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Oxford Cambridge and RSA

A Level Mathematics B (MEI)

H640/02 Pure Mathematics and Statistics

Sample Question Paper Version 2

Date – Morning/Afternoon

Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Book consists of **20** pages. The Question Paper consists of **16** pages.

Formulae A Level Mathematics B (MEI) H640

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r|<1$

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1\, a^{n-1}b + {}^n\mathrm{C}_2\, a^{n-2}b^2 + \quad + {}^n\mathrm{C}_r\, a^{n-r}b^r + \quad + b^n \qquad (n \in \quad),$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \dbinom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \quad + \dfrac{n(n-1) \quad (n-r+1)}{r!}x^r + \qquad (|x|<1,\ n \in \quad)$

Differentiation

f(x)	f (x)
$\tan kx$	$k\sec \ \ kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2\, x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient Rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim\limits_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Integration

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)\left(\mathrm{f}(x)\right)^n \mathrm{d}x = \frac{1}{n+1}\left(\mathrm{f}(x)\right)^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \frac{1}{2})\pi)$

Numerical methods

Trapezium rule: $\int_a^b y\,dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$ **or** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \dfrac{\left(\sum x_i\right)^2}{n} = \sum x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r\, p^r q^{n-r}$ where $q = 1 - p$

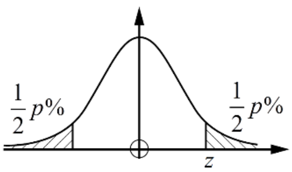
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \dfrac{\sigma}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions.

Section A (23 marks)

1 **In this question you must show detailed reasoning.**

Find the coordinates of the points of intersection of the curve $y = x^2 + x$ and the line $2x + y = 4$. **[5]**

2 Given that $\mathrm{f}(x) = x^3$ and $\mathrm{g}(x) = 2x^3 - 1$, describe a sequence of two transformations which maps the curve $y = \mathrm{f}(x)$ onto the curve $y = \mathrm{g}(x)$. **[4]**

3 Evaluate $\int_0^{\frac{\pi}{12}} \cos 3x \, \mathrm{d}x$, giving your answer in exact form. **[3]**

4 The function $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = x^3 - 4$ for $-1 \le x \le 2$.

For $\mathrm{f}^{-1}(x)$, determine

- the domain
- the range. **[5]**

5 In a particular country, 8% of the population has blue eyes. A random sample of 20 people is selected from this population.

Find the probability that exactly two of these people have blue eyes. **[2]**

6 Each day, for many years, the maximum temperature in degrees Celsius at a particular location is recorded. The maximum temperatures for days in October can be modelled by a Normal distribution. The appropriate Normal curve is shown in **Fig. 6**.

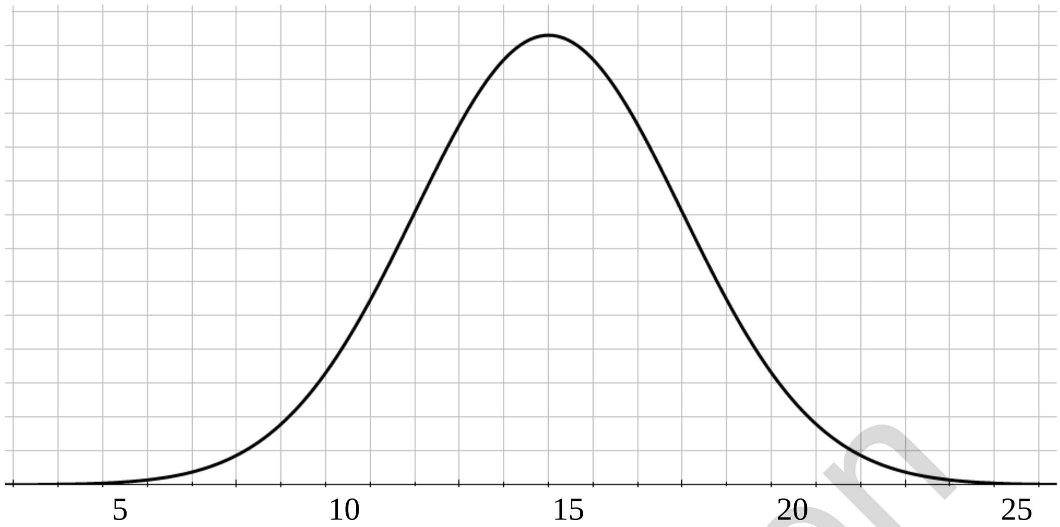


Fig. 6

(a) **(i)** Use the model to write down the mean of the maximum temperatures. **[1]**

