

OCR

Oxford Cambridge and RSA

Friday 12 June 2015 – Morning

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

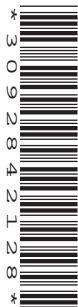
OCR supplied materials:

- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

- 1 Fig. 1 shows part of the curve $y = e^{2x} \cos x$.

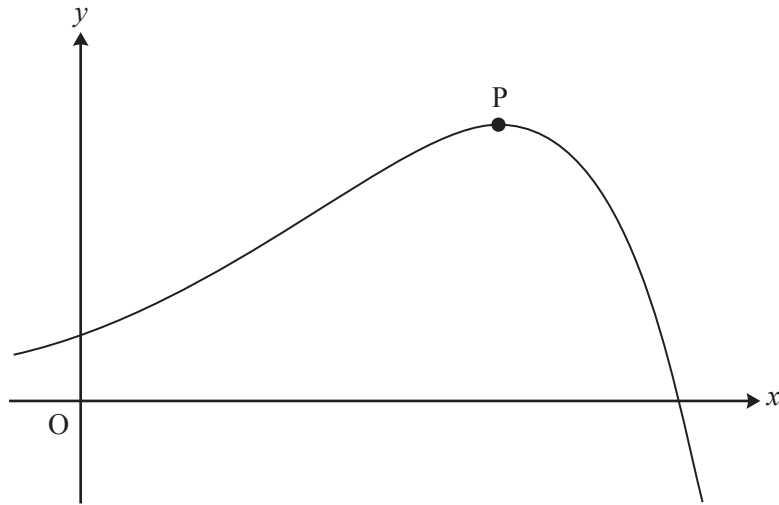


Fig. 1

Find the coordinates of the turning point P.

[6]

2 Find $\int \sqrt[3]{2x-1} dx$.

[4]

3 Find the exact value of $\int_1^2 x^3 \ln x dx$.

[5]

- 4 Fig. 4 shows a cone with its axis vertical. The angle between the axis and the slant edge is 45° . Water is poured into the cone at a constant rate of 5 cm^3 per second. At time t seconds, the height of the water surface above the vertex O of the cone is h cm, and the volume of water in the cone is $V \text{ cm}^3$.

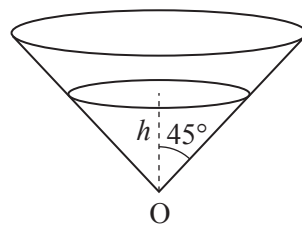


Fig. 4

Find V in terms of h .

Hence find the rate at which the height of water is increasing when the height is 10 cm.

[You are given that the volume V of a cone of height h and radius r is $V = \frac{1}{3}\pi r^2 h$].

[5]

- 5 A curve has implicit equation $y^2 + 2x \ln y = x^2$.

Verify that the point $(1, 1)$ lies on the curve, and find the gradient of the curve at this point.

[6]

6 Solve each of the following equations, giving your answers in exact form.

(i) $6 \arcsin x - \pi = 0.$ [2]

(ii) $\arcsin x = \arccos x.$ [2]

7 (i) The function $f(x)$ is defined by

$$f(x) = \frac{1-x}{1+x}, x \neq -1.$$

Show that $f(f(x)) = x.$

Hence write down $f^{-1}(x).$ [3]

(ii) The function $g(x)$ is defined for all real x by

$$g(x) = \frac{1-x^2}{1+x^2}.$$

Prove that $g(x)$ is even. Interpret this result in terms of the graph of $y = g(x).$ [3]

Section B (36 marks)

- 8 Fig. 8 shows the line $y = 1$ and the curve $y = f(x)$, where $f(x) = \frac{(x-2)^2}{x}$. The curve touches the x -axis at $P(2, 0)$ and has another turning point at the point Q .

