Pearson Edexcel

]Mark Scheme (Results)

January 2019

Pearson Edexcel International GCSE In Mathematics A (4MA1) Higher Tier Paper 1HR

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January 2019
Publications Code 4MA1_1HR_1901_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.

  Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - eeoo – each error or omission
- **No working**

  If no working is shown then correct answers normally score full marks

  If no working is shown then incorrect (even though nearly correct) answers score no marks.
- **With working**

  If there is a wrong answer indicated on the answer line always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.

  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.

  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review. If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.

  If there is a choice of methods shown, then no marks should be awarded, unless the answer on the answer line makes clear the method that has been used.

  If there is no answer on the answer line then check the working for an obvious answer.
- **Ignoring subsequent work**

  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.

It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded in another.

Apart from questions 6, 8, 13b and 24 (where the mark scheme states otherwise) the correct answer, unless clearly obtained from an incorrect method, should be taken to imply a correct method.					
Question	**Working**	**Answer**	**Mark**	**Notes**	
1	$\frac{5}{3}+\frac{11}{4}$		3	M1	converts to improper fractions
	$\frac{20}{12}+\frac{33}{12}$			M1	converts to fractions with the same common denominator
	$\frac{53}{12} = 4\frac{5}{12}$	Shown		A1	Dep on M2
	Alternative method				
	$\frac{2}{3}+\frac{3}{4}=\frac{8}{12}+\frac{9}{12}$			M1	correct method to add proper fractions
	$\frac{17}{12} = 1\frac{5}{12}$			M1	
	$1\frac{5}{12} + 1 + 2 = 4\frac{5}{12}$	Shown		A1	Dep on M2

Question	Working	Answer	Mark	Notes
2	$\frac{3}{4}\times 60$ (= 45) **or** $\frac{1}{4}\times 60 (=15)$ **OR** $\frac{3}{4}\times\frac{3}{5}\left(=\frac{9}{20}\right)$ $\frac{3}{5}\times$"45" (= 27) **or** $\frac{4}{5}\times$"15" (= 12) **OR** $\frac{1}{4}\times\frac{4}{5}\left(=\frac{4}{20}\right)$ $\frac{\text{"27"}+\text{"12"}}{60}$ **OR** $\text{"}\frac{9}{20}\text{"}+\text{"}\frac{4}{20}\text{"}$	$\frac{13}{20}$	4	M1 M1 M1 For a complete method A1 oe
3	$14^2 - 10^2$ (= 96) "96"+ 5^2 (= 121) $\sqrt{\text{"121"}}$	11	4	M1 M1 M1 A1
4	(a =) 40 − 14 (=26) e.g. $\frac{\text{"26"}+b}{2}=30$ **or** 30 + (30 – "26")	26 34	3	M1 Method to find a M1 Method to find b A1

Question	Working	Answer	Mark	Notes
5	30.5 ÷ 8 (= 3.8125) **OR** 60 ÷ 8 (= 7.5) "3.8125" × 60 **OR** 30.5 × "7.5"	228.75	3	M1 M2 for $30.5 \div \frac{8}{60}$ oe M1 A1 accept 229, 228.8
6	$3x + 10 = x + 52$ $3x - x = 52 - 10$ **or** $2x = 42$ **or** $x = 21$ $y = 180 - 2 \times ("21" + 52)$ **or** $y = 180 - 2 \times (3 \times "21" + 10)$ **or** $y = 180 - ("21" + 52) - (3 \times "21" + 10)$	34	4	M1 for equating the expressions for angle P and angle Q M1 for isolating the terms in x M1 for a complete method A1 dep on M2
7	eg $\frac{187}{147}$ **or** $\frac{147}{187}$ **or** $\frac{90}{187}$ **or** $\frac{187}{90}$ eg $90 \div \frac{187}{147}$ **or** $90 \times \frac{147}{187}$ **or** $147 \times \frac{90}{187}$ **or** $147 \div \frac{187}{90}$	71	3	M1 for an appropriate scale factor, candidates may work in either cm or m M1 for a complete method, candidates may work in either cm or m A1 70.7 – 71

