GCE

Mathematics B (MEI)

H640/01: Pure Mathematics and Mechanics

Advanced GCE

Mark Scheme for June 2019

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

Subject-specific Marking Instructions for A Level Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for g. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs	Guidance	
1			**DR**				
			$\int_4^9 \left(2x + x^{\frac{1}{2}}\right) dx = \left[x^2 + \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_4^9$	**M1***	**2.1**	Attempt to integrate using fractional power	
			$\left(9^2 + \frac{2}{3} \times 9^{\frac{3}{2}}\right) - \left(4^2 + \frac{2}{3} \times 4^{\frac{3}{2}}\right)$	**M1 (dep)**	**2.1**	Use of limits must be seen	
			$\left[81 + 18 - 16 - \frac{16}{3}\right] = \frac{233}{3}$	**A1**	**2.1**	**AG** Any interim working must be correct	
				[3]			

Question			Answer	Marks	AOs	Guidance	
2			(Gradient of line segment) = $\frac{5-(-4)}{-1-2} = -3$	**M1**	**3.1a**	Attempt to find gradient – accept sign errors but not reciprocal	
			(Given line is) $y = -3x + 10$ has gradient -3	**M1**	**1.1a**	Finding gradient of given line	
			Same gradient so parallel lines Neither point lies on the line so the lines do not intersect	**A1** **[3]**	**2.2a**	Must make conclusion based on the fact that the two lines are parallel and not the same line	
			Alternative solution				
			Gradient = $\frac{5-(-4)}{-1-2} = -3$	**M1**		Attempt to find gradient – accept sign errors but not reciprocal	
			Equation of the line $y-(-4) = -3(x-2)$ $y = -3x + 2$	**M1**		Finding equation of given line	
			Given line is $y = -3x + 10$ which is parallel [So the lines do not intersect]	**A1**		Conclusion referring to parallel lines	Allow for solving two lines simultaneously and stating there are no solutions.

Question			Answer	Marks	AOs	Guidance	
3	(a)		$1+(-3)(-ax)+\frac{(-3)(-4)}{2}(-ax)^2+...$	**M1**	**1.1a**	Attempt to use the binomial expansion	Allow sign errors, bracket errors, a slip
			Equate coefficients $3a=6a^2$	**M1**	**1.1a**	Equating their coefficients	Allow recovery from missing brackets Their equation should not involve x
			$a=\frac{1}{2}$	**A1** **[3]**	**1.1b**	oe www	
3	(b)	(i)	Valid for $\|x\|<2$	**B1** **[1]**	**2.3**	Accept $\|x\|<\frac{1}{\|a\|}$ for their a	Do not accept $x<2$, $\left\|\frac{1}{2}x\right\|<1$ or $\|x\|\le 2$ or similar
3	(b)	(ii)	$\left[\left(1-\frac{1}{2}x\right)^{-3}\approx\right]1+\frac{3}{2}x+\frac{3}{2}x^2$	**B1** **[1]**	**1.1b**	Cao	

4			Let R_P and R_Q be the normal reactions upwards Assume the person stands on the end beyond Q and that the beam is in equilibrium Taking moments about Q: $50g \times 0.3 + R_P \times 2.1 = 4g \times 0.9$ $R_P = \frac{3.6g - 15g}{2.1}[= -53.2] < 0$ So the beam will tip.	**M1** **A1** **A1** **[3]**	**3.1b** **1.1b** **3.2a**	Finding moment of a force about any point Correct equation from moments about Q Conclusion must be clear from correct working (R_P need not be evaluated but must be clear that it is negative)	If the weight of the person shown between P and Q allow M1 maximum. Could also be obtained from moments about P and resolving to evaluate R_P
			Alternative solution If the person stands x m beyond Q so that the beam is on the point of tipping $R_P = 0$ Taking moments about Q $4g \times 0.9 = 50gx$ giving $x = 0.072$ $0.072 < 0.3$ so this is possible while standing on the beam	**M1** **A1** **A1**		Finding moment of a force about any point Correct equation from moments about Q Conclusion must be clear from correct working and reference to 0.3 m	
			Second alternative solution Largest clockwise moment of the weight of the person about Q is $50g \times 0.3 = 15g[= 147]$ Anticlockwise moment of the weight of the beam is $4g \times 0.9 = 3.6g[= 35.28]$ The moment of the person's weight is larger so the beam will tip	**M1** **A1** **A1**		Finding moment of a force about any point Both correct moments about Q required Conclusion must be clear from a comparison of moments of two forces	SC 2 (omitting g) $0.9 \times 4 < 0.3 \times 50$ $3.6 < 15$ So the beam will tip.