(ii) Explain why the curve indicates that the standard deviation is approximately 3 degrees Celsius. **[1]**

Temperatures can be converted from Celsius to Fahrenheit using the formula $F = 1.8C + 32$, where F is the temperature in degrees Fahrenheit and C is the temperature in degrees Celsius.

(b) For maximum temperature in October in degrees Fahrenheit, estimate

- the mean
- the standard deviation. **[2]**

Answer **all** the questions.
Section B (77 marks)

7 Two events A and B are such that $\mathrm{P}(A) = 0.6$, $\mathrm{P}(B) = 0.5$ and $\mathrm{P}(A \cup B) = 0.85$. Find $\mathrm{P}(A \mid B)$. **[4]**

8 Alison selects 10 of her male friends. For each one she measures the distance between his eyes. The distances, measured in mm, are as follows:

51 57 58 59 61 64 64 65 67 68

The mean of these data is 61.4. The sample standard deviation is 5 232, correct to 3 decimal places.

One of the friends decides he does not want his measurement to be used. Alison replaces his measurement with the measurement from another mal friend This increases the mean to 62.0 and reduces the standard deviation.

Give a possible value for the measurement which has been removed and find the measurement which has replaced it. **[3]**

9 A geyser is a hot spring which erup s from time to time. For two geysers, the duration of each eruption, x minutes, and the waiting time until the next eruption, y minutes, are recorded.

(a) For a random sample of 50 eruptions of the first geyser, the correlation coefficient between x and y is 0.758.
The critical value for a 2-tailed hypothesis test for correlation at the 5% level is 0.279.
Explain whether or not there is evidence of correlation in the population of eruptions. **[2]**

The scatter diagram in **Fig. 9** shows the data from a random sample of 50 eruptions of the second geyser.

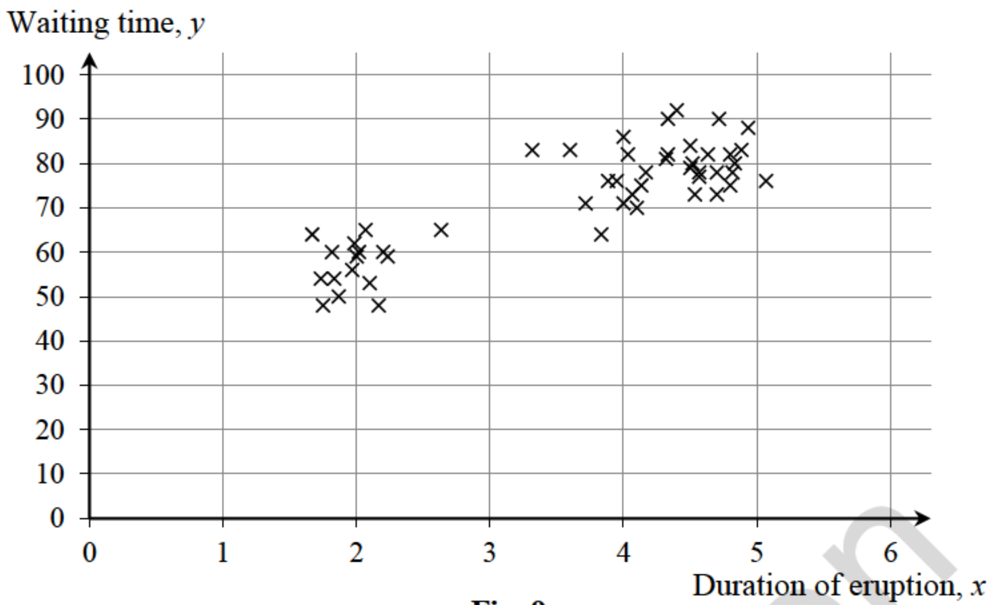


Fig. 9

(b) Stella claims the scatter diagram shows evidence of correlation between duration of eruption and waiting time. Make two comments about Stella's claim. **[2]**

10 A researcher wants to find out how many adults in a large town use the internet at least once a week. The researcher has formulated a suitable question to ask.

