

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

**Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education**

MATHEMATICS

4723

Core Mathematics 3

MARK SCHEME

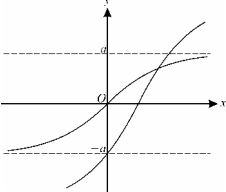
Specimen Paper

MAXIMUM MARK	72
---------------------	-----------

This mark scheme consists of 4 printed pages.

<p>1 <i>EITHER:</i> $4x^2 + 4x + 1 > x^2 - 2x + 1$ i.e. $3x^2 + 6x > 0$ So $x(x+2) > 0$ Hence $x < -2$ or $x > 0$</p> <p><i>OR:</i> Critical values where $2x+1 = \pm(x-1)$ i.e. where $x = -2$ and $x = 0$</p> <p>Hence $x < -2$ or $x > 0$</p>	M1 A1 M1 A1 A1 M1 B1 A1 M1 A1	For squaring both sides For reduction to correct quadratic For factorising, or equivalent For both critical values correct For completely correct solution set For considering both cases, or from graphs For the correct value -2 For the correct value 0 For any correct method for solution set using two critical values For completely correct solution set
<p>2 (i) $\sin x(\frac{1}{2}\sqrt{3}) + \cos x(\frac{1}{2}) + (\sqrt{3})(\cos x(\frac{1}{2}\sqrt{3}) - \sin x(\frac{1}{2}))$ $= \frac{1}{2}\cos x + \frac{3}{2}\cos x = 2\cos x$, as required</p> <hr/> <p>(ii) $\sin 45^\circ + (\sqrt{3})\cos 45^\circ = 2\cos 15^\circ$ Hence $\cos 15^\circ = \frac{1+\sqrt{3}}{2\sqrt{2}}$</p>	M1 A1 M1 A1 M1 A1	For expanding both compound angles For completely correct expansion For using exact values of $\sin 30^\circ$ and $\cos 30^\circ$ For showing given answer correctly For letting $x = 15^\circ$ throughout For any correct exact form
<p>3 (i) $x_2 = \sqrt[3]{7} = 1.9129\dots$ $x_3 = 1.9517\dots$, $x_4 = 1.9346\dots$ $\alpha = 1.94$ to 2dp</p> <hr/> <p>(ii) $x = \sqrt[3]{17-5x} \Rightarrow x^3 + 5x - 17 = 0$</p> <hr/> <p>(iii) <i>EITHER:</i> Graphs of $y = x^3$ and $y = 17 - 5x$ only cross once Hence there is only one real root</p> <p><i>OR:</i> $\frac{d}{dx}(x^3 + 5x - 17) = 3x^2 + 5 > 0$ Hence there is only one real root</p>	B1 M1 A1 M1 A1 M1 A1 M1 A1	For 1.91... seen or implied For continuing the correct process For correct value reached, following x_5 and x_6 both 1.94 to 2dp For letting $x_n = x_{n+1} = x$ (or α) For correct equation stated For argument based on sketching a pair of graphs, or a sketch of the cubic by calculator For correct conclusion for a valid reason For consideration of the cubic's gradient For correct conclusion for a valid reason
<p>4 (i) $\int_0^2 (4x+1)^{-\frac{1}{2}} dx = \left[\frac{1}{2}(4x+1)^{\frac{1}{2}} \right]_0^2 = \frac{1}{2}(3-1) = 1$</p> <hr/> <p>(ii) $\pi \int_0^2 \frac{1}{4x+1} dx = \pi \left[\frac{1}{4} \ln(4x+1) \right]_0^2 = \frac{1}{4} \pi \ln 9$</p>	M1 A1 M1 A1 M1 A1 M1 A1	For integral of the form $k(4x+1)^{\frac{1}{2}}$ For correct indefinite integral For correct use of limits For given answer correctly shown For integral of the form $k \ln(4x+1)$ For correct $\frac{1}{4} \ln(4x+1)$, with or without π Correct use of limits and π For correct (simplified) exact value