Question	Working	Answer	Mark	Notes	
8	eg $8x + 4y = 18 +$ **or** $4x + 2y = 9 -$ $x - 4y = 9$ $4x - 16y = 36$ **or** $4(9 + 4y) + 2y = 9$	$x = 3$, $y = -1.5$	3	M1	correct method to eliminate x or y: coefficients of x or y the same **and** correct operation to eliminate the selected variable (condone any one arithmetic error in multiplication) **or** writing x or y in terms of the other variable and correctly substituting
	eg $4 \times$ "3" $+ 2y = 9$ **or** $4x + 2 \times$ "-1.5" $= 9$ **or** $x = 9 + 4 \times$ "-1.5"			M1	(dep) correct method to find second variable using their value from a correct method to find first variable or for repeating above method to find second variable
				A1	oe, dep first M1
9 (a)		4.8×10^{11}	1	B1	
(b)		$2^{14} \times 3 \times 5^{10}$	3	B3	for the correct answer
				B2	for an answer in the form $2^{m} \times 3 \times 5^{n}$, where m and n are positive integers
				B1	for at least 2 correct steps in repeated prime factorisation (including tree diagram)
(c)		29 296 875	1	B1	Accept 3×5^{10}, 2.9296875×10^{7}

Question	Working	Answer	Mark	Notes
10	$\pi\times\left(\frac{12}{2}\right)^2$ (=113….) or $\pi\times\left(\frac{12}{2}-2\right)^2$ (= 50.2…) or $\pi\times\left(\frac{12}{2}\right)^2\div 2$ (=56.5...) or $\pi\times\left(\frac{12}{2}-2\right)^2\div 2$ (= 25.1…)	10π	3	M1
	eg $(\pi\times 6^2-\pi\times 4^2)\div 2$ oe			M1 for a complete method
				A1

Question	Working	Answer	Mark	Notes
11	12×5.5 (= 66) $\frac{"66"+18}{20}$	4.2	3	M1 M1 for a complete method A1
12 (a)		$\frac{n}{2n-1}$	2	M1 for $2n \pm k$ oe as the denominator A1 oe
(b)	$(2n-1)^2 = 4n^2 - 4n + 1$ $4(n^2 - n) + 1$ **or** $\frac{4n^2-4n+1}{4} = n^2 - n + \frac{1}{4}$	Proved	3	M1 or $(2n+1)^2 = 4n^2 + 4n + 1$ ft on $2n \pm k$ (k non zero) M1 or $4(n^2 + n) + 1$ **or** $\frac{4n^2+4n+1}{4} = n^2 + n + \frac{1}{4}$ A1 Conclusion

Question		Working	Answer	Mark	Notes	
13	(a)		$3x^2 - 2x - 8$	2	B2	(B1 for at least 1 correct non zero term)
	(b)	"$3x^2 - 2x - 8$" = 0	$-\frac{4}{3}, 2$	3	M1	Dep on at least B1, ft on M marks only dep on $\frac{dy}{dx}$ being a 3 term quadratic
		$(3x + 4)(x - 2)$ (=0)			M1	
		or				
		$x = \frac{2 \pm \sqrt{100}}{2 \times 3}$ or $x = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 3 \times (-8)}}{2 \times 3}$				
					A1	(dep 2nd M1)
	(c)	At x = 2, $y = 2^3 - 2^2 - 8 \times 2 + 12$ (= 0)	Shown	2	M1	Substitutes at least one of $-\frac{4}{3}$ **or** 2 **or** their answer from (b) into $(y =) x^3 - x^2 - 8x + 12$
		or at $x = -\frac{4}{3}$,				
		$y = \left(-\frac{4}{3}\right)^3 - \left(-\frac{4}{3}\right)^2 - 8 \times \left(-\frac{4}{3}\right) + 12$ $\left(= \frac{500}{27}\right)$				
					A1	must show that (2,0) is a turning point on the curve and give concluding statement