For each of the following methods of taking a sample of the adults in the town, give a reason why the method may be biased.

Method A: Ask people walking along a particular street between 9 am and 5 pm on one Monday.

Method B: Put the question through every letter box in the town and ask people to send back answers.

Method C: Put the question on the local council website for people to answer online. **[3]**

11 Suppose x is an irrational number, and y is a rational number, so that $y = \dfrac{m}{n}$,

where m and n are integers and $n \neq 0$.
Prove by contradiction that $x + y$ is not rational. **[4]**

12 **Fig. 12** shows the curve $2x^3 + y^3 = 5y$.

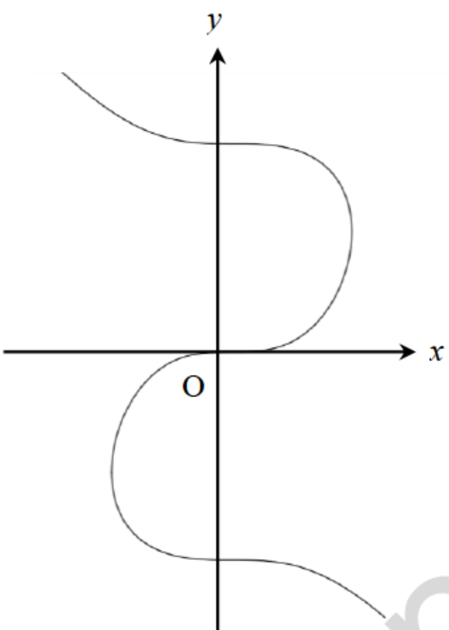


Fig. 12

(a) Find the gradient of the curve $2x^3 + y^3 = 5y$ at the point $(1, 2)$, giving your answer in exact form. **[4]**

(b) Show that all the stationary points of the curve lie on the y-axis. **[2]**

13 Evaluate $\displaystyle\int_0^1 \frac{1}{1+\sqrt{x}}\,dx$, giving your answer in the form $a + b\ln c$, where a, b and c are integers.

[6]

14 In a chemical reaction, the mass m grams of a chemical at time t minutes is modelled by the differential equation

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \frac{m}{t(1+2t)} .$$

At time 1 minute, the mass of the chemical is 1 gram.

(a) Solve the differential equation to show that $m = \dfrac{3t}{(1+2t)}$. **[8]**

(b) Hence

(i) find the time when the mass is 1.25 grams, **[2]**

(ii) show what happens to the mass of the chemical as t becomes large. **[2]**

15 A quality control department checks the lifetimes of batteries produced by a company.

The lifetimes, x minutes, for a random sample of 80 'Superstrength' batteries are shown in the table below.

Lifetime	$160 \le x < 165$	$165 \le x < 168$	$168 \le x < 170$	$170 \le x < 172$	$172 \le x < 175$	$175 \le x < 180$
Frequency	5	14	20	21	16	4

(a) Estimate the proportion of these batteries which have a lifetime of at least 174.0 minutes. **[2]**

(b) Use the data in the table to estimate
- the sample mean,
- the sample standard deviation. **[3]**

The data in the table on the previous page are represented in the following histogram, **Fig 15**:

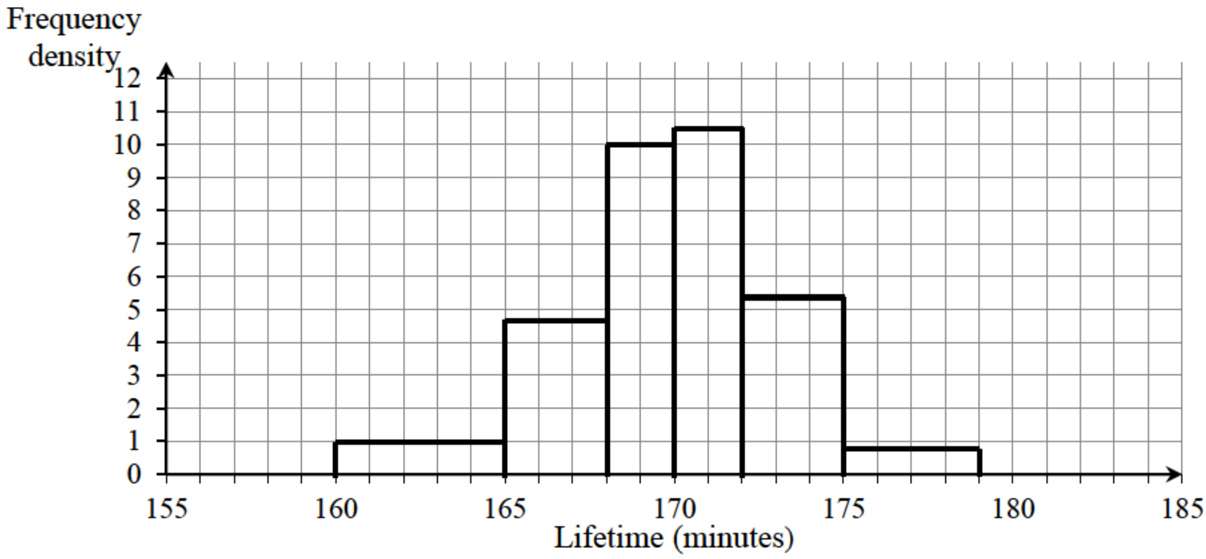


Fig. 15

A quality control manager models the data by a Normal distribution with the mean and standard deviation you calculated in part **(b)**.

(c) Comment briefly on whether the histogram supports this choice of model. **[2]**

(d) **(i)** Use this model to estimate the probability that a randomly selected battery will have a lifetime of more than 174.0 minutes.

(ii) Compare your answer with your answer to part **(a)**. **[3]**

The company also manufactures 'Ultrapower' batteries, which are stated to have a mean lifetime of 210 minutes.

(e) A random sample of 8 Ultrapower batteries is selected. The mean lifetime of these batteries is 207.3 minutes.

Carry out a hypothesis test at the 5% level to investigate whether the mean lifetime is as high as stated. You should use the following hypotheses $H_0 : \mu = 210$, $H_1 : \mu < 210$, where μ represents the population mean for Ultrapower batteries.

You should assume that the population is Normally distributed with standard deviation 3.4.

[5]

TURN OVER FOR THE NEXT QUESTION

16 **Fig. 16.1**, **Fig. 16.2** and **Fig. 16.3** show some data about life expectancy, including some from the pre-release data set.

