Mark Scheme 4753
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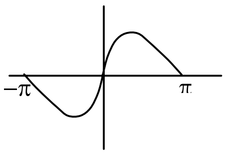
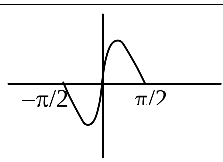
Section A

1 $y = (1 + 6x)^{1/3}$		
$\Rightarrow \frac{dy}{dx} = \frac{1}{3}(1+6x)^{-2/3}.6$	M1	Chain rule
	B1	$\frac{1}{3}(1+6x)^{-2/3}$ or $\frac{1}{3}u^{-2/3}$
$= 2(1 + 6x)^{-2/3}$ $= 2[(1 + 6x)^{1/3}]^{-2}$	A1	any correct expression for the derivative
$= \frac{2}{y^2}$ *	E1	www
or $y^3 = 1 + 6x$	M1	Finding x in terms of y
$\Rightarrow x = (y^3 - 1)/6$	A1	
$\Rightarrow dx/dy = 3y^2/6 = y^2/2$	B1	$y^2/2$ o.e.
$\Rightarrow dy/dx = 1/(dx/dy) = 2/y^2$ *	E1	
or $y^3 = 1 + 6x$	M1	together with attempt to differentiate implicitly
$\Rightarrow 3y^2\, dy/dx = 6$	A1	$3y^2\, dy/dx$
	A1	$= 6$
$\Rightarrow dy/dx = 6/3y^2 = 2/y^2$ *	E1	
	[4]	
2 (i) When $t = 0$, $P = 5 + a = 8$	M1	substituting $t = 0$ into equation
$\Rightarrow a = 3$	A1	
When $t = 1$, $5 + 3e^{-b} = 6$	M1	Forming equation using their a
$\Rightarrow e^{-b} = 1/3$		
$\Rightarrow -b = \ln 1/3$	M1	Taking lns on correct re-arrangement (ft their a)
$\Rightarrow b = \ln 3 = 1.10$ (3 s.f.)	A1ft	
(ii) 5 million	B1	or $P = 5$
	[6]	
3 (i) $\ln (3x^2)$	B1	$2\ln x = \ln x^2$
	B1	$\ln x^2 + \ln 3 = \ln 3x^2$
(ii) $\ln 3x^2 = \ln(5x + 2)$		
$\Rightarrow 3x^2 = 5x + 2$	M1	Anti-logging
$\Rightarrow 3x^2 - 5x - 2 = 0$*	E1	
(iii) $(3x + 1)(x - 2) = 0$	M1	Factorising or quadratic formula
$\Rightarrow x = -1/3$ or 2	A1cao	
$x = -1/3$ is not valid as $\ln(-1/3)$ is not defined	B1ft	ft on one positive and one negative root
	[7]	

4 (i) $\frac{dV}{dt} = 2$ **(ii)** tan 30 = 1/√3 = r/h ⇒ $h = \sqrt{3}\, r$ ⇒ $V = \frac{1}{3}\pi r^2.\sqrt{3}r = \frac{\sqrt{3}}{3}\pi r^3$ * $\frac{dV}{dr} = \sqrt{3}\pi r^2$ **(iii)** When $r = 2$, $dV/dr = 4\sqrt{3}\pi$ $\frac{dV}{dt} = \frac{dV}{dr}.\frac{dr}{dt}$ ⇒ $2 = 4\sqrt{3}\pi\ dr/dt$ ⇒ $dr/dt = 1/(2\sqrt{3}\pi)$ or 0.092 cm s^{-1}	B1 M1 E1 B1 M1 M1 A1cao [7]	 Correct relationship between r and h in any form From exact working only o.e. e.g. $(3\sqrt{3}/3)\pi\, r^2$ or $\frac{dr}{dt} = \frac{dr}{dV}.\frac{dV}{dt}$ substituting 2 for dV/dt and r = 2 into their dV/dr
5(i) $y^3 = 2xy + x^2$ ⇒ $3y^2\frac{dy}{dx} = 2x\frac{dy}{dx} + 2y + 2x$ ⇒ $(3y^2 - 2x)\frac{dy}{dx} = 2y + 2x$ ⇒ $\frac{dy}{dx} = \frac{2(x+y)}{3y^2 - 2x}$ * **(ii)** $\frac{dx}{dy} = \frac{3y^2 - 2x}{2(x+y)}$	B1 B1 M1 E1 B1cao [5]	$3y^2\frac{dy}{dx} =$ $2x\frac{dy}{dx} + 2y + 2x$ collecting dy/dx terms on one side www
6(i) $y = 1 + 2\sin x$ $y \leftrightarrow x$ ⇒ $x = 1 + 2\sin y$ ⇒ $x - 1 = 2\sin y$ ⇒ $(x - 1)/2 = \sin y$ ⇒ $y = \arcsin(\frac{x-1}{2})$ * Domain is $-1 \le x \le 3$ **(ii)** A is (π/2, 3) B is (1, 0) C is (3, π/2)	M1 A1 E1 B1 B1cao B1cao B1ft [7]	Attempt to invert Allow π/2 = 1.57or better ft on their A

