Mark Scheme 4753
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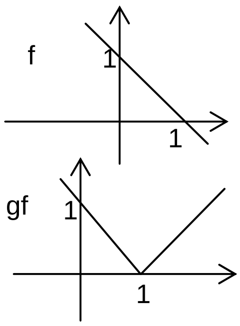
Section A

1 (i) ½ $(1 + 2x)^{-1/2} \times 2$ $= \dfrac{1}{\sqrt{1+2x}}$	M1 B1 A1 [3]	chain rule ½ $u^{-1/2}$ or ½ $(1 + 2x)^{-1/2}$ oe, but must resolve ½ × 2 = 1
(ii) $y = \ln(1-e^{-x})$ $\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{1}{1-e^{-x}}.(-e^{-x})(-1)$ $= \dfrac{e^{-x}}{1-e^{-x}}$ $= \dfrac{1}{e^{x}-1}$ *	M1 B1 A1 E1 [4]	chain rule $\dfrac{1}{1-e^{-x}}$ or $\dfrac{1}{u}$ if substituting $u = 1-e^{-x}$ $\times(-e^{-x})(-1)$ or e^{-x} www (may imply $\times e^{x}$ top and bottom)
2 gf(x) = $\lvert 1-x \rvert$ 	B1 B1 B1 [3]	 intercepts must be labelled line must extend either side of each axis condone no labels, but line must extend to left of y axis
3(i) Differentiating implicitly: $(4y+1)\dfrac{dy}{dx} = 18x$ $\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{18x}{4y+1}$ When $x = 1$, $y = 2$, $\dfrac{dy}{dx} = \dfrac{18}{9} = 2$	M1 A1 M1 A1cao [4]	$(4y+1)\dfrac{dy}{dx} = \ldots$ allow $4y+1\dfrac{dy}{dx} = \ldots$ condone omitted bracket if intention implied by following line. $4y\dfrac{dy}{dx}+1$ M1 A0 substituting $x = 1$, $y = 2$ into their derivative (provided it contains x's and y's). Allow unsupported answers.
(ii) $\dfrac{dy}{dx} = 0$ when $x = 0$ $\Rightarrow \quad 2y^2 + y = 1$ $\Rightarrow \quad 2y^2 + y - 1 = 0$ $\Rightarrow \quad (2y - 1)(y + 1) = 0$ $\Rightarrow \; y = ½$ or $y = -1$ So coords are (0, ½) and (0, –1)	B1 M1 A1 A1 [4]	$x = 0$ from their numerator = 0 (must have a denominator) Obtaining correct quadratic and attempt to factorise or use quadratic formula $y = \dfrac{-1 \pm \sqrt{1-4\times -2}}{4}$ cao allow unsupported answers provided quadratic is shown

4(i) $T = 25 + ae^{-kt}$. When $t = 0$, $T = 100$ $\Rightarrow$ $100 = 25 + ae^0$ $\Rightarrow$ $a = 75$ When $t = 3$, $T = 80$ $\Rightarrow$ $80 = 25 + 75e^{-3k}$ $\Rightarrow$ $e^{-3k} = 55/75$ $\Rightarrow$ $-3k = \ln(55/75)$, $k = -\ln(55/75)/3$ $= 0.1034$	M1 A1 M1 M1 A1cao [5]	substituting $t = 0$ and $T = 100$ into their equation (even if this is an incorrect version of the given equation) substituting $t = 3$ and $T = 80$ into (their) equation taking lns correctly at any stage 0.1 or better or $-\frac{1}{3}\ln(\frac{55}{75})$ o.e. if final answer
(ii) (A) $T = 25 + 75e^{-0.1034\times 5}$ $= 69.72$ (B) 25°C	M1 A1 B1cao [3]	substituting $t = 5$ into their equation 69.5 to 70.5, condone inaccurate rounding due to value of k.
5 $n = 1$, $n^2 + 3n + 1 = 5$ prime $n = 2$, $n^2 + 3n + 1 = 11$ prime $n = 3$, $n^2 + 3n + 1 = 19$ prime $n = 4$, $n^2 + 3n + 1 = 29$ prime $n = 5$, $n^2 + 3n + 1 = 41$ prime $n = 6$, $n^2 + 3n + 1 = 55$ not prime so statement is false	M1 E1 [2]	One or more trials shown finding a counter-example – must state that it is not prime.
6 (i) $-\pi/2 < \arctan x < \pi/2$ $\Rightarrow$ $-\pi/4 < f(x) < \pi/4$ $\Rightarrow$ range is $-\pi/4$ to $\pi/4$	M1 A1cao [2]	$\pi/4$ or $-\pi/4$ or 45 seen not $\leq$
(ii) $y = ½ \arctan x$ $x \leftrightarrow y$ $x = ½ \arctan y$ $\Rightarrow$ $2x = \arctan y$ $\Rightarrow$ $\tan 2x = y$ $\Rightarrow$ $y = \tan 2x$ *either* $\frac{dy}{dx} = 2\sec^2 2x$	M1 A1cao M1 A1cao	tan(arctan y or x) = y or x derivative of tan is $\sec^2$ used
or $y = \frac{\sin 2x}{\cos 2x} \Rightarrow \frac{dy}{dx} = \frac{2\cos^2 2x + 2\sin^2 2x}{\cos^2 2x}$ $= \frac{2}{\cos^2 2x}$	M1 A1cao	quotient rule (need not be simplified but mark final answer)
When $x = 0$, $dy/dx = 2$	B1 [5]	www
(iii) So gradient of $y = ½ \arctan x$ is ½ .	B1ft [1]	ft their '2', but not 1 or 0 or ∞