5			N2L gives $4000-800=1200a$ $a=\frac{8}{3}$ m s^{-2} Using $v=u+at$ gives $v=\frac{8}{3}\times 9=24$ m s^{-1}	**M1** **A1** **M1** **A1** **[4]**	**3.1b** **1.1b** **3.3** **1.1b**	Both forces present; allow sign errors Soi Using suvat equation(s) leading to a value for v **FT** their a	Allow for $3200=1200a$ seen

6	**(a)**		$\text{LHS}=\dfrac{\sin^2\theta-(1-\cos\theta)}{(1-\cos\theta)\sin\theta}$ $=\dfrac{(1-\cos^2\theta)+\cos\theta-1}{(1-\cos\theta)\sin\theta}$ $=\dfrac{\cos\theta(1-\cos\theta)}{\sin\theta(1-\cos\theta)}$ $=\cot\theta$	**M1** **B1** **M1** **A1** **[4]**	**2.1** **2.1** **2.1** **2.1**	Attempt to write LHS as a single fraction Use of identity $\sin^2\theta=1-\cos^2\theta$ Algebraic manipulation eg factorising the numerator **AG** Complete proof	Where candidates manipulate the entire statement, allow M1 for eliminating or combining fractions eg multiplying through by $(1-\cos\theta)\sin\theta$ M1 for algebraic manipulation leading to a known identity B1 identity obtained. A1 Complete proof
			Alternative solution $\dfrac{\sin\theta}{(1-\cos\theta)}\dfrac{(1+\cos\theta)}{(1+\cos\theta)}=\dfrac{\sin\theta(1+\cos\theta)}{\sin^2\theta}$ So LHS becomes $\dfrac{(1+\cos\theta)}{\sin\theta}-\dfrac{1}{\sin\theta}=\dfrac{\cos\theta}{\sin\theta}=\cot\theta$	**M1** **B1** **M1** **A1**		Attempt to change the denominator of the fraction Use of trig identity Combining the fractions **AG** Complete proof	
6	**(b)**		Uses $\dfrac{1}{\tan\theta}=3\tan\theta$ $\tan\theta=\pm\dfrac{1}{\sqrt{3}}$ $\theta=\dfrac{\pi}{6},\ \dfrac{5\pi}{6}$	**M1** **M1** **A1** **[3]**	**1.1a** **1.1a** **1.1b**	soi. Also allow for equivalent equation in $\cot\theta$ Method must be clearly using the given answer in **(a)** Do not allow if additional answers in the interval; ignore additional values outside the interval.	Allow positive root only for second M mark

7			$v = 3.5t - 0.5t^2 \Rightarrow \frac{dv}{dt} = 3.5 - t$ When $t = 8$, $a = 3.5 - 8 = -4.5$ And $v = 3.5 \times 8 - 0.5 \times 8^2 = -4$ Either "Velocity is negative and decreasing, so the speed is increasing". Or "As both the velocity and acceleration are negative, the speed is increasing" or similar	**M1** **A1** **B1** **A1** **[4]**	**3.1b** **2.1** **3.1b** **3.2a**	Attempt to differentiate to find a Evaluating a when t = 8 Evaluating v when t = 8 Argued from negative a and v	Allow A1 for establishing that a < 0 from correct expression without evaluating. Allow B1for establishing $v < 0$ from correct expression without evaluating.
			Special case When $t = 8$, $v = 3.5 \times 8 - 0.5 \times 8^2 = -4$ Evaluating v either side of $t = 8$ Statement referring to correct values	**B1** **B1** **B1**		Evaluating v when t = 8 Two correct values Argued from correct working	Method is not a full argument so max 3/4 marks.