Question	Working	Answer	Mark	Notes	
14 (a)		97	1	B1	96 - 98
(b)		Correct graph	2	M1	for at least 4 points plotted correctly at end of interval **or** for all 6 points plotted consistently within each interval at the correct height
				A1	accept curve or line segments accept curve that is not joined to (0, 0)
(c)		14	2	M1	A line drawn at CF = 60 to meet at least one curve or sight of "55" or "69"
				A1	13 - 15 ft candidate's CFD

Question		Working	Answer	Mark	Notes	
15	(a)		$81x^8y^{20}$	2	B2	(B1 two terms correct in a product of 3 terms)
	(b)	$4n(n^2+2n-15)$ **or** $(4n^2-12n)(n+5)$ **or** $(4n^2+20n)(n-3)$	$4n^3+8n^2-60n$	2	M1	For a correct partial expansion (may be unsimplified e.g $4n(n^2+5n-3n-15)$)
					A1	
	(c)		$(2c-3d)(2c+3d)$	1	B1	
	(d)	$\frac{(4-x)(3-x)}{x(4-x)}$ or $\frac{(x-4)(x-3)}{x(4-x)}$	$\frac{3-x}{x}$	3	M1	for either numerator or denominator factorised correctly
					M1	for both numerator and denominator factorised correctly
					A1	oe

Question	Working	Answer	Mark	Notes
16 (a)	$\frac{2}{12}\times\frac{1}{11}$	$\frac{1}{66}$	2	M1 A1
(b)	Any two of $\frac{7}{12}\times\frac{3}{11}\left(=\frac{21}{132}\right)$ **or** $\frac{7}{12}\times\frac{2}{11}\left(=\frac{14}{132}\right)$ **or** $\frac{3}{12}\times\frac{2}{11}\left(=\frac{6}{132}\right)$	$\frac{41}{66}$	3	M1 for any two correct
	$2\times\frac{7}{12}\times\frac{3}{11}+2\times\frac{7}{12}\times\frac{2}{11}+2\times\frac{3}{12}\times\frac{2}{11}$			M1 for a complete method A1 oe
	Alternative method $\frac{7}{12}\times\frac{6}{11}\left(=\frac{42}{132}\right)$ **and** $\frac{3}{12}\times\frac{2}{11}\left(=\frac{6}{132}\right)$	$\frac{41}{66}$		M1 both correct
	$1-"\frac{2}{12}\times\frac{1}{11}"-\frac{7}{12}\times\frac{6}{11}-\frac{3}{12}\times\frac{2}{11}$			M1 for a complete method A1 **SC** B2 for an answer of $\frac{41}{72}$ oe

Question	Working	Answer	Mark	Notes
17 (a)	$2\pi r^2 + 2\pi r \times 2r$	$6r^2$	2	M1 A1
(b)	S.A. $6\pi r^2 : 4\pi r^2 = 3 : 2$	Shown	3	M1 ft their answer from (a), must be in terms of r. Ratios could be seen as fractions throughout eg $\frac{3}{2}$
	$V_c : V_s = 2\pi r^3 : \frac{4}{3}\pi r^3$			M1
	$= 3 \times 2 : 4 = 3 : 2$			A1 oe eg ratios could be $\frac{3}{2}:1$

Question	Working	Answer	Mark	Notes
18	$\frac{\sqrt{8}}{\sqrt{8}-2}\times\frac{\sqrt{8}+2}{\sqrt{8}+2}$ $\frac{\sqrt{8}\left(\sqrt{8}+2\right)}{8-4}=\frac{8+2\sqrt{8}}{4}=\frac{8+4\sqrt{2}}{4}$ $=2+\sqrt{2}$	Shown	3	M1 or $\frac{2\sqrt{2}}{2\sqrt{2}-2}$ or $\frac{\sqrt{2}}{\sqrt{2}-1}$ M1 or $\frac{\sqrt{2}}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}$ A1 (dep on M2) Conclusion - need not state the value of n
19	Angle BCE = 73° Angle BDE = 73° Angle DEB = 73° **and** Angle DCB = 180−73 (=107°) Angle DEB = 73° **and** Angle DBE = 180−73×2 (=34°) Angle DCE = 34° eg Alternate segment theorem Opposite angles of a cyclic quadrilateral sum to 180° Alternate angles are equal Angles in the Same segment are equal Angles in a triangle sum to 180	34	5	M1 angles may be written on the diagram M1 A1 B2 for a full set of reasons relevant to their method (B1 for at least one relevant circle theorem)

