Mark Scheme 4754
June 2005

1. (a) Please mark in red and award part marks on the right side of the script, level with the work that has earned them.
   (b) If a part of a question is completely correct, or only *one* accuracy mark has been lost, the total mark or slightly reduced mark should be put in the margin at the end of the section, shown as, for example, 7 or 7 – 1, without any ringing. Otherwise, part marks should be shown as in the mark scheme, as M1, A1, B1, etc.
   (c) The total mark for the question should be put in the right hand margin at the end of each question, and ringed.

2. Every page of the script should show evidence that it has been assessed, even if the work has scored no marks.

3. Do not assume that, because an answer is correct, so is the intermediate working; nor that, because an answer is wrong, no marks have been earned.

4. Errors, slips, etc. should be marked clearly where they first occur by underlining or ringing. Missing work should be indicated by a caret (∧).
   - For correct work, use ✓,
   - For incorrect work, use X,
   - For correct work after and error, useˋ✓
   - For error in follow through work, useˋˋ✓

5. An ‘M’ mark is earned for a correct method (or equivalent method) for that part of the question. A method may contain incorrect working, but there must be sufficient evidence that, if correct, it would have given the correct answer.

   An ‘A’ mark is earned for accuracy, but cannot be awarded if the corresponding M mark has not been earned. An A mark shown as A1 f.t. or A1 ✓ shows that the mark has been awarded following through on a previous error.

   A ‘B’ mark is an accuracy mark awarded independently of any M mark.

   ‘E’ marks are accuracy marks dependent on an M mark, used as a reminder that the answer has been given in the question and must be fully justified.

6. If a question is misread or misunderstood in such a way that the nature and difficulty of the question is unaltered, follow the work through, awarding all marks earned, but deducting one mark once, shown as MR – 1, from any accuracy or independent marks earned in the affected work. If the question is made easier by the misread, then deduct more marks appropriately.

7. Mark deleted work if it has not been replaced. If it has been replaced, ignore the deleted work and mark the replacement.

8. Other abbreviations:

c.a.o. : correct answer only
b.o.d. : benefit of doubt (where full work is not shown)
X
: work of no mark value between crosses
X
s.o.i. : seen or implied
s.c. : special case (as defined in the mark scheme)
w.w.w : without wrong working

Procedure

1. Before the Examiners' Meeting, mark at least 10 scripts of different standards and bring them with you to the meeting. List any problems which have occurred or that you can foresee.

2. After the meeting, mark 7 scripts and the 3 photocopied scripts provided and send these to your team leader. Keep a record of the marks, and enclose with your scripts a stamped addressed envelope for their return. Your team leader will contact you by telephone or email as soon as possible with any comments. You must ensure that the corrected marks are entered on to the mark sheet.

3. By a date agreed at the standardisation meeting prior to the batch 1 date, send a further sample of about 40 scripts, from complete centres. You should record the marks for these scripts on your marksheets. They will not be returned to you, but you will receive feedback on them. If all is well, you will then be given clearance to send your batch 1 scripts and marksheets to Cambridge.

4. Towards the end of the marking period, your team leader will request a final sample of about 60 scripts. This sample will consist of complete centres and will not be returned to you. The marks must be entered on the mark sheets before sending the scripts, and should be sent, with the remainder of your marksheets, to the office by the final deadline.

5. Please contact your team leader by telephone or email in case of difficulty. Contact addresses and telephone numbers will be found in your examiner packs.

