



The curve touches the x-axis at the point A (1, 0) and cuts the x-axis at the point B.

(a) (i) Use the factor theorem to show that $x - 3$ is a factor of

$$\mathrm{p}(x) = x^3 - 5x^2 + 7x - 3$$ *(2 marks)*

(ii) Hence find the coordinates of B. *(1 mark)*

(b) The point M, shown on the diagram, is a minimum point of the curve with equation $y = x^3 - 5x^2 + 7x - 3$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. *(2 marks)*

(ii) Hence determine the x-coordinate of M. *(3 marks)*

(c) Find the value of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ when $x = 1$. *(2 marks)*

(d) (i) Find $\int \left(x^3 - 5x^2 + 7x - 3\right) \mathrm{d}x$. *(4 marks)*

(ii) Hence determine the area of the shaded region bounded by the curve and the coordinate axes. *(4 marks)*

5 Express each of the following in the form $m + n\sqrt{3}$, where m and n are integers:

(a) $\left(\sqrt{3} + 1\right)^2$; *(2 marks)*

(b) $\frac{\sqrt{3} + 1}{\sqrt{3} - 1}$. *(3 marks)*

Turn over ▶

6 The cubic polynomial $p(x)$ is given by $p(x) = (x-2)(x^2+x+3)$.

(a) Show that $p(x)$ can be written in the form $x^3 + ax^2 + bx - 6$, where a and b are constants whose values are to be found. *(2 marks)*

(b) Use the Remainder Theorem to find the remainder when $p(x)$ is divided by $x+1$. *(2 marks)*

(c) Prove that the equation $(x-2)(x^2+x+3) = 0$ has only one real root and state its value. *(3 marks)*

7 Solve each of the following inequalities:

(a) $3(x-1) > 3 - 5(x+6)$; *(3 marks)*

(b) $x^2 - x - 6 < 0$. *(4 marks)*

8 A line has equation $y = mx - 1$, where m is a constant.

A curve has equation $y = x^2 - 5x + 3$.

(a) Show that the x-coordinate of any point of intersection of the line and the curve satisfies the equation

$$x^2 - (5+m)x + 4 = 0$$ *(1 mark)*

(b) Find the values of m for which the equation $x^2 - (5+m)x + 4 = 0$ has equal roots. *(4 marks)*

(c) Describe geometrically the situation when m takes either of the values found in part (b). *(1 mark)*

END OF QUESTIONS