

Mark Scheme (Final) Summer 2007

GCE

GCE Mathematics (6663/01)





General Principal for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

 $(x^{2} + bx + c) = (x + p)(x + q)$, where |pq| = |c|, leading to x = ...

 $(ax^2 + bx + c) = (mx + p)(nx + q)$, where |pq| = |c| and |mn| = |a|, leading to x = ...

2. Formula

Attempt to use $\underline{correct}$ formula (with values for a, b and c).

3. <u>Completing the square</u>

Solving $x^2 + bx + c = 0$: $(x \pm p)^2 \pm q \pm c$, $p \neq 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \rightarrow x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \rightarrow x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an <u>exact</u> answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these <u>may</u> not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does <u>not</u> cover this, please contact your team leader for advice.

June 2007 6663 Core Mathematics C1 Mark Scheme

Question number		Scheme	Marks	
1.	9 – 5	or $3^2 + 3\sqrt{5} - 3\sqrt{5} - \sqrt{5} \times \sqrt{5}$ or $3^2 - \sqrt{5} \times \sqrt{5}$ or $3^2 - (\sqrt{5})^2$	M1	
	= <u>4</u>		A1cso	(2) 2
	M1	for an attempt to multiply out. There must be at least 3 correct terms. Allo only, no arithmetic errors.	w one sign slij	p
	e.g.	$3^2 + 3\sqrt{5} - 3\sqrt{5} + (\sqrt{5})^2$ is M1A0		
		$3^2 + 3\sqrt{5} + 3\sqrt{5} - (\sqrt{5})^2$ is M1A0 as indeed is $9 \pm 6\sqrt{5} - 5$		
	BUT	$9 + \sqrt{15} - \sqrt{15} - 5 = 4$ is M0A0 since there is more than a sign error.		
		$6+3\sqrt{5}-3\sqrt{5}-5$ is M0A0 since there is an arithmetic error.		
		If all you see is 9 ± 5 that is M1 but please check it has not come from inco	orrect working	
		Expansion of $(3+\sqrt{5})(3+\sqrt{5})$ is M0A0		
	A1cso	for 4 only. Please check that no incorrect working is seen.		
		Correct answer only scores both marks.		

Question number	Scheme	Marks	
2.	(a) Attempt $\sqrt[3]{8}$ or $\sqrt[3]{(8^4)}$	M1	
	= <u>16</u>	A1	(2)
	$= 16(b) 5 x^{\frac{1}{3}}$ 5, $x^{\frac{1}{3}}$	B1, B1	(2)
			4
(a)	M1 for: 2 (on its own) or $(2^3)^{\frac{4}{3}}$ or $\sqrt[3]{8}$ or $(\sqrt[3]{8})^4$ or 2^4 or $\sqrt[3]{8^4}$ or $\sqrt[3]{4096}$ 8^3 or 512 or $(4096)^{\frac{1}{3}}$ is M0 A1 for 16 only		
(b)	1^{st} B1 for 5 on its own or × something.		
	So e.g. $\frac{5x^{\frac{4}{3}}}{x}$ is B1 But $5^{\frac{1}{3}}$ is B0 An expression showing cancelling is not sufficient		
	(see first expression of QC0184500123945 the mark is scored for the second	nd expression)	
	$2^{nd} B1$ for $x^{\frac{1}{3}}$		
	Can use ISW (incorrect subsequent working) e.g $5x^{\frac{4}{3}}$ scores B1B0 but it may lead to $\sqrt[3]{5x^4}$ which we ignore as ISW.		
	Correct answers only score full marks in both parts.		