Section B

7(i) $2x - x \ln x = 0$ $\Rightarrow$ $x(2 - \ln x) = 0$ $\Rightarrow$ $(x = 0)$ or $\ln x = 2$ $\Rightarrow$ at A, $x = e^2$	M1 A1 [2]	Equating to zero
(ii) $\frac{dy}{dx} = 2 - x.\frac{1}{x} - \ln x.1$ $= 1 - \ln x$ $\frac{dy}{dx} = 0 \Rightarrow 1 - \ln x = 0$ $\Rightarrow$ $\ln x = 1$, $x = e$ When $x = e$, $y = 2e - e \ln e = e$ So B is (e, e)	M1 B1 A1 M1 A1cao B1ft [6]	Product rule for $x \ln x$ $d/dx (\ln x) = 1/x$ $1 - \ln x$ o.e. equating their derivative to zero $x = e$ $y = e$
(iii) At A, $\frac{dy}{dx} = 1 - \ln e^2 = 1 - 2$ $= -1$ At C, $\frac{dy}{dx} = 1 - \ln 1 = 1$ $1 \times -1 = -1 \Rightarrow$ tangents are perpendicular	M1 A1cao E1 [3]	Substituting x=1 or their e^2 into their derivative -1 and 1 www
(iv) Let $u = \ln x$, $dv/dx = x$ $\Rightarrow v = ½ x^2$ $\int x \ln x dx = \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x^2.\frac{1}{x}dx$ $= \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x dx$ $= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + c$ * $A = \int_1^e (2x - x \ln x)dx$ $= \left[x^2 - \frac{1}{2}x^2 \ln x + \frac{1}{4}x^2\right]_1^e$ $= (e^2 - ½e^2\ln e + ¼ e^2) - (1 - ½1^2\ln 1 + ¼ 1^2)$ $= \frac{3}{4}e^2 - \frac{5}{4}$	M1 A1 E1 B1 B1 M1 A1 cao [7]	Parts: $u = \ln x$, $dv/dx = x \Rightarrow v = ½ x^2$ correct integral and limits $\left[x^2 - \frac{1}{2}x^2 \ln x + \frac{1}{4}x^2\right]$ o.e. substituting limits correctly

8 (i) $f(-x) = \frac{\sin(-x)}{2-\cos(-x)}$ $= \frac{-\sin(x)}{2-\cos(x)}$ $= -f(x)$  	M1 A1 B1 [3]	substituting $-x$ for x in $f(x)$ Graph completed with rotational symmetry about O.
(ii) $f'(x) = \frac{(2-\cos x)\cos x - \sin x . \sin x}{(2-\cos x)^2}$ $= \frac{2\cos x - \cos^2 x - \sin^2 x}{(2-\cos x)^2}$ $= \frac{2\cos x - 1}{(2-\cos x)^2}$ * $f'(x) = 0$ when $2\cos x - 1 = 0$ $\Rightarrow \cos x = ½ , x = \pi/3$ When $x = \pi/3$, $y = \frac{\sin(\pi/3)}{2-\cos(\pi/3)} = \frac{\sqrt{3}/2}{2-1/2}$ $= \frac{\sqrt{3}}{3}$ So range is $-\frac{\sqrt{3}}{3} \le y \le \frac{\sqrt{3}}{3}$	M1 A1 E1 M1 A1 M1 A1 B1ft [8]	Quotient or product rule consistent with their derivatives Correct expression numerator = 0 Substituting their $\pi/3$ into y o.e. but exact ft their $\frac{\sqrt{3}}{3}$
(iii) $\int_0^{\pi} \frac{\sin x}{2-\cos x} dx$ let $u = 2 - \cos x$ $\Rightarrow du/dx = \sin x$ When $x = 0$, $u = 1$; when $x = \pi$, $u = 3$ $= \int_1^3 \frac{1}{u} du$ $= [\ln u]_1^3$ $= \ln 3 - \ln 1 = \ln 3$	M1 B1 A1ft A1cao	$\int \frac{1}{u} du$ $u = 1$ to 3 $[\ln u]$
or $= [\ln(2-\cos x)]_0^{\pi}$ $= \ln 3 - \ln 1 = \ln 3$	M2 A1 A1 cao [4]	$[k \ln (2 - \cos x)]$ $k = 1$
(iv)  	B1ft [1]	Graph showing evidence of stretch s.f. ½ in x – direction
(v) Area is stretched with scale factor ½ So area is ½ ln 3	M1 A1ft [2]	soi ½ their ln 3