Section B

7(i) Asymptote when $1 + 2x^3 = 0$ $\Rightarrow$ $2x^3 = -1$ $\Rightarrow$ $x = -\frac{1}{\sqrt[3]{2}}$ $= -0.794$	M1 A1 A1cao [3]	 oe, condone $\pm\frac{1}{\sqrt[3]{2}}$ if positive root is rejected must be to 3 s.f.
(ii) $\frac{dy}{dx} = \frac{(1+2x^3).2x - x^2.6x^2}{(1+2x^3)^2}$ $= \frac{2x+4x^4-6x^4}{(1+2x^3)^2}$ $= \frac{2x-2x^4}{(1+2x^3)^2}$ * $dy/dx = 0$ when $2x(1 - x^3) = 0$ $\Rightarrow$ $x = 0, y = 0$ or $x = 1$, $y = 1/3$	M1 A1 E1 M1 B1 B1 B1 B1 [8]	Quotient or product rule: (udv–vdu M0) $2x(1+2x^3)^{-1} + x^2(-1)(1+2x^3)^{-2}.6x^2$ allow one slip on derivatives correct expression – condone missing bracket if if intention implied by following line derivative = 0 $x = 0$ or 1 – allow unsupported answers $y = 0$ and 1/3 SC–1 for setting denom = 0 or extra solutions (e.g. $x = -1$)
(iii) $A = \int_0^1 \frac{x^2}{1+2x^3}dx$	M1	Correct integral and limits – allow $\int_1^0$
either $= \left[\frac{1}{6}\ln(1+2x^3)\right]_0^1$ $= \frac{1}{6}\ln 3$ *	M1 A1 M1 E1	$k \ln(1 + 2x^3)$ $k = 1/6$ substituting limits dep previous M1 www
or let $u = 1 + 2x^3 \Rightarrow du = 6x^2dx$ $\Rightarrow$ $A = \int_1^3 \frac{1}{6}.\frac{1}{u}du$ $= \left[\frac{1}{6}\ln u\right]_1^3$ $= \frac{1}{6}\ln 3$ *	 M1 A1 M1 E1 [5]	 $\frac{1}{6u}$ $\frac{1}{6}\ln u$ substituting correct limits (but must have used substitution) www

8 (i) $x\cos 2x = 0$ when $x = 0$ or $\cos 2x = 0$ $\Rightarrow$ $2x = \pi/2$ $\Rightarrow$ $x = \frac{1}{4}\pi$ $\Rightarrow$ P is $(\pi/4, 0)$	M1 M1 A1 [3]	$\cos 2x = 0$ or $x = \frac{1}{2}\cos^{-1}0$ $x = 0.785$.. or 45 is M1 M1 A0
(ii) $f(-x) = -x\cos(-2x)$ $= -x\cos 2x$ $= -f(x)$ Half turn symmetry about O.	M1 E1 B1 [3]	$-x\cos(-2x)$ $= -x\cos 2x$ Must have two of: rotational, order 2, about O, (half turn = rotational order 2)
(iii) $f'(x) = \cos 2x - 2x\sin 2x$	M1 A1 [2]	product rule
(iv) $f'(x) = 0 \Rightarrow \cos 2x = 2x\sin 2x$ $\Rightarrow$ $2x\dfrac{\sin 2x}{\cos 2x} = 1$ $\Rightarrow$ $x\tan 2x = \frac{1}{2}$ *	 M1 E1 [2]	 $\dfrac{\sin}{\cos} = \tan$ www
(v) $f'(0) = \cos 0 - 2.0.\sin 0 = 1$ $f''(x) = -2\sin 2x - 2\sin 2x - 4x\cos 2x$ $= -4\sin 2x - 4x\cos 2x$ $\Rightarrow$ $f''(0) = -4\sin 0 - 4.0.\cos 0 = 0$	B1ft M1 A1 E1 [4]	allow ft on (their) product rule expression product rule on $(2)x\sin 2x$ correct expression – mark final expression www
(vi) Let $u = x$, $dv/dx = \cos 2x$ $\Rightarrow v = \frac{1}{2}\sin 2x$ $\int_0^{\pi/4} x\cos 2x\,dx = \left[\frac{1}{2}x\sin 2x\right]_0^{\pi/4} - \int_0^{\pi/4}\frac{1}{2}\sin 2x\,dx$ $= \frac{\pi}{8} + \left[\frac{1}{4}\cos 2x\right]_0^{\pi/4}$ $= \frac{\pi}{8} - \frac{1}{4}$ Area of region enclosed by curve and x-axis between $x = 0$ and $x = \pi/4$	M1 A1 A1 M1 A1 B1 [6]	Integration by parts with $u = x$, $dv/dx = \cos 2x$ $\left[\frac{1}{4}\cos 2x\right]$ - sign consistent with their previous line substituting limits – dep using parts www or graph showing correct area – condone P for $\pi/4$.