SECTION A

1 $3\cos\theta + 4\sin\theta = R\cos(\theta - \alpha)$ $= R(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$ $\Rightarrow R\cos\alpha = 3, R\sin\alpha = 4$ $\Rightarrow R^2 = 3^2 + 4^2 = 25, R = 5$ $\tan\alpha = 4/3 \Rightarrow \alpha = 0.927$ $f(\theta) = 7 + 5\cos(\theta - 0.927)$ $\Rightarrow$ Range is 2 to 12 Greatest value of $\dfrac{1}{7+3\cos\theta+4\sin\theta}$ is ½ .	**B1** **M1** **A1** **M1** **A1** **B1ft** **[6]**	R=5 tan α=4/3 oe ft their R 0.93 or 53.1° or better their cos (θ - 0. 927) = 1 or -1 used (*condone use of graphical calculator)* 2 and 12 seen cao simplified
2 $\sqrt{4+2x} = 2(1+\frac{1}{2}x)^{\frac{1}{2}}$ $= 2\{1+\frac{1}{2}.(\frac{1}{2}x)+\frac{\frac{1}{2}.(-\frac{1}{2})}{2!}(\frac{1}{2}x)^2+\frac{\frac{1}{2}.(-\frac{1}{2}).(-\frac{3}{2})}{3!}(\frac{1}{2}x)^3+...\}$ $= k\left(1+\frac{1}{4}x-\frac{1}{32}x^2+\frac{1}{128}x^3+...\right)$ $= \left(2+\frac{1}{2}x-\frac{1}{16}x^2+\frac{1}{64}x^3+...\right)$ Valid for $-2 < x < 2$.	**M1** **M1** **A2,1,0** **A1cao** **B1cao** **[6]**	Taking out 4 oe correct binomial coefficients $\frac{1}{4}x, -\frac{1}{32}x^2, +\frac{1}{128}x^3$
3 $\sec^2\theta = 4$ $\Rightarrow \dfrac{1}{\cos^2\theta} = 4$ $\Rightarrow \cos^2\theta = ¼$ $\Rightarrow \cos\theta = ½$ or $-½$ $\Rightarrow \theta = \pi/3, 2\pi/3$ **OR** $\sec^2\theta = 1 + \tan^2\theta$ $\Rightarrow \tan^2\theta = 3$ $\Rightarrow \tan\theta = \sqrt{3}$ or $-\sqrt{3}$ $\Rightarrow \theta = \pi/3, 2\pi/3$	**M1** **M1** **A1 A1** **M1** **M1** **A1 A1** **[4]**	sec θ = 1/cos θ used ± ½ allow unsupported answers ± √3 allow unsupported answers

4 $V = \int \pi y^2 dx$	**M1**	Correct formula
$= \int_0^1 \pi(1+e^{-2x})dx$	**M1**	$k\int_0^1 \left(1+e^{-2x}\right)dx$
$= \pi\left[x - \frac{1}{2}e^{-2x}\right]_0^1$	**B1**	$\left[x - \frac{1}{2}e^{-2x}\right]$
$= \pi(1 - ½ \mathrm{e}^{-2} + ½)$	**M1**	substituting limits. Must see *0* used. Condone omission of π
$= \pi(1½ \ - ½ \mathrm{e}^{-2})$	**A1** **[5]**	o.e. but must be exact
5 2cos 2x = 2(2cos^2x – 1) = 4cos^2x – 2	**M1**	Any double angle formula used
⇒ 4cos^2x – 2 = 1 + cos x		
⇒ 4cos^2x – cos x – 3 = 0	**M1**	*getting a quadratic in cos x*
⇒ (4cos$x$ + 3)(cos x – 1) = 0	**M1dep**	attempt to solve
	A1	for -3/4 and 1
⇒ cos x = –3/4 or 1		
⇒ x = 138.6° or 221.4°	**B1 B1**	139,221 or better
or 0	**B1**	www
	[7]	-1 extra solutions in range
6 (i) $y^2 - x^2 = (t + 1/t)^2 - (t - 1/t)^2$	**M1**	Substituting for x and y in terms of t oe
$= t^2 + 2 + 1/t^2 - t^2 + 2 - 1/t^2$		
$= 4$	**E1** **[2]**	
(ii) EITHER dx/dt = 1 + 1/t^2, dy/dt = 1 – 1/t^2	**B1**	For both results
⇒ $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$	**M1**	
$= \frac{1-1/t^2}{1+1/t^2}$		
$= \frac{t^2-1}{t^2+1} = \frac{(t-1)(t+1)}{t^2+1}$ *	**E1**	
OR $2y\frac{dy}{dx} - 2x = 0$	**B1**	
⇒ $\frac{dy}{dx} = \frac{x}{y} = \frac{t-1/t}{t+1/t}$	**M1**	
$= \frac{t^2-1}{t^2+1} = \frac{\left(t-1\right)\left(t+1\right)}{t^2+1}$	**E1**	
OR $y = \sqrt{(4+x^2)}$,		
⇒ $\frac{dy}{dx} = \frac{x}{\sqrt{4+x^2}}$	**B1**	
$= \frac{t-1/t}{\sqrt{4+t^2-2+1/t^2}}$		
$= \frac{t-1/t}{\sqrt{\left(t+1/t^2\right)}} = \frac{t-1/t}{\left(t+1/t\right)}$	**M1**	
$= \frac{t^2-1}{t^2+1} = \frac{\left(t-1\right)\left(t+1\right)}{t^2+1}$	**E1**	
⇒ dy/dx = 0 when t = 1 or –1	**M1**	
t = 1, ⇒ (0, 2)	**A1 A1**	
t = –1 ⇒ (0, –2)	**[6]**	