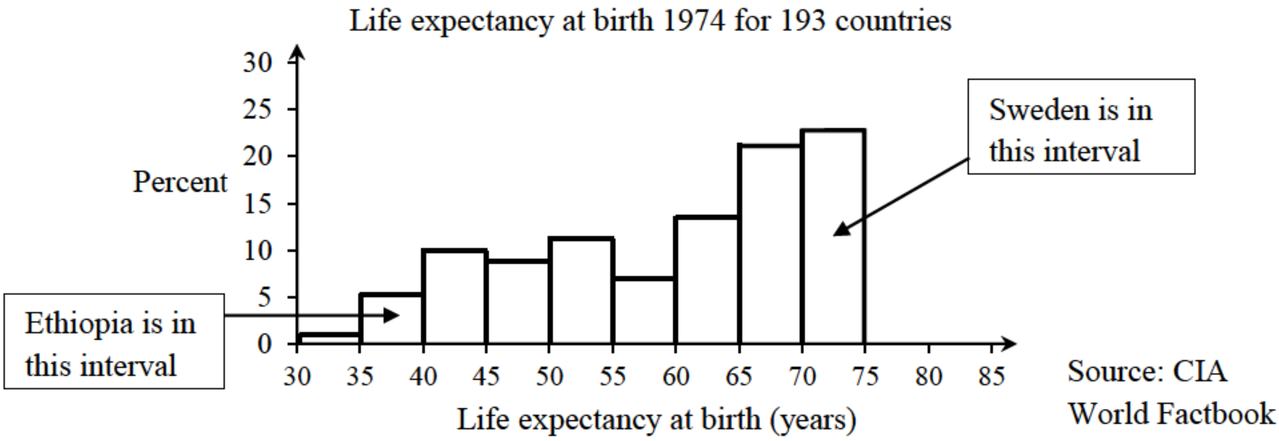


Fig. 16.1

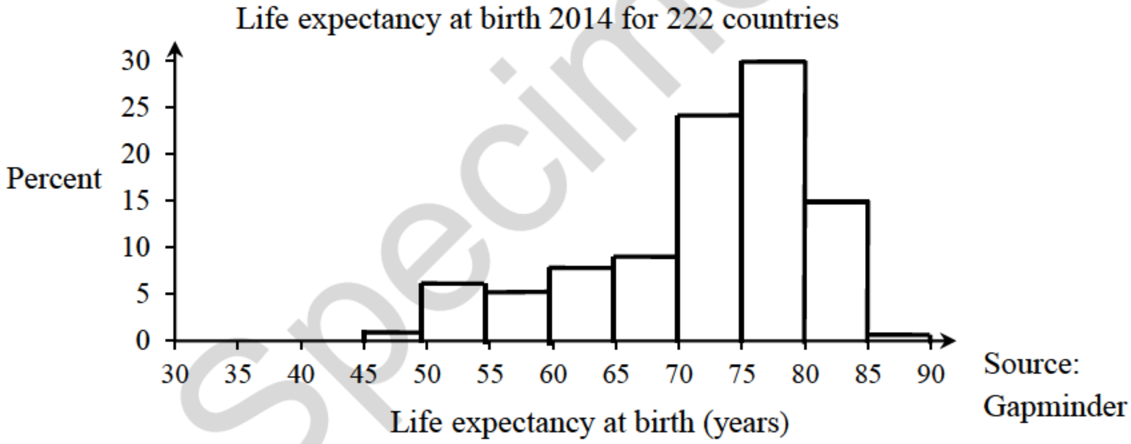
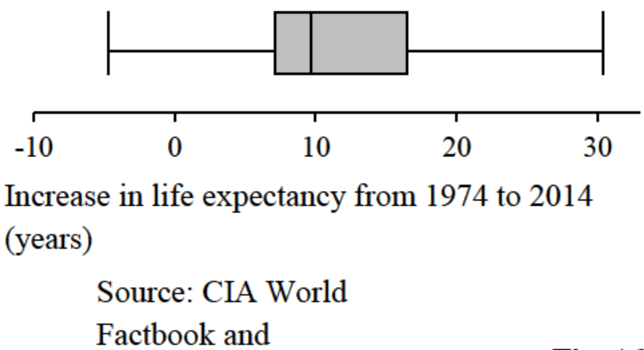


Fig. 16.2



Increase in life expectancy for 193 countries from 1974 to 2014	
Number of values	193
Minimum	- 4.618
Lower quartile	6.9576
Median	9.986
Upper quartile	15.873
Maximum	30.742

Fig. 16.3

(a) Comment on the shapes of the distributions of life expectancy at birth in 2014 and 1974. **[2]**

(b) **(i)** The minimum value shown in the box plot is negative.
What does a negative value indicate? **[1]**

(ii) What feature of **Fig 16.3** suggests that a Normal distribution would **not** be an appropriate model for increase in life expectancy from one year to another year? **[1]**

(iii) Software has been used to obtain the values in the table in **Fig. 16.3**.
Decide whether the level of accuracy is appropriate. Justify your answer. **[1]**

(iv) John claims that for half the people in the world their life expectancy has improved by 10 years or more.
Explain why **Fig. 16.3** does **not** provide conclusive evidence for John's claim. **[1]**

(c) Decide whether the maximum increase in life expectancy from 1974 to 2014 is an outlier.
Justify your answer. **[3]**

Here is some further information from the pre-release data set.

