A-level

FURTHER MATHEMATICS 7367/1

Paper 1

Mark scheme

June 2021

Version: 1.0 Final Mark Scheme

216A7367/1/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking Instructions	AO	Marks	Typical solution
1	Circles correct answer	1.1b	B1	2450
	Total		**1**	

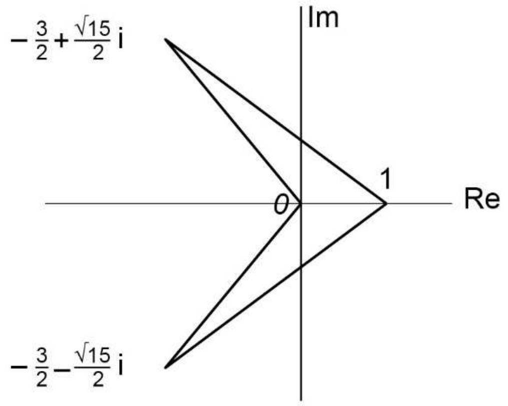
Q	Marking Instructions	AO	Marks	Typical solution
2	Circles correct answer	2.2a	B1	10
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical solution
3	Circles correct answer	2.2a	B1	$y = \frac{2}{x}$
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical solution
4	Uses appropriate hyperbolic identity or substitutes using exponential form	1.1a	M1	$3\tanh^2 x - 2\operatorname{sech} x = 2$ $3 - 3\operatorname{sech}^2 x - 2\operatorname{sech} x - 2 = 0$ $0 = 3\operatorname{sech}^2 x + 2\operatorname{sech} x - 1$ $\operatorname{sech} x = \frac{1}{3}$ or -1 But $\operatorname{sech} x > 0 \therefore \operatorname{sech} x = \frac{1}{3}$ $\cosh x = 3 \Rightarrow x = \pm\cosh^{-1} 3$ $x = \pm\ln(3 + \sqrt{8})$
	Solves a quadratic or quartic equation and selects positive root	2.2a	M1	
	Obtains correct value(s) of $\operatorname{sech} x$ or $\cosh x$ or e^x	1.1b	B1	
	Expresses x in logarithmic form which contains one of 3 or $\sqrt{8}$ or $2\sqrt{2}$	1.1a	M1	
	Obtains correct values of x Condone missing ± in working prior to final answer Condone $x = \pm\ln(3 + 2\sqrt{2})$	1.1b	A1	
	Total		**5**	

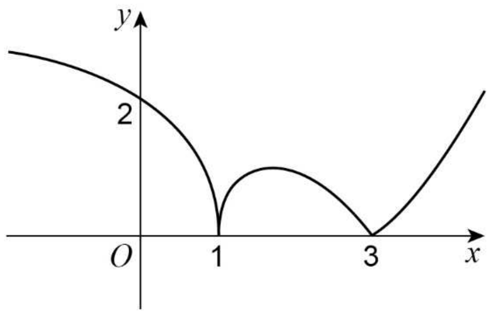
Q	Marking Instructions	AO	Marks	Typical solution
5	Demonstrates the result for $n=1$ **and** states that it is true for $n=1$	1.1b	B1	Let $n=1$ then $\mathbf{M}^1 = \begin{bmatrix} 3 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \mathbf{M}$ so the result is true for $n=1$
	States the assumption that the result true for $n=k$ Condone use of n instead of k	2.4	B1	Assume the result if true for $n=k$:
	Writes $\mathbf{M}^{k+1}$ as $\mathbf{M}\mathbf{M}^k$ or $\mathbf{M}^k\mathbf{M}$ Condone use of n instead of k	3.1a	M1	$\mathbf{M}^{k+1} = \begin{bmatrix} 3 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 3^k & 3^k-1 & -3^k+1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	Calculates $\mathbf{M}^{k+1}$ correctly (fully simplified) Condone use of n instead of k	1.1b	A1	$= \begin{bmatrix} 3^{k+1} & 3^{k+1}-1 & -3^{k+1}+1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Hence true for $n=k+1$
	Completes a rigorous argument by stating that It is true for $n=1$; that if it is true for $n=k$ then it is true for $n=k+1$ And hence (by induction) true for all integers $n \geq 1$ Do not condone use of n instead of k in the inductive step	2.1	R1	It is true for $n=1$. If it is true for $n=k$ then it is true for $n=k+1$. Hence true by induction for all integers $n \geq 1$
	Total		**5**	