SECTION B

7 (i) $\int \frac{t}{1+t^2}dt$ = ½ ln(1 + t^2) + c **OR** $\int \frac{t}{1+t^2}dt$ let $u = 1 + t^2$, du = 2tdt $= \int \frac{1/2}{u}du$ = ½ ln u + c = ½ ln(1 + t^2) + c	**M1** **A2** **M1** **A1** **A1** **[3]**	k ln(1 + t^2) ½ ln(1 + t^2) [+c] substituting $u = 1 + t^2$ condone no c
(ii) $\frac{1}{t(1+t^2)} = \frac{A}{t} + \frac{Bt+C}{1+t^2}$ $\Rightarrow$ 1 = A(1+ t^2) + (Bt + C)t $t = 0 \Rightarrow 1 = A$ coeff^t of t^2 $\Rightarrow 0 = A + B$ $\Rightarrow B = -1$ coeff^t of t $\Rightarrow 0 = C$ $\Rightarrow \frac{1}{t(1+t^2)} = \frac{1}{t} - \frac{t}{1+t^2}$	**M1** **M1** **A1** **A1** **A1** **[5]**	Equating numerators substituting or equating coeff^{t}s dep 1st M1 A = 1 B = –1 C = 0
(iii) $\frac{dM}{dt} = \frac{M}{t(1+t^2)}$ $\Rightarrow \int \frac{1}{M}dM = \int \frac{1}{t(1+t^2)}dt = \int [\frac{1}{t} - \frac{t}{1+t^2}]dt$ $\Rightarrow$ ln M = ln t – ½ ln(1 + t^2) + c $= \ln(\frac{e^c t}{\sqrt{1+t^2}})$ $\Rightarrow M = \frac{Kt}{\sqrt{1+t^2}}$ * where $K = e^c$	**M1** **B1** **A1ft** **M1** **M1** **E1** **[6]**	Separating variables and substituting their partial fractions ln M = … ln t – ½ ln(1 + t^2) + c combining ln t and ½ ln(1 + t^2) $K = e^c$ o.e.
(iv) t = 1, M = 25 $\Rightarrow$ 25 = K/√2 $\Rightarrow$ K = 25√2 = 35.36 As $t \to \infty$, $M \to K$ So long term value of M is 35.36 grams	**M1** **A1** **M1** **A1ft** **[4]**	25√2 or 35 or better soi ft their K.
8 (i) P is (0, 10, 30) Q is (0, 20, 15) R is (–15, 20, 30) $\Rightarrow \overrightarrow{PQ} = \begin{pmatrix} 0-0 \\ 20-10 \\ 15-30 \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \\ -15 \end{pmatrix}$ * $\Rightarrow \overrightarrow{PR} = \begin{pmatrix} -15-0 \\ 20-10 \\ 30-30 \end{pmatrix} = \begin{pmatrix} -15 \\ 10 \\ 0 \end{pmatrix}$ *	**B2,1,0** **E1** **E1** **[4]**	

(ii) $\begin{pmatrix}2\\3\\2\end{pmatrix}.\begin{pmatrix}0\\10\\-15\end{pmatrix}=0+30-30=0$ $\begin{pmatrix}2\\3\\2\end{pmatrix}.\begin{pmatrix}-15\\10\\0\end{pmatrix}=-30+30+0=0$ $\Rightarrow$ $\begin{pmatrix}2\\3\\2\end{pmatrix}$ is normal to the plane $\Rightarrow$ equation of plane is $2x + 3y + 2z = c$ At P (say), $x = 0$, $y = 10$, $z = 30$ $\Rightarrow$ $c = 2\times0 + 3\times10 + 2\times30 = 90$ $\Rightarrow$ equation of plane is $2x + 3y + 2z = 90$	**M1** **E1** **M1** **M1dep** **A1 cao** **[5]**	Scalar product with 1 vector in the plane OR vector x product oe $2x + 3y + 2z = c$ **or** an appropriate vector form substituting to find c **or** completely eliminating parameters
(iii) S is $(-7\frac{1}{2}, 20, 22\frac{1}{2})$ $\overrightarrow{OT}=\overrightarrow{OP}+\frac{2}{3}\overrightarrow{PS}$ $=\begin{pmatrix}0\\10\\30\end{pmatrix}+\frac{2}{3}\begin{pmatrix}-7\frac{1}{2}\\10\\-7\frac{1}{2}\end{pmatrix}=\begin{pmatrix}-5\\16\frac{2}{3}\\25\end{pmatrix}$ So T is $(-5,16\frac{2}{3},25)$ *	**B1** **M1** **A1ft** **E1** **[4]**	**Or** $\frac{1}{3}(\overrightarrow{OP}+\overrightarrow{OR}+\overrightarrow{OQ})$ oe ft their S **Or** $\frac{1}{3}\begin{pmatrix}0\\10\\30\end{pmatrix}+\frac{2}{3}\begin{pmatrix}-7\frac{1}{2}\\20\\22\frac{1}{2}\end{pmatrix}$ ft their S
(iv) $\mathbf{r}=\begin{pmatrix}-5\\16\frac{2}{3}\\25\end{pmatrix}+\lambda\begin{pmatrix}2\\3\\2\end{pmatrix}$ At C (–30, 0, 0): $-5 + 2\lambda = -30$, $16\frac{2}{3}+3\lambda=0$, $25 + 2\lambda = 0$ 1st and 3rd eqns give $\lambda = -12\frac{1}{2}$, not compatible with 2nd. So line does not pass through C.	**B1,B1** **M1** **A1** **E1** **[5]**	$\begin{pmatrix}-5\\16\frac{2}{3}\\25\end{pmatrix}+\ldots\ldots+\lambda\begin{pmatrix}2\\3\\2\end{pmatrix}$ Substituting coordinates of C into vector equation At least 2 relevant correct equations for λ oe www