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Question number	Scheme	Marks	
3.	(a) $\left(\frac{dy}{dx}\right) = 6x^1 + \frac{4}{2}x^{-\frac{1}{2}}$ or $\left(6x + 2x^{-\frac{1}{2}}\right)$	M1 A1	(2)
	(b) $6 + -x^{-\frac{3}{2}}$ or $6 + -1 \times x^{-\frac{3}{2}}$	M1 A1ft	(2)
	(a) $\left(\frac{dy}{dx}\right) = 6x^{1} + \frac{4}{2}x^{-\frac{1}{2}}$ or $\left(6x + 2x^{-\frac{1}{2}}\right)$ (b) $6x - x^{-\frac{3}{2}}$ or $6x - 1 \times x^{-\frac{3}{2}}$ (c) $x^{3} + \frac{8}{3}x^{\frac{3}{2}} + C$ A1: $\frac{3}{3}x^{3}$ or $\frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}$ A1: both, simplified and $+C$	M1 A1 A1	(3)
			7
(a)	M1 for <u>some</u> attempt to differentiate: $x^n \to x^{n-1}$ Condone missing $\frac{dy}{dx}$ or $y = \dots$		
	A1 for both terms correct, as written or better. No + <i>C</i> here. Of course $\frac{2}{\sqrt{x}}$ is	acceptable.	
(b)	M1 for some attempt to differentiate again. Follow through their $\frac{dy}{dx}$, at least of or correct follow through.	one term correc	t
	A1f.t. as written or better, follow through must have 2 distinct terms and simplified	ed e.g. $\frac{4}{4} = 1$.	
(c)	M1 for some attempt to integrate: $x^n \to x^{n+1}$. Condone misreading $\frac{dy}{dx}$ or $\frac{d^2y}{dx^2}$ (+ <i>C</i> alone is not sufficient)		
	1 st A1 for either $\frac{3}{3}x^3$ or $\frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}$ (or better) $\frac{2}{3} \times 4x^{\frac{3}{2}}$ is OK here too but not for 2 nd	A1.	
	$2^{\text{nd}} \text{ A1 for } \underline{\text{both}} x^3 \text{ and } \frac{8}{3}x^{\frac{3}{2}} \text{ or } \frac{8}{3}x\sqrt{x}$ i.e. simplified terms $\underline{\text{and}} +C$ all on one l	ine.	
	$2\frac{2}{3}$ instead of $\frac{8}{3}$ is OK		

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Question number	Scheme	Marks
4.	(a) Identify $a = 5$ and $d = 2$ (May be implied)	B1
	$(u_{200} =)a + (200 - 1)d$ (= 5 + (200 - 1)×2)	M1
	= <u>403(p)</u> or (£) <u>4.03</u>	A1 (3)
	(b) $(S_{200} =) \frac{200}{2} [2a + (200 - 1)d]$ or $\frac{200}{2} (a + "\text{their } 403")$	M1
	$=\frac{200}{2} [2 \times 5 + (200 - 1) \times 2] \text{ or } \frac{200}{2} (5 + \text{"their 403"})$	A1
	= <u>40 800</u> or <u>£408</u>	A1 (3)
(a)	B1 can be implied if the correct answer is obtained. If 403 is <u>not</u> obtained th a and d must be clearly identified as $a = 5$ and $d = 2$.	en the values of
	This mark can be awarded at any point.	
	M1 for attempt to use <i>n</i> th term formula with $n = 200$. Follow through their <i>a</i> Must have use of $n = 200$ and one of <i>a</i> or <i>d</i> correct or correct follow through	
	Must be 199 not 200.	
	A1 for 403 or 4.03 (i.e. condone missing £ sign here). Condone £403 here.	
N.B.	$a = 3$, $d = 2$ is B0 and $a + 200d$ is M0 <u>BUT</u> $3 + 200 \times 2$ is B1M1 and A1 Answer only of 403 (or 4.03) scores 3/3.	if it leads to 403.
(b)	M1 for use of correct sum formula with $n = 200$. Follow through their <i>a</i> and Must have some use of $n = 200$, and some of <i>a</i> , <i>d</i> or <i>l</i> correct or correct fo	
	1 st A1 for any correct expression (i.e. must have $a = 5$ and $d = 2$) but can f.t. the	e
	2^{nd} A1 for 40800 or £408 (i.e. the £ sign is required before we accept 408 this times the formula of the	ne).
	40800p is fine for A1 but £40800 is A0.	
ALT	Listing	
(a)	They might score B1 if $a = 5$ and $d = 2$ are clearly identified. Then award M1A1	together for 403.
(b)	$\sum_{r=1}^{200} (2r+3)$. Give M1 for $2 \times \frac{200}{2} \times (201) + 3k$ (with $k > 1$), A1 for $k = 200$ and A2	1 for 40800.