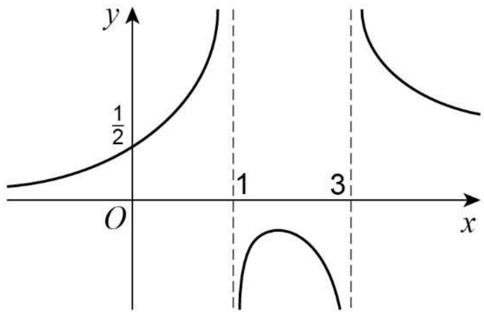
Q	Marking Instructions	AO	Marks	Typical solution
6(a)	Defines z and z^* in terms of two variables for example x and y	1.1a	M1	Let $z = x + \mathrm{i}y$ then $z^* = x - \mathrm{i}y$ $2z - z^* = x + 3\mathrm{i}y$ $(2z - z^*)^* = x - 3\mathrm{i}y$ $z^2 = x^2 - y^2 + 2\mathrm{i}xy$ Re: $x = x^2 - y^2$ ....① Im: $-3y = 2xy$ ....② ② $\Rightarrow y = 0$ or $x = -\frac{3}{2}$ If $y = 0$ then ① $\Rightarrow x = 0$ or 1 If $x = -\frac{3}{2}$ then ① $\Rightarrow -\frac{3}{2} = \frac{9}{4} - y^2$ and $y = \pm\frac{\sqrt{15}}{2}$ So the only solutions are $z = 0,\ z = 1,$ $z = -\frac{3}{2} + \frac{\sqrt{15}}{2}\mathrm{i}$ and $z = -\frac{3}{2} - \frac{\sqrt{15}}{2}\mathrm{i}$ Hence these are the only solutions there are exactly four solutions
	Obtains correct expressions for $(2z - z^*)^*$ and z^2	1.1b	A1	
	Uses their expressions for $(2z - z^*)^*$ and z^2 to form a pair of simultaneous equations	3.1a	M1	
	Deduces that the second equation implies the result " $y = 0$ or $x = -\frac{3}{2}$ "	2.2a	A1	
	Deduces that $y = 0$ Implies the result "$x = 0$ or 1" PI $z = 0$ and $z = 1$	2.2a	A1	
	Obtains any two correct solutions in the form $z = \cdots$	1.1b	A1	
	Produces a clear argument to conclude that there are exactly four solutions stating them in the form $z = \cdots$	2.1	R1	
	Total		**7**	

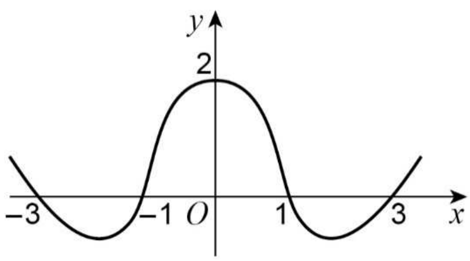
Q	Marking Instructions	AO	Marks	Typical solution
6(b)(i)	Shows their four points correctly on Argand diagram Condone no labelling	1.1b	B1F	
	Connects their four points to obtain shape with correct symmetry	1.1b	B1F	
	Total		**2**	

Q	Marking Instructions	AO	Marks	Typical solution
6(b)(ii)	Produces a clear argument to show the required result	2.1	R1	Area of upper triangle = Area of lower triangle $= \frac{1}{2} \times 1 \times \frac{\sqrt{15}}{2} = \frac{\sqrt{15}}{4}$ Total area $= 2 \times \frac{\sqrt{15}}{4} = \frac{\sqrt{15}}{2}$ as required
	Total		**1**	