COMPREHENSION

<table>
<tr><td>1. The masses are measured in units.
The ratio is dimensionless</td><td>B1
B1
[2]</td><td></td></tr>
<tr><td>2. Converting from base 5,
$3.03232 = 3 + \frac{0}{5} + \frac{3}{5^2} + \frac{2}{5^3} + \frac{3}{5^4} + \frac{2}{5^5}$
= 3.14144</td><td>M1
A1
[2]</td><td></td></tr>
<tr><td>3.
<table>
<tr><th>n</th><th>x_n</th></tr>
<tr><td>0</td><td>0.5</td></tr>
<tr><td>1</td><td>0.8</td></tr>
<tr><td>2</td><td>0.512</td></tr>
<tr><td>3</td><td>0.7995392</td></tr>
<tr><td>4</td><td>0.5128840565</td></tr>
<tr><td>5</td><td>0.7994688035</td></tr>
</table></td><td>B1</td><td>Condone variations in last digits</td></tr>
<tr><td>4.
$\frac{\phi}{1} = \frac{1}{\phi - 1}$
$\Rightarrow \phi^2 - \phi = 1 \quad \Rightarrow \phi^2 - \phi - 1 = 0$
Using the quadratic formula gives
$\phi = \frac{1 \pm \sqrt{5}}{2}$</td><td>M1
E1</td><td>
Or complete verification B2</td></tr>
<tr><td>5.
$\frac{1}{\phi} = \frac{1}{\frac{1+\sqrt{5}}{2}} = \frac{2}{1+\sqrt{5}}$
$= \frac{2}{\sqrt{5}+1} \times \frac{\sqrt{5}-1}{\sqrt{5}-1}$
$= \frac{2(\sqrt{5}-1)}{(\sqrt{5})^2 - 1} = \frac{2(\sqrt{5}-1)}{4} \quad = \frac{\sqrt{5}-1}{2}$
OR
$\frac{1}{\phi} = \phi - 1$
$= \frac{\sqrt{5}+1}{2} - 1 = \frac{\sqrt{5}-1}{2}$</td><td>M1
M1
E1
M1
M1
E1
[3]</td><td>Must discount ±
Must discount ±
Substituting for ϕ and simplifying</td></tr>
</table>

6. Let $r = \dfrac{a_{n+1}}{a_n}$ and $r = \dfrac{a_n}{a_{n-1}}$ $a_{n+1} = 2a_n + 3a_{n-1}$ dividing through by a_n. $\Rightarrow r = 2 + \dfrac{3}{r}$ $\Rightarrow r^2 - 2r - 3 = 0$ $\Rightarrow (r-3)(r+1) = 0$ $\Rightarrow$ r = 3 (discounting -1)	**M1** **M1** **A1** **E1** **[4]**	For either ratio used $r = 2 + \dfrac{3}{r}$ SC B2 if dividing terms as far as $a_9/a_8 = a_{10}/a_9$ =3.00
7. The length of the next interval = l, where $\dfrac{0.0952...}{l} = 4.669...$ $\Rightarrow$ l = 0.0203 So next bifurcation at 3.5437… + 0.0203…≈ 3.564	**M1** **A1** **DM1** **A1** **[4]**	