Question number	Scheme	Marks	
5.	(a) Translation parallel to <i>x</i> -axis Top branch intersects +ve <i>y</i> -axis Lower branch has no intersections No obvious overlap	M1 A1	
	$\left(0,\frac{3}{2}\right)$ or $\frac{3}{2}$ marked on y- axis	B1 (3)	
S.C.	(b) $x = -2$, $y = 0$ [Allow ft on first B1 for $x = 2$ when translated "the wrong way" but must be compatible with their sketch.]	B1, B1 (2) 5	
(a)	 M1 for a horizontal translation – two branches with one branch cutting y – axis only. If one of the branches cuts both axes (translation up and across) this is M0. A1 for a horizontal translation to left. Ignore any figures on axes for this mark. B1 for correct intersection on positive y-axis. More than 1 intersection is B0. x=0 and y = 1.5 in a table alone is insufficient unless intersection of their sketch is with +ve y-axis. A point marked on the graph overrides a point given elsewhere. 		
(b)	1 st B1 for $x = -2$. NB $x \neq -2$ is B0. Can accept $x = +2$ if this is compatible with their sketch. Usually they will have M1A0 in part (a) (and usually B0 too) 2 nd B1 for $y = 0$.		
S.C.	If $x = -2$ and $y = 0$ and some other asymptotes are also given award B1B0 The asymptote equations should be clearly stated in part (b). Simply marking $x = -2$ or $y = 0$		
	on the sketch is insufficient <u>unless</u> they are clearly marked "asymptote $x = -2$ " etc.		

Question number	Scheme	Marks
6.	(a) $2x^2 - x(x-4) = 8$	M1
	$x^2 + 4x - 8 = 0 \tag{(*)}$	A1cso (2)
	(b) $x = \frac{-4 \pm \sqrt{4^2 - (4 \times 1 \times -8)}}{2}$ or $(x+2)^2 \pm 4 - 8 = 0$	M1
	$x = -2 \pm (any correct expression)$	A1
	$\sqrt{48} = \sqrt{16}\sqrt{3} = 4\sqrt{3}$ or $\sqrt{12} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$	B1
	$y = (-2 \pm 2\sqrt{3}) - 4$ M: Attempt at least one y value	M1
	$x = -2 + 2\sqrt{3}, y = -6 + 2\sqrt{3}$ $x = -2 - 2\sqrt{3}, y = -6 - 2\sqrt{3}$	A1 (5)
		7
(a)	M1 for correct attempt to form an equation in x only. Condone sign errors/slip	ps but attempt at
	this line must be seen. E.g. $2x^2 - x^2 \pm 4x = 8$ is OK for M1.	
	A1cso for correctly simplifying to printed form. No incorrect working seen. The	= 0 <u>is</u> required.
	These two marks can be scored in part (b). For multiple attempts pic	k best.
(b)	1 st M1 for use of correct formula. If formula is not quoted then a fully correct su	bstitution is
	required. Condone missing $x = \text{ or just} + \text{ or } - \text{ instead of } \pm \text{ for } M1$.	
	For completing the square must have as printed or better.	
	If they have $x^2 - 4x - 8 = 0$ then M1 can be given for $(x-2)^2 \pm 4 - 8 = 0$.	
	1 st A1 for $-2 \pm any$ correct expression. (The \pm is required but $x = is$ not)	
	B1 for simplifying the surd e.g. $\sqrt{48} = 4\sqrt{3}$. Must reduce to $b\sqrt{3}$ so $\sqrt{16}\sqrt{3}$	or $\sqrt{4}\sqrt{3}$ are OK.
	2^{nd} M1 for attempting to find at least one <i>y</i> value. Substitution into one of the give	
	and an attempt to solve for y.	-
	2^{nd} A1 for correct y answers. Pairings need <u>not</u> be explicit but they must say whi	ch is x and which y .
	Mis-labelling x and y loses final A1 only.	