t	7	7.5	7.9	8	8.1	8.5	9
v	0	-1.875	-3.555	-4	-4.455	-5.88	-9

8	**(a)**		Arithmetic sequence with $a = 9300$ $a_{10} = a + 9d = 9300 + 9d = 3900$ $d = -600$ $a_{20} = a + 19d = 9300 - 19 \times 600 = -2100$ [So 20th term is negative]	**M1** **A1** **A1** **[3]**	**3.1a** **1.1b** **2.1**	Using formula for term of AP with values substituted soi Allow for -2100 without comment	eg d = 600 stated but later subtracted Also allow for earlier negative term found and comment that 20^{th} is less
8	**(b)**		S is increasing as long as the extra terms are positive First negative term when $a_n < 0$ $9300 - 600(n-1) < 0 \Rightarrow \frac{9300}{600} < n - 1 \Rightarrow n > 16.5$ So maximum sum after 16 terms $S_{16} = \frac{16}{2}\left(2 \times 9300 - 600(16-1)\right) = 76\,800$	**M1** **A1** **M1** **A1** **[4]**	**3.1a** **2.2a** **1.1a** **1.1b**	Attempt to find first negative term with 9300 and their d Allow $n > 16$ or $n \geq 17$ oe Using sum with their n and their d cao	(or last positive term) Also allow M1A1 for establishing $a_{16} = 300$ and $a_{17} = -300$
			Alternative solution $S = \frac{n}{2}\left(2 \times 9300 - 600(n-1)\right)\left[= 9600n - 300n^2\right]$ [To find max S treating S and n as continuous] $\frac{dS}{dn} = 9600 - 600n = 0$ Max sum when $n = 16$ $S_{16} = \frac{16}{2}\left(2 \times 9300 - 600(16-1)\right) = 76800$	**M1** **M1** **A1** **A1**		Using the sum formula with 9300 and their d substituted Setting derivative to zero and solve Allow for $n = 16$ Cao	Also allow M1 for other methods for finding the max A1 for n = 16 eg M1A1 for $S_n = k - 300(n-16)^2$
			Second alternative solution $S_{15} = 76\,500$ $S_{16} = 76\,800$ $S_{17} = 76\,500$ max total is 76800	**M1** **M1** **A1** **A1**		Using the sum formula with 9300 and their d to find at least two totals Evaluating S_{15}, S_{16} and S_{17} Award for S_{16} as maximum identified from correct working Cao	Allow BC FT their d: three consecutive totals around their maximum eg $d = -540$ needs 17^{th}, 18^{th}, 19^{th} totals

9	**(a)**		Vertical motion $u_y = 100\sin 25°,\quad v_y = 0$ $v^2 = u^2 + 2as$ $0 = (100\sin 25°)^2 - 2\times 9.8\times s$ $s = \frac{42.26^2}{19.6} = 91.1\text{ m}$ (3sf)	**M1** **B1** **A1** **[3]**	**3.3** **1.1a** **3.2a**	Use of suvat equation(s) with v = 0 leading to a value for s. Allow sign errors Correct component of velocity soi cao	eg from t = 4.31 s
9	**(b)**		Vertical motion $u_y = 100\sin 25°,\quad y = 0$ $s = ut + \frac{1}{2}at^2 \Rightarrow 0 = (100\sin 25°)t - 4.9t^2$ $t\left(100\sin 25° - 4.9t\right) = 0 \Rightarrow t = 0$ or $8.62...$ $x = (100\cos 25°)\times 8.62... = 781.678...$ Range is 782 m (3sf)	**M1** **A1** **M1** **A1** **[4]**	**3.3** **1.1a** **1.1a** **1.1b**	Use of suvat equation(s) with y = 0 and their $u_y \neq 100$ leading to a value for t. Allow sign errors Correct value or expression for t from correct working Use of horizontal motion equation with $u_x = 100\cos 25°$ **FT** their value for t	Can be BC Allow for sin/cos interchange used throughout part (b).