	Question total		**10**	

Q	Marking Instructions	AO	Marks	Typical solution
7(a)	Reflects the part of the graph which lies below the x-axis in the x-axis Condone any continuous curve	3.1a	M1	
	Obtains the correct graph including all the intercepts	1.1b	A1	
	Total		**2**	

Q	Marking Instructions	AO	Marks	Typical solution
7(b)	Shows correct asymptotes	2.2a	B1	
	Draws graph of correct shape: 1) Curve above the x-axis to left of x = 1 and an increasing function 2) Curve below x-axis between x = 1 and x = 3, with a local maximum 3) Curve above the x-axis to right of x = 3 and a decreasing function Condone one wrong asymptote	3.1a	M1	
	Obtains correct graph including intercept (ignore other details such as the y-values for minimum and maximum values of x) Condone one wrong asymptote	1.1b	A1F	
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical solution
7(c)	Sketches and reflects the part of the graph which lies to the right of the y-axis in the y-axis Condone reasonable attempt at the graph being symmetric about the y-axis	1.1a	M1	
	Correctly shows all the intercepts on their sketch	1.1b	A1	
	Total		**2**	

	Question total		**7**	

Q	Marking Instructions	AO	Marks	Typical solution
8	Uses Newton's second law to form a four term differential equation. Must have correct terms, condone wrong signs PI	3.3	M1	$4\frac{\mathrm{d}v}{\mathrm{d}t} = 1.8 + 30t^{\frac{1}{2}} - 0.08v^2$ $\frac{\mathrm{d}v}{\mathrm{d}t} = 0.45 + 7.5t^{\frac{1}{2}} - 0.02v^2$ $v_{70.5} \cong 54 + 0.5(\dot{v}_{70})$ $= 54 + 0.5(0.45 + 7.5 \times 70^{\frac{1}{2}} - 0.02 \times 54^2)$ $= 54 + 0.5(4.8795\ldots)$ $= 56.439751$ $v_{71} \cong 56.439751 + 0.5(\dot{v}_{70.5})$ $v_{71} \cong 56.439751 + 0.5(-0.2857\ldots)$ $= 56.2969$ Velocity after 71 seconds = 56.30 m s^{-1}
	Obtains correct expression for $\frac{\mathrm{d}v}{\mathrm{d}t}$ or $4\frac{\mathrm{d}v}{\mathrm{d}t}$ PI	1.1b	A1	
	Substitutes correct values into their first Euler equation	1.1a	M1	
	Obtains value for $v_{70.5}$ which rounds to 56.4	1.1b	A1	
	Uses Euler's method exactly twice	3.1a	M1	
	Obtains correct answer to required degree of accuracy Condone lack of units	3.2a	A1	
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical solution
9	Defines $\mathrm{f}(x)$ and $\mathrm{g}(x)$ **or** uses the correct $\mathrm{f}(x)$ and $\mathrm{g}(x)$ with l'Hopital's rule	3.1a	M1	Let $\mathrm{f}(x) = x,\ \ \mathrm{g}(x) = \mathrm{e}^x$ Then $x\mathrm{e}^{-x} = \frac{\mathrm{f}(x)}{\mathrm{g}(x)}$ and $\mathrm{f}(x)$ and $\mathrm{g}(x)$ both tend to ∞ as $x \to \infty$ $\therefore \lim_{x\to\infty}(x\mathrm{e}^{-x}) = \lim_{x\to\infty}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \lim_{x\to\infty}\left(\frac{\mathrm{f}'(x)}{\mathrm{g}'(x)}\right)$ $\mathrm{f}'(x) = 1$ and $\mathrm{g}'(x) = \mathrm{e}^x$ $\therefore \lim_{x\to\infty}(x\mathrm{e}^{-x}) = \lim_{x\to\infty}\left(\frac{1}{\mathrm{e}^x}\right) = \lim_{x\to\infty}(\mathrm{e}^{-x})$ $= 0$ as required
	Explains how $\mathrm{f}(x)$ and $\mathrm{g}(x)$ fulfil the requirements for l'Hôpital's rule	2.4	E1	
	Obtains $\frac{\mathrm{f}'(x)}{\mathrm{g}'(x)} = \mathrm{e}^{-x}$	1.1b	A1	
	Uses correct reasoning to obtain the required limit The explanation of the requirements for l'Hôpital's rule is not needed for this mark	2.1	R1	
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical solution