Question number	Scheme	Marks
7.	(a) Attempt to use discriminant $b^2 - 4ac$	M1
	$k^{2} - 4(k+3) > 0 \implies k^{2} - 4k - 12 > 0$ (*)	A1cso (2)
	(b) $k^2 - 4k - 12 = 0 \implies$	
	$(k \pm a)(k \pm b)$, with $ab = 12$ or $(k =)\frac{4 \pm \sqrt{4^2 - 4 \times 12}}{2}$ or $(k-2)^2 \pm 2^2 - 12$	M1
	k = -2 and 6 (both)	A1
	$k < -2, k > 6$ or $(-\infty, -2); (6, \infty)$ M: choosing "outside"	M1 A1ft (4)
		6
(a)	M1 for use of $b^2 - 4ac$, one of <i>b</i> or <i>c</i> must be correct. Or full attempt using completing the square that leads to a 3TQ in <i>k</i> e.g. $\left[\left[x + \frac{k}{2}\right]^2 = \right] \frac{k^2}{4} - (k+3)$ A1cso Correct argument to printed result. Need to state (or imply) that $b^2 - 4ac > 1$ incorrect working seen. Must have >0. If > 0 just appears with $k^2 - 4(k+1)$ If >0 appears on last line only with no explanation give A0. $b^2 - 4ac$ followed by $k^2 - 4k - 12 > 0$ only is insufficient so M0A0 e.g. $k^2 - 4 \times 1 \times k + 3$ (missing brackets) can get M1A0 but $k^2 + 4(k+3)$ is M0A0 (Using $\sqrt{b^2 - 4ac} > 0$ is M0.	3) > 0 that is OK.
(b)	1^{st} M1for attempting to find critical regions. Factors, formula or complete 1^{st} A1for $k = 6$ and -2 only 2^{nd} M1for choosing the outside regions 2^{nd} A1f.t.as printed or f.t. their (non identical) critical values $6 < k < -2$ is M1A0 but ignore if it follows a correct version $-2 < k < 6$ is M0A0 whatever their diagram looks likeCondone use of x instead of k for critical values and final answers in (b).	ng the square.
	Treat this question as 3 two mark parts. If part (a) is seen in (b) or vice versa mark	s can be awarded.

Question number	Scheme	Marks	
8.	(a) $(a_2 = \underline{)3k + 5}$ [must be seen in part (a) or labelled $a_2 =]$	B1	(1)
	(b) $(a_3 =)3(3k+5)+5$	M1	
	$=\underline{9k+20} \tag{(*)}$	A1cso	(2)
	(c)(i) $a_4 = 3(9k + 20) + 5 (= 27k + 65)$	M1	
	$\sum_{r=1}^{4} a_r = k + (3k+5) + (9k+20) + (27k+65)$	M1	
	(ii) $= 40k + 90$	A1	
	= 10(4k+9) (or explain why divisible by 10)		(4) 7
(b)	M1 for attempting to find a_3 , follow through their $a_2 \neq k$. A1cso for simplifying to printed result with no incorrect working seen.		
(c)	1 st M1 for attempting to find a_4 . Can allow a slip here e.g. $3(9k + 20)$ [i.e.	e. forgot +5]	
	2 nd M1 for attempting sum of 4 relevant terms, follow through their (a) an	d (b).	
	Must have 4 terms starting with <i>k</i> .		
	Use of arithmetic series formulae at this point is M0A0A0		
	1 st A1 for simplifying to $40k + 90$ or better		
	2 nd A1ft for taking out a factor of 10 or dividing by 10 or an explanation in	words true $\forall k$.	
	Follow through their sum of 4 terms provided that both Ms are		
	scored and their sum <u>is</u> divisible by 10.		
	A comment is <u>not</u> required.		
	e.g. $\frac{40k+90}{10} = 4k+9$ is OK for this final A1.		
S.C.	$\sum_{r=2}^{5} a_r = 120k + 290 = 10(12k + 29)$ can have M1M0A0A1ft.		