10	**(a)**		$7\cos x - 2\sin x = R\cos(x+\alpha)$ $\Rightarrow 7 = R\cos\alpha,\ 2 = R\sin\alpha$	**M1**	**1.1a**	Forming two equations soi. Allow sign errors.	
			$R = \sqrt{7^2+2^2} = \sqrt{53}$	**B1**	**1.1b**	Allow even if from equations with sin/cos interchange; must be exact	
			$\alpha = \arctan\frac{2}{7} = 0.278...$	**M1** **A1** **[4]**	**1.1a** **1.1b**	**FT** their equations cao (3sf) Must be in radians for the A mark.	Allow M1M1 for $\alpha = \arctan\left(\pm\frac{2}{7}\right)$ $\alpha = 15.9°$ gets M1M1A0
10	**(b)**		$y = \frac{1}{7\cos x - 2\sin x} = \frac{1}{\sqrt{53}}\sec(x+\alpha)$ In either order				
			Stretch scale factor $\frac{1}{\sqrt{53}}$ in the y-direction	**B1** **B1**	**1.1a** **1.1b**	Stretch in the y-direction for correct scale factor **FT** their value for R	Do not allow enlargement instead of stretch
			Translation $\begin{pmatrix} -0.278 \\ 0 \end{pmatrix}$	**B1** **[3]**	**1.1b**	**FT** their value for α. Allow for translation to the left by 0.278	Do not allow for "shift" or "slide" used instead.

11	**(a)**		Component in **i** direction zero $\Rightarrow k = -4$	**B1** **[1]**	**3.3**	Allow $(-4\mathbf{i}+5\mathbf{j})$ seen instead of k	Do not allow for $k = -4\mathbf{i}$ or similar.
11	**(b)**		Weight $= -0.8g\mathbf{j}$	**B1**	**1.2**	Allow if seen	
			N2L $5\mathbf{j}+3\mathbf{j}-0.8g\mathbf{j}=0.8\mathbf{a}$	**M1**	**1.1a**	Condone missing weight	
			$[\Rightarrow \mathbf{a}=0.2\mathbf{j}]$ $\mathbf{v} = (4\mathbf{i} + 7\mathbf{j}) + 0.2\mathbf{j} \times 10$	**M1**	**1.1a**	Using their **a** in a vector suvat equation(s)	
			velocity is $(4\mathbf{i} + 9\mathbf{j})$ m s^{-1}	**A1** **[4]**	**2.5**	Must be in correct vector form	Accept fully correct column vector
			Alternative solution Acceleration is vertical so consider only vertical motion $5+3-0.8g=0.8a$	**M1** **B1**		Applying N2L in 1-dimension Condone missing weight Including the weight (consistent sign convention)	
			$[\Rightarrow a=0.2]$				
			$v=u+at=7+0.2\times 10=9$	**M1**		Using suvat equation(s) in the vertical direction	
			So the velocity is $(4\mathbf{i} + 9\mathbf{j})$ m s^{-1}	**A1** **[4]**		Must be in vector form	Accept fully correct column vector

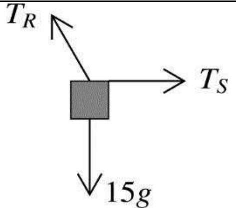
Due to the error on the paper, **all** candidates will get 4 marks for question 12(a) and all candidates will get 2 marks for 12 (b).

All assessors will need to be instructed to put a SEEN annotation on both and then award full marks in RM.

The only instance where full marks would not be awarded is where a candidate has only put their name on the script and has not attempted any question. This would then need to be a 0.

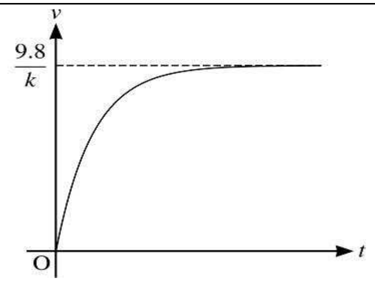
The MS for the correct question is given below for completeness.