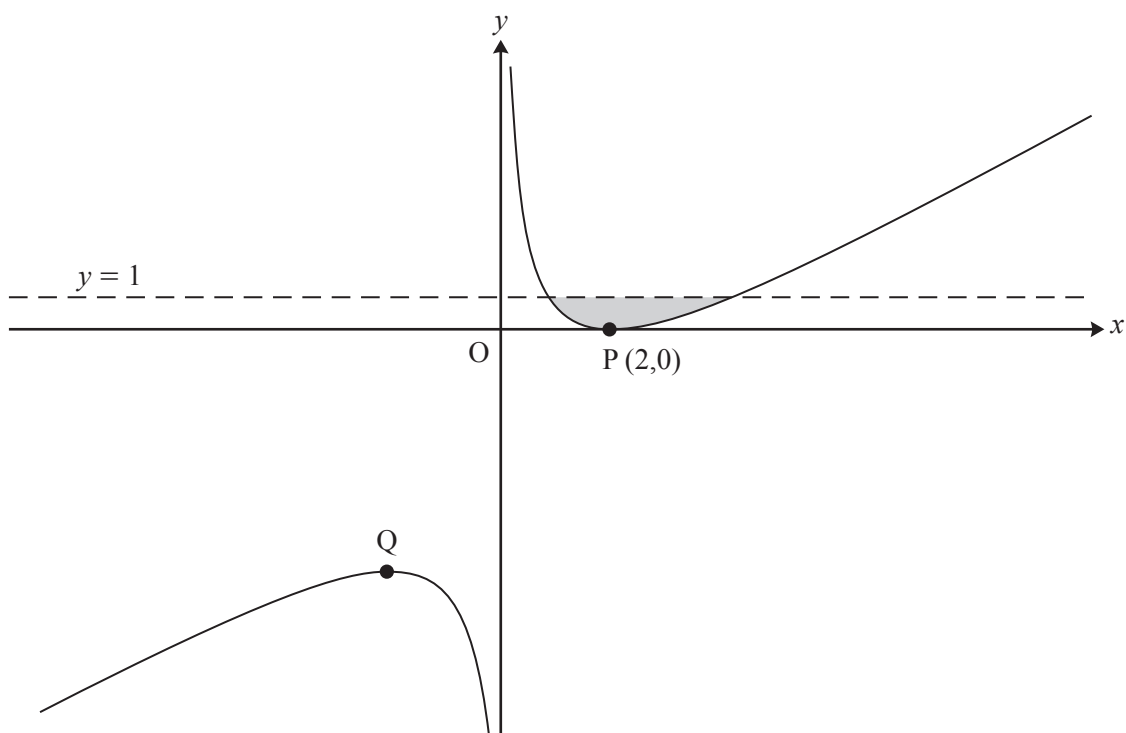


Fig. 8

- (i) Show that $f'(x) = 1 - \frac{4}{x^2}$, and find $f''(x)$.

Hence find the coordinates of Q and, using $f''(x)$, verify that it is a maximum point. [7]

- (ii) Verify that the line $y = 1$ meets the curve $y = f(x)$ at the points with x -coordinates 1 and 4. Hence find the exact area of the shaded region enclosed by the line and the curve. [6]

The curve $y = f(x)$ is now transformed by a translation with vector $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$. The resulting curve has equation $y = g(x)$.

- (iii) Show that $g(x) = \frac{x^2 - 3x}{x + 1}$. [3]

- (iv) Without further calculation, write down the value of $\int_0^3 g(x) dx$, justifying your answer. [2]

9 Fig. 9 shows the curve $y = f(x)$, where

$$f(x) = (e^x - 2)^2 - 1, \quad x \in \mathbb{R}.$$

The curve crosses the x -axis at O and P, and has a turning point at Q.

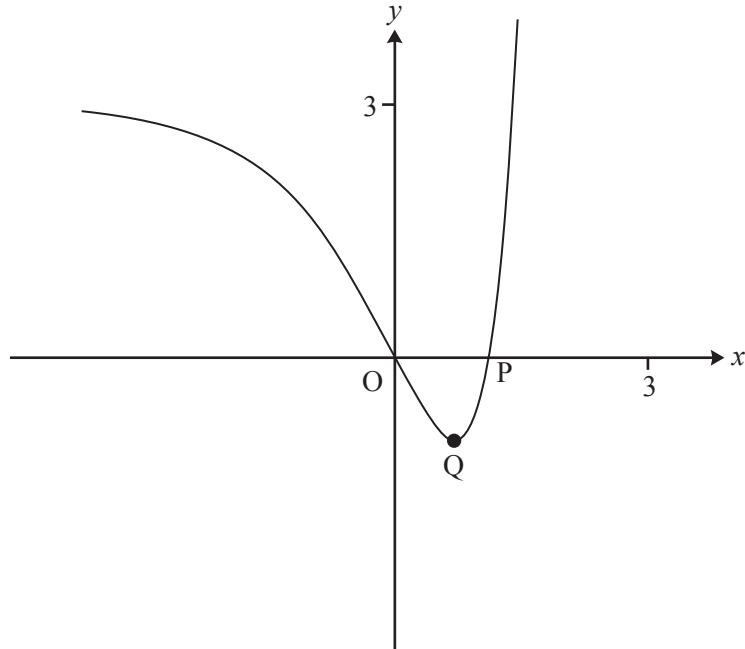


Fig. 9

- (i) Find the exact x -coordinate of P. [2]
- (ii) Show that the x -coordinate of Q is $\ln 2$ and find its y -coordinate. [4]
- (iii) Find the exact area of the region enclosed by the curve and the x -axis. [5]

The domain of $f(x)$ is now restricted to $x \geq \ln 2$.

- (iv) Find the inverse function $f^{-1}(x)$. Write down its domain and range, and sketch its graph on the copy of Fig. 9. [7]

END OF QUESTION PAPER

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