Question	Working	Answer	Mark	Notes
20	Let N be the midpoint of BC Let sides of cube have length 2a cm $AN^2 = 4a^2 + a^2 (= 5a^2)$ or $AM^2 = 4a^2 + a^2 + 4a^2 (= 9a^2)$ eg $\tan MAN = \frac{2a}{\sqrt{"5a^2"}}$ or $\sin MAN = \frac{2a}{\sqrt{"9a^2"}}$	41.8	4	B1 for recognising that required angle is MAN (could be marked on a diagram) M1 any $a > 0$ (a could be a number or a letter) M1 correct trig statement for angle MAN, any $a > 0$ (a could be a number or a letter) A1 41.8 - 41.82
21	$x^2 = 5^2 + y^2 - 2\times5\times y\cos 60^o$ $(y-1)^2 = 5^2 + y^2 - 5y$ **or** $x^2 = 5^2 + (x+1)^2 - 5x - 5$ $y^2 - 2y + 1 = 25 + y^2 - 5y$ **or** $x^2 = 5^2 + x^2 + 2x + 1 - 5x - 5$ $5y - 2y = 25 - 1$ **or** $y = 8$ **or** $3x = 21$ **or** $x = 7$	20	5	M1 recognising need for the cosine rule M1 M1 for expansion of $(y-1)^2$ or $(x+1)^2$ in a correct equation M1 for correct linear equation with correct isolation of terms A1

Question	Working	Answer	Mark	Notes
22	eg $\overrightarrow{EX} = \overrightarrow{ED} + \overrightarrow{DC} + \overrightarrow{CX}$ or $\overrightarrow{EX} = \overrightarrow{EF} + \overrightarrow{FA} + \overrightarrow{AX}$		4	M1 a correct statement for $\overrightarrow{EX}$
	$\overrightarrow{DC} = -\mathbf{b} + \mathbf{a}$ or $\overrightarrow{CX} = -\mathbf{b} + \mathbf{a}$ or $\overrightarrow{FA} = -\mathbf{b} + \mathbf{a}$			M1
	$\overrightarrow{EX} = \mathbf{a} + 2(-\mathbf{b} + \mathbf{a})$			M1 for a complete method which gives a correct but unsimplified expression for $\overrightarrow{EX}$
		$3\mathbf{a} - 2\mathbf{b}$		A1

Question	Working	Answer	Mark	Notes
23 (a)	$y = \frac{\sqrt{x^2 + k^2}}{x}$, $x^2y^2 = x^2 + k^2$ $x^2(y^2 - 1) = k^2$		3	M1 for squaring and rearranging correctly to the form $x^2(y^2 - 1) = k^2$
	$\frac{k}{\sqrt{p^2 - 1}} = k$			M1 (dep) for "$f^{-1}(p)$" = k
		$\sqrt{2}$		A1
	Alternative method			
	$p = f(k)$			M1
	$p = \frac{\sqrt{k^2 + k^2}}{k}$			M1
		$\sqrt{2}$		A1
(b)	$(gf(a) =)\left(\frac{\sqrt{a^2 + k^2}}{a}\right)^2$ or $(gf(x) =)\left(\frac{\sqrt{x^2 + k^2}}{x}\right)^2$		3	M1
	$ka^2 - a^2 = k^2$			M1 (dep) for rearranging gf = k and isolating correctly the terms in a^2
		$\frac{k}{\sqrt{k-1}}$		A1 oe eg $\sqrt{\frac{k^2}{k-1}}$

14