Question number	Scheme	Marks	
9.	(a) $f(x) = \frac{6x^3}{3} - \frac{10x^2}{2} - 12x \ (+C)$	M1 A1	
	$x = 5: \qquad 250 - 125 - 60 + C = 65 \qquad C = 0$	M1 A1	(4)
	(b) $x(2x^2-5x-12)$ or $(2x^2+3x)(x-4)$ or $(2x+3)(x^2-4x)$	M1	
	= x(2x+3)(x-4) (*)	A1cso	(2)
	(c) Shape	B1	
	Through origin	B1	
	$\left(-\frac{3}{2},0\right)$ and (4,0)	B1	(3)
			9
(a)	1 st M1 for attempting to integrate, $x^n \rightarrow x^{n+1}$	1	
	1^{st} A1 for all x terms correct, need not be simplified. Ignore + C here.		
	2^{nd} M1 for some use of $x = 5$ and $f(5)=65$ to form an equation in C based on their f	integration.	
	There must be some visible attempt to use $x = 5$ and $f(5)=65$. No + <i>C</i> is M	0.	
	2^{nd} A1 for $C = 0$. This mark cannot be scored unless a suitable equation is seen.		
(b)	 M1 for attempting to take out a correct factor or to verify. Allow usual errors of They must get to the equivalent of one of the given partially factorised expression verifying, x(2x²+3x-8x-12) i.e. with no errors in signs. A1cso for proceeding to printed answer with no incorrect working seen. Commer This mark is <u>dependent upon a fully correct solution to part (a)</u> so M1A1M0A0M Will be common or M1A1M1A0M1A0. To score 2 in (b) they must score 4 in (a) 	pressions or, if at <u>not</u> required 1A0 for (a) &	
(c)	1 st B1 for positive x^3 shaped curve (with a max and a min) positioned anywhere. 2 nd B1 for any curve that passes through the origin (B0 if it only touches at the or 3 rd B1 for the two points <u>clearly</u> given as coords or values marked in appropriate Ignore any extra crossing points (they should have lost first B1). Condone (1.5, 0) if clearly marked on –ve <i>x</i> -axis. Condone (0, 4) etc if ma Curve can <u>stop</u> (i.e. not pass through) at (-1.5, 0) and (4, 0).	places on <i>x</i> ax	
	A point on the graph overrides coordinates given elsewhere.		