10	Defines the improper integral as a limit	2.4	E1	$\int_0^8 \ln(x)\,\mathrm{d}x = \lim_{h\to 0}\int_h^8 \ln(x)\,\mathrm{d}x$
	Selects and uses the method of integration by parts. Implied by stating and using the formula for the integral of $\ln x$	3.1a	M1	$u = \ln x \quad v' = 1$ $u' = \frac{1}{x} \quad v = x$
	Obtains the correct integral with or without c. Condone no limits	1.1b	A1	$\int_0^8 \ln(x)\,\mathrm{d}x = \lim_{h\to 0}\left([x\ln(x)]_h^8 - \int_h^8 1\mathrm{d}x\right)$
	Substitutes 8 correctly into their two-term expression for the integral	1.1a	M1	$= \lim_{h\to 0}([x\ln(x) - x]_h^8)$ $= \{8\ln(8) - 8\} - \lim_{h\to 0}\{h\ln(h) - h\}$
	Applies the limiting process correctly, using $\lim_{h\to 0}\{h\ln(h)\} = 0$ This does not have to be stated explicitly	2.2a	M1	As $\lim_{h\to 0}\{h\ln(h)\} = 0$
	Obtains correct value OE, explicitly stating $\lim_{h\to 0}\{h\ln(h)\} = 0$ NMS = 0	1.1b	A1	Then $\int_0^8 \ln(x)\,\mathrm{d}x = 8\ln(8) - 8$
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical solution
11(a)	Obtains scalar (or vector) product of the direction vectors PI by seeing AWRT $103°$	1.1a	M1	$\begin{bmatrix}2\\3\\-1\end{bmatrix}\cdot\begin{bmatrix}-2\\1\\1\end{bmatrix}=-2$ Moduli of vectors are $\sqrt{14}$ and $\sqrt{6}$ Let α be angle between lines $\cos\alpha=\frac{-2}{\sqrt{14}\sqrt{6}}=\frac{-1}{\sqrt{21}}$ Angle between lines $=180-\alpha=77.4°$
	Divides their scalar product (or their magnitude of vector product) by product of the magnitudes PI by AWRT $103°$	1.1a	M1	
	Deduces the correct angle, correct to at least 1dp	2.2a	A1	
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical solution
11(b)(i)	Finds vector product of direction vectors	3.1a	M1	$\mathbf{n}_1=\begin{bmatrix}2\\3\\-1\end{bmatrix}\times\begin{bmatrix}-2\\1\\1\end{bmatrix}=4\begin{bmatrix}1\\0\\2\end{bmatrix}$ $d=\begin{bmatrix}2\\2\\3\end{bmatrix}\bullet\begin{bmatrix}1\\0\\2\end{bmatrix}=8$ $\mathbf{r}\bullet\begin{bmatrix}1\\0\\2\end{bmatrix}=8$
	Obtains correct result (or multiple of it)	1.1b	A1	
	Takes scalar product of their normal vector ($\mathbf{n}_1$) and a point in the plane	2.2a	M1	
	Obtains correct result in the correct form (or multiple of it)	1.1b	A1	
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical solution
11(b)(ii)	Obtains the correct distance for their vector equation ACF	2.2a	B1F	Distance to origin $=\frac{8}{\sqrt{5}}$
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical solution
11(c)	Forms two direction vectors from the three points **and** identifies the need to take the cross product of them	3.1a	M1	$\overrightarrow{AB} = \begin{bmatrix} -3 \\ 6 \\ -6 \end{bmatrix} = -3\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$, $\overrightarrow{AC} = \begin{bmatrix} -1 \\ 5 \\ -7 \end{bmatrix}$
	Obtains correct result (or multiple of it)	1.1b	A1	$\mathbf{n}_2 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} \times \begin{bmatrix} -1 \\ 5 \\ -7 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 3 \end{bmatrix}$ Let β be angle between planes
	Finds scalar (or vector) product of their normal vectors	1.1a	M1	$\cos\beta = \dfrac{\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \bullet \begin{bmatrix} 4 \\ 5 \\ 3 \end{bmatrix}}{\sqrt{5}\sqrt{50}} = \dfrac{2}{\sqrt{10}}$
	Obtains correct angle (accept $129.2°$)	1.1b	A1	$\beta = 50.8°$
	Total		**4**	