Country	Life expectancy at birth in 2014
Ethiopia	60.8
Sweden	81.9

(d) **(i)** Estimate the change in life expectancy at birth for Ethiopia between 1974 and 2014.

(ii) Estimate the change in life expectancy at birth for Sweden between 1974 and 2014.

(iii) Give **one** possible reason why the answers to parts **(i)** and **(ii)** are so different. **[4]**

Fig. 16.4 shows the relationship between life expectancy at birth in 2014 and 1974.

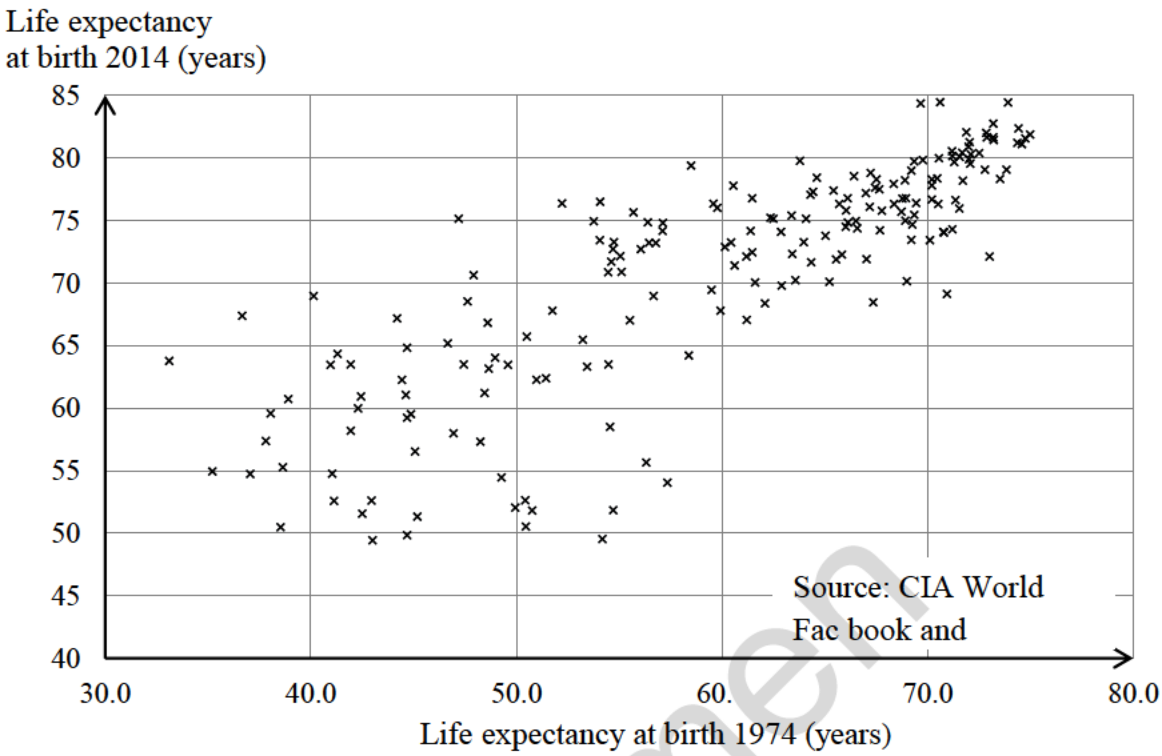


Fig. 16 4

A spreadsheet gives the following linear model for all the data in **Fig 16.4**.

(Life expectancy at birth 2014) = 30 98 + 0.67 × (Life expectancy at birth 1974)

The life expectancy at birth in 1974 or the region that now constitutes the country of South Sudan was 37.4 years. The value f r this country in 2014 is not available.

(e) **(i)** Use the linear model to estimate the life expectancy at birth in 2014 for South Sudan. **[2]**

(ii) Give two reasons why your answer to part **(i)** is not likely to be an accurate estimate for the life expectancy at birth in 2014 for South Sudan.
You should refer to **both** information from **Fig 16.4 and** your knowledge of the large data set. **[2]**

(f) In how many of the countries represented in **Fig. 16.4** did life expectancy drop between 1974 and 2014? Justify your answer. **[3]**

END OF QUESTION PAPER

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