12	**(a)**		$x = 4t^2, \quad y = 8t$ $PQ = 4 + 4t^2$ $PR^2 = (4t^2 - 4)^2 + (8t)^2$ $= 16t^4 + 32t^2 + 16$ $= (4 + 4t^2)^2 = PQ^2$ So equidistant	**B1** **M1** **M1** **A1** **[4]**	**2.1** **2.1** **2.1** **2.1**	Use of distance formula Must show some algebraic manipulation Correct expression for PR or PR^2 Conclusion comparing PQ and PR or PQ^2 and PR^2	
12	**(b)**		Substitute $t = \frac{y}{8}$ to obtain $x = 4\left(\frac{y}{8}\right)^2$ $y^2 = 16x$	**M1** **A1** **[2]**	**1.1a** **1.1b**	Rearranged equation must be used Allow any equivalent form including $y = \pm 4\sqrt{x}$	allow use of $t = \frac{\sqrt{x}}{2}$ oe used for M1A0.

13	(a)			B1 [1]	1.1a	Three forces in approximately correct directions, with arrows and labels; tensions must be distinct Accept W or mg for the weight; condone missing g for this mark	
13	(b)		Resolve vertically: $T_R\cos 30° = 15g$ $T_R = \dfrac{15\times 9.8}{\cos 30°} = 98\sqrt{3} = 170$ N (3sf)	M1 A1 [2]	3.3 1.1b	Forming equilibrium equation (allow sin/cos interchange) Oe	
13	(c)		Resolve horizontally: $T_S = T_R\sin 30°$ $T_S = 84.9$ N (3sf)	M1 A1 [2]	1.1a 1.1b	Allow sin/cos interchange if consistent with (b) Oe	Accept $T_S = 15g\tan 30°$

14	**(a)**		Sector area $=\frac{1}{2}r^2x$ and Triangle $=\frac{1}{2}r^2\sin x$ Area segment $=\frac{1}{2}r^2(x-\sin x)$ $\frac{1}{2}r^2(x-\sin x)=0.05\pi r^2$ $x-\sin x=2\times 0.05\times\pi \Rightarrow x-\sin x-\frac{1}{10}\pi=0$	**M1** **A1** **B1** **A1** **[4]**	2.1 2.1 2.1 2.1	Both areas seen segment area found $0.05\pi r^2$ oe seen **AG** Must be fully shown and correct rearrangement	
14	**(b)**		$x_{n+1}=x_n-\dfrac{x_n-\sin x_n-\frac{1}{10}\pi}{1-\cos x_n}$	**B1** **[1]**	**1.1a**	$x_{n+1}=$ must be seen Algebraic form must be seen Derivative must be worked out	Condone x used instead of x_n in the fraction part.
14	**(c)**		$x_0=1.2$ $x_1=1.27245...$ $x_2=1.26895...$ $x_3=1.26894...$ So root is 1.269 to 3 dp	**M1** **A1** **A1** **[3]**	**1.1a** **1.1b** **2.2b**	3 iterations recorded The first 3 iterations correct Allow 1.27, 1.269 or more decimal places if correct.	Root is 1.268947865 to 9 dp

15	(a)		$\int \frac{1}{9.8-kv}\,dv = \int 1\,dt$	M1	3.1a	Attempt to separate variables and integrate	Allow up to 3M marks for an attempt with an incorrect rearrangement.
				A1	1.1b	Correct rearrangement	
			$-\frac{1}{k}\ln(9.8-kv) = t + c$	A1	1.1b	Correctly integrated – must include +c somewhere	
			$9.8 - kv = Ae^{-kt}$	M1	1.1a	Attempt to write in the form v = (Clearing natural logs; may be done after evaluation of the constant)	
			When $t=0,\ v=0$ giving $0 = \frac{1}{k}(9.8 - Ae^{-0})$ $A = 9.8$	M1	3.1a	Using initial conditions leading to a value for the constant; may be done before clearing logs	
			Hence $v = \frac{9.8}{k}(1-e^{-kt})$	A1	1.1b	Correct value for A (or $c = -\frac{1}{k}\ln 9.8$)	
				A1 **[7]**	1.1b	Must be in the form $v = \ldots$	
			Alternative using a Further Maths method $\frac{dv}{dt} + kv = 9.8$	M1		Writing in the correct form and using integrating factor	
			Integrating factor $e^{\int P(t)\,dt} = e^{kt}$	A1		Correct integrating factor	
			$ve^{kt} = \int \left(9.8e^{kt}\right) dt = \frac{9.8}{k}e^{kt} + c$	A1		Correct equation	
			$v = \frac{9.8}{k} + ce^{-kt}$	M1		Attempt to write in the form v =	
			When $t=0,\ v=0$ giving $c = -\frac{9.8}{k}$	M1		Using initial conditions leading to a value for the constant	
			Hence $v = \frac{9.8}{k}(1-e^{-kt})$	A1		Correct value for c	
				A1		Must be in the form $v = \ldots$	