	Question total		**12**	

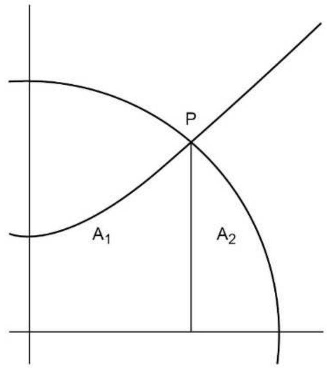
Q	Marking Instructions	AO	Marks	Typical solution
12(a)	Calculates $\lvert\mathbf{A}\rvert$ using the determinants of three 2x2 matrices	1.1a	M1	$\lvert\mathbf{A}\rvert = 1\begin{vmatrix}-2 & p\\ 5 & -11\end{vmatrix} - 5\begin{vmatrix}4 & p\\ 8 & -11\end{vmatrix} + 3\begin{vmatrix}4 & -2\\ 8 & 5\end{vmatrix}$
	Obtains correct $\lvert\mathbf{A}\rvert$	1.1b	A1	$\lvert\mathbf{A}\rvert = (22-5p) - 5(-44-8p) + 3(20+16)$ $\lvert\mathbf{A}\rvert = 350 + 35p = 35(10+p)$
	Obtains matrix of minors/cofactors with four elements correct	1.1a	M1	Cofactors: $\begin{bmatrix}22-5p & 44+8p & 36\\ 70 & -35 & 35\\ 5p+6 & 12-p & -22\end{bmatrix}$
	Obtains fully correct matrix of cofactors	1.1b	A1	
	Transposes their matrix of cofactors (with at most one further error) & divides by their determinant	1.1b	A1F	$\mathbf{A}^{-1} = \dfrac{1}{350+35p}\begin{bmatrix}22-5p & 70 & 5p+6\\ 44+8p & -35 & 12-p\\ 36 & 35 & -22\end{bmatrix}$ $p \neq -10$
	Obtains fully correct answer including $p \neq -10$	2.1	R1	
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical solution
12(b)(i)	Uses their $\mathbf{A}^{-1}$ to form a product to find the coordinates of the point of intersection. Must include $\begin{bmatrix} 5 \\ 24 \\ -30 \end{bmatrix}$ or Eliminates one variable to form two simultaneous equations in two variables	3.1a	M1	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{35(10+p)} \begin{bmatrix} 22-5p & 70 & 5p+6 \\ 44+8p & -35 & 12-p \\ 36 & 35 & -22 \end{bmatrix} \begin{bmatrix} 5 \\ 24 \\ -30 \end{bmatrix}$ $= \frac{1}{35(10+p)} \begin{bmatrix} 110-25p+1680-150p-180 \\ 220-40p\ \ 840\ \ 360\ \ 30p \\ 180+840+660 \end{bmatrix}$ $= \frac{1}{35(10+p)} \begin{bmatrix} 1610-175p \\ -980+70p \\ 1680 \end{bmatrix}$ $= \frac{1}{10+p} \begin{bmatrix} 46-5p \\ -28+2p \\ 48 \end{bmatrix}$ $x = \frac{46-5p}{10+p}$; $y\ \frac{-28+2p}{10+p}$; $z\ \frac{48}{10+p}$ Point of intersection is: $\left(\frac{46-5p}{10+p}, \frac{-28+2p}{10+p}, \frac{48}{10+p}\right)$
	One component correct from their $\mathbf{A}^{-1}$, can be unsimplified or Obtains one correct value for x, y or z , can be unsimplified	1.1b	A1F	
	Two components correct from their $\mathbf{A}^{-1}$, can be unsimplified or Obtains a second correct value for x, y or z , can be unsimplified	1.1b	A1F	
	All three correct, like terms collected, but can be unsimplified Condone any form of the answer	2.1	R1	
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical solution
12(b)(ii)	Obtains the scalar product of normal vectors of two planes	3.1a	M1	$\begin{matrix} x & +5y & +3z & = & 5 \\ 4x & -2y & +2z & = & 24 \\ 8x & +5y & -11z & = & -30 \end{matrix}$ $\begin{bmatrix}1\\5\\3\end{bmatrix} \cdot \begin{bmatrix}4\\-2\\2\end{bmatrix} = 4 - 10 + 6 = 0$ So the planes represented by the first two equations are perpendicular $\begin{bmatrix}1\\5\\3\end{bmatrix} \times \begin{bmatrix}4\\-2\\2\end{bmatrix} = \begin{bmatrix}16\\10\\-22\end{bmatrix} = 2\begin{bmatrix}8\\5\\-11\end{bmatrix}$ As the vector perpendicular to the first two planes is a multiple of the normal vector of the third plane, the three planes are mutually perpendicular
	Calculates that the scalar product is 0 **and** interprets this as meaning the two planes are perpendicular.	3.2a	A1	
	Obtains vector product of the **same** two normal vectors or Obtains the scalar products of the other two pairs of normal vector combinations	3.1a	M1	
	Clearly shows the vector product is a multiple of third plane's normal vector and interprets this as meaning that the three planes are **mutually** perpendicular or States that all three scalar products are zero and interprets this as meaning that the three planes are **mutually** perpendicular	3.2a	R1	
	Total		**4**	

