Question 1

(i)

x	6	17	9	20	13	15	11	14
y	6	13	10	11	9	7	12	15
Rank x	8	2	7	1	5	3	6	4
Rank y	8	2	5	4	6	7	3	1
d	0	0	2	-3	-1	-4	3	3
d^2	0	0	4	9	1	16	9	9

M1 for attempt at ranking (allow all ranks reversed)

M1 for d^2

$\Sigma d^2 = 48$

A1 CAO for Σd^2

$$r_s = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6\times 48}{8\times 63}$$

$= 0.429$ (to 3 s.f.) [*allow* 0.43 to 2 s.f.]

M1 for method for r_s

A1 f.t. for $|r_s| < 1$

NB No ranking scores zero

5

(ii)

H_0: no association between X and Y in the population

H_1: some positive association between X and Y in the population

B1 for H_0

B1 for H_1

B1 for population SOI

NB H_0 H_1 <u>not</u> ito ρ

3

One tail test critical value at 5% level is 0.6429

B1 for ± 0.6429

Since $0.429 < 0.6429$, there is insufficient evidence to reject H_0,

M1 for sensible comparison with c.v., provided that $|r_s| < 1$

i.e. conclude that there is not enough evidence to show positive association between the two judges' scores.

A1 for conclusion in context f.t. their r_s and sensible cv

3

(iii)

A bivariate Normal distribution is required.

B1

Scatter diagram.

G1 labelled axes
G1 correct points

Suitable discussion

E1
E1

5

TOTAL **16**

Question 2

(i)	Counts have a uniform average rate of occurrence All counts are independent	E1 E1	**2**
(ii)	Variance = 3.4	B1	**1**
(iii)	(*A*) *Either* $P(X = 3) = 0.5584 - 0.3397 = 0.2187$ *Or* $P(X = 3) = e^{-3.4}\frac{3.4^3}{3!} = 0.2186$ (*B*) Using tables: $P(X \geq 3) = 1 - P(X \leq 2)$ $= 1 - 0.3397$ $= 0.6603$	M1 for use of tables or calculation A1 M1 for $1 - P(X \leq 2)$ M1 correct use of Poisson tables A1	**2** **3**
(iv)	$\lambda = 12 \times 3.4 = 40.8$ $P(X = 40) = e^{-40.8}\frac{40.8^{40}}{40!} = 0.0625$	B1 for mean M1 for calculation A1	**3**
(v)	Mean no. per hour = $12 \times 3.4 = 40.8$ Using Normal approx. to the Poisson, $X \sim N(40.8, 40.8)$ $P(X \geq 40) = P\left(Z > \frac{39.5 - 40.8}{\sqrt{40.8}}\right)$ $= P(Z > -0.2035) = \Phi(0.2035)$ $= 0.5806$	B1 for Normal approx. B1 for correct parameters (SOI) B1 for correct continuity corr. M1 for probability using correct tail A1 CAO (3 s.f.)	**5**
(vi)	Overall mean = 4.8 $P(X \geq 8) = 1 - P(X \leq 7)$ $= 1 - 0.8867 = 0.1133$	B1 for 4.8 M1 A1	**3**
		TOTAL	**19**

Question 3

(i)	(*A*) P($X < 65$) = $P\left(Z < \frac{65-63}{5.2}\right)$ = P($Z < 0.3846$) = $\Phi(0.3846) = 0.6497$	M1 for standardizing M1 for structure A1 CAO (min 3 s.f.), NB When a candidate's answers suggest that (s)he appears to have neglected to use the difference column of the Normal distribution tables penalise the first occurrence only	**3**
	(*B*) P(60< X < 65) = $P\left(\frac{60-63}{5.2} < Z < \frac{65-63}{5.2}\right)$ = P(–0.5769 < Z < 0.3846) = $\Phi(0.3846) - (1 - \Phi(0.5769))$ = 0.6497 – (1 – 0.7181) = 0.3678	M1 for standardizing both M1 for correct structure A1 CAO 3s.f.	**3**
(ii)	P(All 5 between 60 and 65) = $0.3678^5 = 0.00673$	M1 A1 FT (min 2sf)	**2**
(iii)	From tables Φ^{-1} (0.95) = 1.645 $\frac{k-63}{5.2} = -1.645$ $x = 63 - 5.2 \times 1.645 = 54.45$ mins	B1 for ±1.645 seen M1 for correct equation in k A1 CAO	**3**
(iv)	H_0: $\mu = 63$ minutes; H_1: $\mu < 63$ minutes. Where μ denotes the population mean time on the new course.	B1 for use of 63 B1 for both correct B1 for definition of μ	**3**
	Test statistic = $\frac{61.7-63}{5.2/\sqrt{15}} = \frac{-1.3}{1.3426}$ = –0.968	M1 must include $\sqrt{15}$ A1	
	5% level 1 tailed critical value of z = 1.645 –0.968 > –1.645 so not significant. There is not sufficient evidence to reject H_0	B1 for ±1.645 M1 for sensible comparison leading to a conclusion	
	There is insufficient evidence to conclude that the new course results in lower times.	A1 FT for correct conclusion in words in context	**5**
			19

Question 4

(i)

H_0: no association between category of runner and type of running;
H_1: some association between category of runner and type of running;

EXPECTED	Junior	Senior	Veteran
Track	5.13	7.84	6.03
Road	6.48	9.90	7.62
Both	5.40	8.25	6.35

M1 A2 for expected values (to 2 dp)
(allow A1 for at least one row or column correct)

CONTRIBUTN	Junior	Senior	Veteran
Track	2.9257	0.0032	2.6949
Road	0.9468	0.3663	2.5190
Both	0.3615	0.3694	0.0192

M1 for valid attempt at $(O-E)^2/E$
A1 for all correct
NB These M1A1 marks cannot be implied by a correct final value of X^2

$X^2 = 10.21$

M1 for summation
A1 for X^2

Refer to X_4^2

B1 for 4 deg of f

Critical value at 5% level = 9.488

B1 CAO for cv

Result is significant

B1 FT their 'sensible' X^2

There is evidence to suggest that there is some association between category of runner and type of running.
NB if H_0 H_1 reversed, or 'correlation' mentioned, do not award first B1or final E1

E1 must be consistent with their X^2

1 **7** **4**

(ii)

- Juniors appear be track runners more often than expected and road less often than expected. — E1 E1
- Seniors tend to be as expected ín all three categories of running. — E1 E1
- Veterans tend to be road runners more than expected and track runners less than expected. — E1 E1

6

TOTAL **18**