15	(b)			B1 B1 [2]	1.1b 1.1b	Graph through (0, 0) with the correct general shape or correct for their v. Horizontal asymptote labelled $\frac{9.8}{k}$	Ignore any graph for $t<0$ Any graph involving $\sqrt{t}$ must be concave
15	(c)		As $t\to\infty,\ v\to\frac{9.8}{k}=7$ $k=1.4$	M1 A1 [2]	3.4 1.1b	Recognises the limiting value Any form from correct v	Allow M1 for their v used, provided it has a non-zero limiting value
			Alternative solution Limiting value when $\frac{dv}{dt}=0$ $k=\frac{9.8}{7}=1.4$	M1 A1		Could be implied	
15	(d)		$v=3.5\Rightarrow 1-e^{-1.4t}=\frac{3.5}{7}$ Hence $-1.4t=\ln 0.5\Rightarrow t=0.495$	B1 [1]	3.4	May substitute in other versions of the equation relating v and t Allow for 0.50 or awrt 0.495	No FT from incorrect expression for v

16	(a)		Resolve down the plane $2g\sin 20° = 2a$ $a = g\sin 20°[= 3.352]$ $s = ut + \frac{1}{2}at^2$ with $s = 0.7,\ u = 1.4$ and their a $0.7 = 1.4t + \frac{1}{2}(g\sin 20°)t^2$ $(4.9\sin 20°)t^2 + 1.4t - 0.7 = 0$ Time taken is 0.352 s	**M1** **A1** **M1** **A1** **A1** **[5]**	**3.1b** **3.3** **3.3** **3.4** **3.2a**	N2L with no other forces with attempt to resolving the weight Allow $a = -g\sin 20°$ if it is clear that up the slope is the positive direction. Using suvat equation(s) leading to an equation for t; allow sign errors Correct equation using their 0.5a Allow coefficient of t^2 1.7 or better Cao Must select correct root if two roots given	Allow sin/cos interchange for M mark Solution BC is sufficient Allow if positive root only seen
16	(b)		Friction (and/or air resistance) would have the effect of slowing the particle so ignoring friction underestimates time	**B1** **[1]**	**3.5a**	Needs to indicate why ignoring resistance produces an underestimate	eg "friction will slow it down" for B1
16	(c)		$s = 0.7,\ u = 1.4,\ t = 0.8 \Rightarrow 0.7 = 1.4\times 0.8 + \frac{1}{2}a\times 0.8^2$ $a = \frac{0.7 - 1.12}{0.32} = -1.3125$ (−1.31 to 3 sf)	**M1** **A1** **[2]**	**3.3** **1.1b**	Use of suvat equation(s) to find a; allow sign errors Allow $a = 1.3125$ if sign convention clear	eg both u and s negative
16	(d)		N2L down the plane $2g\sin 20° - F = 2\times(-1.3125)$ $(F = 9.32859...)$ Resolve perpendicular to the plane $R = 2g\cos 20°$ $(R = 18.4\ldots)$ Use of $F = \mu R$ $\mu = \frac{9.32859...}{18.4...} = 0.506$ (3sf)	**M1** **A1** **M1** **A1** **M1** **A1** **[6]**	**3.1b** **1.1b** **3.3** **1.1b** **3.4** **1.1b**	All forces present and weight resolved; allow sin/cos interchange Fully correct equation; F need not be evaluated here No extra forces Fully correct equation; R need not be evaluated here Allow for their F and R used Accept 0.506 or 0.507 Must be 3sf	Notice R may be found first. Allow sin/cos interchange in their first equation but the second equation must be consistent to award M1

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