	Question total		**14**	

Q	Marking Instructions	AO	Marks	Typical solution
13	Finds the image of the general point for one order of application of S and T or Recalls that the matrix for S represents a stretch parallel to the x-axis	1.2	B1	S then T $\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3x \\ y \end{bmatrix}$ $\begin{bmatrix} 3x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ -5 \end{bmatrix} = \begin{bmatrix} 3x \\ y-5 \end{bmatrix}$ T then S $\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ -5 \end{bmatrix} = \begin{bmatrix} x \\ y-5 \end{bmatrix}$ $\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y-5 \end{bmatrix} = \begin{bmatrix} 3x \\ y-5 \end{bmatrix}$ These are the same So Kamla is correct
	Finds the image of the general point for the alternative order of application of S and T or Explains that S only affects x or T only affects y	2.4	B1	
	Completes a rigorous argument to show that Kamla is correct	2.1	R1	
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical solution
14	Find x-coordinate of P using simultaneous equations	3.1a	M1	 At P, $y^2 - x^2 = 16$ and $x^2 + y^2 = 32$ $x = 2\sqrt{2}$ $A_1 = \int_0^{2\sqrt{2}} (x^2+16)^{\frac{1}{2}}\,dx$ Let $x = 4\sinh u$ Then $(x^2+16)^{\frac{1}{2}} = 4\cosh u$ and $\frac{dx}{du} = 4\cosh u$ $A_1 = \int_0^{x=2\sqrt{2}} 16\cosh^2 u\,du$ $= 8\int_0^{x=2\sqrt{2}} (\cosh 2u + 1)\,du$ $= [4\sinh 2u + 8u]_0^{x=2\sqrt{2}}$ When $x = 2\sqrt{2}$, $\sinh u = \frac{\sqrt{2}}{2}$ and $\cosh u = \frac{\sqrt{6}}{2}$ $\therefore \sinh 2u = 2\sinh u\cosh u = \sqrt{3}$ So $A_1 = 4\sqrt{3} + 8\sinh^{-1}\left(\frac{\sqrt{2}}{2}\right)$ $A_1 = 4\sqrt{3} + 8\ln\left(\frac{\sqrt{2}+\sqrt{6}}{2}\right)$ OP makes an angle of $\frac{\pi}{3}$ with the x-axis So area of sector $= \frac{1}{2} \times 32 \times \frac{\pi}{3} = \frac{16\pi}{3}$ $A_2 = \frac{16\pi}{3} - \frac{1}{2} \times 2\sqrt{2} \times 2\sqrt{6}$ $A_2 = \frac{16\pi}{3} - 4\sqrt{3}$ Required area $= A_1 + A_2$ $= \frac{16\pi}{3} + 8\ln\left(\frac{\sqrt{2}+\sqrt{6}}{2}\right)$ as required
	Obtains correct x-coordinate of P	1.1b	A1	
	Splits region into two or more parts, at least one of which is given as an integral. All integrals with correct limits. Follow through their x-coordinate of P	3.1a	M1	
	Makes appropriate substitution to obtain A_1	3.1a	M1	
	Obtains correct integrand in terms of u Condone incorrect/omission of limits	1.1b	A1	
	Uses hyperbolic identity to integrate	3.1a	M1	
	Deduces that $\sinh 2u = \sqrt{3}$	2.2a	M1	
	Obtains correct value of A_1	1.1b	A1	
	Subtracts area of triangle from area of sector to obtain value of A_2 or makes appropriate substitution to obtain A_2	3.1a	M1	
	Deduces that OP makes an angle of $\frac{\pi}{3}$ with the x-axis or $[\sin 2w]_{\pi/6}^{\pi/2} = \frac{-\sqrt{3}}{2}$	2.2a	M1	
	Obtains correct value of A_2	1.1b	A1	
	Uses a rigorous argument by adding together the two areas	2.1	R1	
	Total		**12**	