<p>5 (i) 200 °C</p> <hr/> <p>(ii) $150 = 200 - 180e^{-0.1t} \Rightarrow e^{-0.1t} = \frac{50}{180}$ Hence $-0.1t = \ln \frac{5}{18} \Rightarrow t = 12.8$</p> <hr/> <p>(iii) $\frac{d\theta}{dt} = 18e^{-0.1t}$ Hence rate is $18e^{-0.1 \times 12.8} = 5.0$ °C per minute</p>	<p>B1 1</p> <hr/> <p>M1 M1 A1 3</p> <hr/> <p>M1 A1 M1 A1 4</p>	<p>For value 200</p> <hr/> <p>For isolating the exponential term For taking logs correctly For correct value 12.8 (minutes)</p> <hr/> <p>For differentiation attempt For correct derivative For using their value from (ii) in their θ For value 5.0(0)</p>
<p>6 (i) Domain of f^{-1} is $x \geq 1$ Range is $x \geq 0$</p> <hr/> <p>(ii) If $y = 1 + \sqrt{x}$, then $x = (y-1)^2$ Hence $f^{-1}(x) = (x-1)^2$</p> <hr/> <p>(iii) The graphs intersect on the line $y = x$ Hence x satisfies $x = (x-1)^2$ i.e. $x^2 - 3x + 1 = 0 \Rightarrow x = \frac{3 \pm \sqrt{5}}{2}$ So $x = \frac{1}{2}(3 + \sqrt{5})$ as x must be greater than 1</p>	<p>B1 2</p> <hr/> <p>M1 A1 2</p> <hr/> <p>B1 B1 M1 A1 4</p>	<p>For the correct set, in any notation Ditto</p> <hr/> <p>For changing the subject, or equivalent For correct expression in terms of x</p> <hr/> <p>For stating or using this fact For either $x = f(x)$ or $x = f^{-1}(x)$ For solving the relevant quadratic equation For showing the given answer fully</p>
<p>7 (i) $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$</p> <hr/> <p>(ii) $\frac{8t}{1-t^2} + 3 \times \frac{1}{t} \times (1+t^2) = 0$ Hence $8t^2 + 3(1-t^2)(1+t^2) = 0$ i.e. $3t^4 - 8t^2 - 3 = 0$, as required</p> <hr/> <p>(iii) $(3t^2 + 1)(t^2 - 3) = 0$ Hence $t = \pm\sqrt{3}$ So $x = \frac{1}{3}\pi, \frac{2}{3}\pi, \frac{4}{3}\pi, \frac{5}{3}\pi$</p>	<p>B1 1</p> <hr/> <p>B1 B1 M1 A1 4</p> <hr/> <p>M1 A1 A1 A1 4</p>	<p>For correct RHS stated</p> <hr/> <p>For $\cot x = \frac{1}{t}$ seen For $\sec^2 x = 1 + t^2$ seen For complete substitution in terms of t For showing given equation correctly</p> <hr/> <p>For factorising or other solution method For $t^2 = 3$ found correctly For any two correct angles For all four correct and no others</p>

<p>8 (i) $\frac{dy}{dx} = \frac{2 \ln x}{x}$</p> $\frac{d^2y}{dx^2} = \frac{x(2/x) - 2 \ln x}{x^2} = \frac{2 - 2 \ln x}{x^2}$ <hr/> <p>(ii) For maximum gradient, $2 - 2 \ln x = 0 \Rightarrow x = e$</p> <p>Hence P is $(e, 1)$</p> <p>The gradient at P is $\frac{2}{e}$</p> <p>Tangent at P is $y - 1 = \frac{2}{e}(x - e)$</p> <p>Hence, when $x = 0$, $y = -1$ as required</p>	<p>M1 A1 M1 A1</p> <p>M1 A1 A1✓ A1✓ M1 A1</p>	<p>For relevant attempt at the chain rule</p> <p>For correct result, in any form</p> <p>For relevant attempt at quotient rule</p> <p>4 For correct simplified answer</p> <hr/> <p>For equating second derivative to zero</p> <p>For correct value e</p> <p>For stating or using the y-coordinate</p> <p>For stating or using the gradient at P</p> <p>For forming the equation of the tangent</p> <p>6 For correct verification of $(0, -1)$</p> <p style="text-align: center;">10</p>									
<p>9 (i) $a = \frac{1}{2}\pi$</p> <hr/> <p>(ii) $x = \tan(\frac{1}{4}\pi) = 1$</p> <hr/> <p>(iii)</p>  <p>Asymptotes are $y = \pm 2a$</p> <hr/> <table border="1" data-bbox="268 1176 574 1288"> <thead> <tr> <th>x</th> <th>$\tan^{-1} x$</th> <th>$2 \tan^{-1}(x-1)$</th> </tr> </thead> <tbody> <tr> <td>1.535</td> <td>0.993</td> <td>0.983</td> </tr> <tr> <td>1.545</td> <td>0.996</td> <td>0.998</td> </tr> </tbody> </table> <p>Hence graphs cross between 1.535 and 1.545</p> <hr/> <p>(v) Relevant values of $(\tan^{-1} x)^2$ are (approximately) 0, 0.0600, 0.2150, 0.4141, 0.6169 $\frac{1}{12}\{0 + 4(0.0600 + 0.4141) + 2 \times 0.2150 + 0.6169\}$ Hence required approximation is 0.245</p>	x	$\tan^{-1} x$	$2 \tan^{-1}(x-1)$	1.535	0.993	0.983	1.545	0.996	0.998	<p>B1</p> <p>M1 A1✓</p> <p>B1 B1</p> <p>B1</p> <p>M1 A1</p> <p>M1 M1 A1</p>	<p>1 For correct exact value stated</p> <hr/> <p>2 For correct answer, following their a</p> <hr/> <p>3 For correct statement of asymptotes</p> <hr/> <p>2 For correct details and explanation</p> <hr/> <p>3 For correct (2 or 3sf) answer</p> <p style="text-align: center;">4</p>
x	$\tan^{-1} x$	$2 \tan^{-1}(x-1)$									
1.535	0.993	0.983									
1.545	0.996	0.998									