Question number	Scheme	Marks
10.	(a) $x = 1$: $y = -5 + 4 = -1$, $x = 2$: $y = -16 + 2 = -14$ (can be given	1 st B1 for – 1
	in (b) or (c))	2 nd B1 for - 14
	$PQ = \sqrt{\left(2-1\right)^2 + \left(-14 - (-1)\right)^2} = \sqrt{170} $ (*)	M1 A1cso (4)
	(b) $y = x^3 - 6x^2 + 4x^{-1}$	M1
	$\frac{dy}{dx} = 3x^2 - 12x - 4x^{-2}$	M1 A1
	$x = 1$: $\frac{dy}{dx} = 3 - 12 - 4 = -13$ M: Evaluate at one of the points	M1
	$x = 2: \frac{dy}{dx} = 12 - 24 - 1 = -13$: Parallel A: Both correct + conclusion	A1 (5)
	(c) Finding gradient of normal $\left(m = \frac{1}{13}\right)$	M1
	$y1 = \frac{1}{13}(x - 1)$	M1 A1ft
	x - 13y - 14 = 0 o.e.	A1cso (4)
		13
(a)	M1 for attempting PQ or PQ^2 using their P and their Q. Usual rules about que	-
	We must see attempt at $1^2 + (y_P - y_Q)^2$ for M1. $PQ^2 = $ etc could be N	11A0.
(b)	A1cso for proceeding to the correct answer with no incorrect working seen. 1^{st} M1 for multiplying by x^2 , the x^3 or $-6x^2$ must be correct.	
	2 nd M1 for some correct differentiation, at least one term must be correct as printed	1.
	1st A1 for a fully correct derivative.These 3 marks can be awarded anywhere when first seen.	
	3^{rd} M1 for attempting to substitute $x = 1$ or $x = 2$ in their derivative. Substituting i 2^{rd} A1 for -13 from both substitutions and a brief comment.	n y is M0.
	The -13 must come from their derivative.	
(c)	1^{st} M1for use of the perpendicular gradient rule. Follow through their – 1 2^{nd} M1for full method to find the equation of the normal or tangent at P. Iquoted allow slips in substitution, otherwise a correct substitution is	f formula is s required.
	1^{st} A1ft for a correct expression. Follow through their – 1 and their changes 2^{nd} A1cso for a correct equation with = 0 and integer coefficients. This mark is dependent upon the – 13 coming from their derivative Tangant can get MOM1A0A0, shanged gradient can get MOM1A1/	in (b) hence cso.
	Tangent can get M0M1A0A0, changed gradient can get M0M1A1A Condone confusion over terminology of tangent and normal, mark gradient and equ	
MR	Allow for $-\frac{4}{x}$ or (x+6) but not omitting $4x^{-1}$ or treating it as $4x$.	

Question number	Scheme	Marks	
11.	(a) $y = -\frac{3}{2}x(+4)$ Gradient $= -\frac{3}{2}$	M1 A1	(2)
	(b) $3x + 2 = -\frac{3}{2}x + 4$ $x = \dots, \frac{4}{9}$	M1, A1	
	$y = 3\left(\frac{4}{9}\right) + 2 = \frac{10}{3}\left(=3\frac{1}{3}\right)$	A1	(3)
	(c) Where $y = 1$, $l_1 : x_A = -\frac{1}{3}$ $l_2 : x_B = 2$ M: Attempt one of these	M1 A1	
	Area = $\frac{1}{2}(x_B - x_A)(y_P - 1)$	M1	
	$=\frac{1}{2} \times \frac{7}{3} \times \frac{7}{3} = \frac{49}{18} = 2\frac{13}{18}$ o.e.	A1	(4)
			9
(a)	M1 for an attempt to write $3x + 2y - 8 = 0$ in the form $y = mx + c$ or a full method that leads to $m =$, e.g find 2 points, and attempt gradient u e.g. finding $y = -1.5x + 4$ alone can score M1 (even if they go on to say $m =$ A1 for $m = -\frac{3}{2}$ (can ignore the +c) or $\frac{dy}{dx} = -\frac{3}{2}$	= 4)	
(b)	M1 for forming a suitable equation in one variable and attempting to solve lead 1^{st} A1 for any exact correct value for x 2^{nd} A1 for any exact correct value for y (These 3 marks can be scored anywhere, they may treat (a) and (b) as a single for the second se	-	<i>y</i> =
(c)	1 st M1 for attempting the <i>x</i> coordinate of <i>A</i> or <i>B</i> . One correct value seen scores M 1 st A1 for $x_A = -\frac{1}{3}$ and $x_B = 2$	1.	
	2^{nd} M1 for a full method for the area of the triangle – follow through their x_A, x_B, y_B	\mathcal{Y}_P .	
	e.g. determinant approach $\frac{1}{2}\begin{vmatrix} 2 & -\frac{1}{3} & \frac{4}{9} & 2\\ 1 & 1 & \frac{10}{3} & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 2 - \dots - (-\frac{1}{3} \dots) \end{vmatrix}$		
	2^{nd} A1 for $\frac{49}{18}$ or an exact equivalent.		
	All accuracy marks require answers as single fractions or mixed numbers not nece terms.	ssarily in low	est