Q	Marking Instructions	AO	Marks	Typical solution
15(a)	Uses a rigorous argument to obtain the required result	2.1	R1	Tension in AP = 24m(0.5)=12m Tension in BP = 10m(1.2)=12m So tensions are equal
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical solution
15(b)	Obtains one correct tension	1.1a	B1	$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 10m(1.2 - x) - 24m(0.5 + x) + 6.664m$ $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 34x = 6.664$
	Uses Newton's second law to form a **four** term differential equation with at least two terms correct (allow equivalent notation for derivatives) Condone sign errors on the terms	3.1b	M1	
	Completes a rigorous argument to give the required differential equation	2.1	R1	
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical solution
15(c)	Obtains correct 2nd order DE	2.2a	B1	$m\ddot{x} = 10mv + 6.664m - 34mx$ But $v = -\dot{x}$ So $\ddot{x} + 10\dot{x} + 34x = 6.664$ $\lambda^2 + 10\lambda + 34 = 0$ $\lambda = -5 \pm 3\mathrm{i}$ CF: $x = A\mathrm{e}^{-5t}\cos 3t + B\mathrm{e}^{-5t}\sin 3t$ PI: $x = 0.196$ General Solution: $x = A\mathrm{e}^{-5t}\cos 3t + B\mathrm{e}^{-5t}\sin 3t + 0.196$ $t = 0, x = 0.4 \Longrightarrow A = 0.204$ $\dot{x} =$ $-5A\mathrm{e}^{-5t}\cos 3t - 3A\mathrm{e}^{-5t}\sin 3t$ $-5B\mathrm{e}^{-5t}\sin 3t + 3B\mathrm{e}^{-5t}\cos 3t$ $0 = -5A + 3B$ $B = 0.34$ $x = 0.204\mathrm{e}^{-5t}\cos 3t + 0.34\mathrm{e}^{-5t}\sin 3t + 0.196$
	Obtains correct solution to their **three** term Auxiliary Equation	1.1a	M1	
	Obtains their correct Complementary Function	1.1b	A1F	
	Obtains correct Particular Integral ACF	1.1b	B1	
	Obtains correct general solution (ft their CF, but must have two unknowns)	2.2a	A1F	
	Uses $x = 0.4$ when $t = 0$ to obtain correct A ACF	3.3	B1	
	Sets their correct $\dot{x} = 0$ when $t = 0$	1.1a	M1	
	Obtains correct B ACF	1.1b	A1	
	Obtains correct final equation ACF	2.1	R1	
	Total		**9**	

	Question total		**13**	

	Paper total		**100**	