

Mathematics

Advanced GCE

Unit **4723**: Core Mathematics 3

Mark Scheme for January 2011

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1	<u>Either:</u> Obtain $\frac{1}{3}a$	B1	condone $ x = \frac{1}{3}a$
	Attempt solution of linear eqn	M1	with signs of $3x$ and $5a$ different; allow M1 only if a given particular value and no recovery occurs; allow M1 only if a in terms of x attempted; allow M1 only if double inequality attempted but with no recovery to state actual values of x
	Obtain $-3a$	A1	3 as final answer
	<u>Or:</u> Obtain $9x^2 + 24ax + 16a^2 = 25a^2$	B1	
	Attempt solution of 3-term quad eqn	M1	as far as substitution into correct quadratic formula or correct factorisation of their quadratic; allow M1 only if a given particular value
	Obtain $-3a$ and $\frac{1}{3}a$	A1	(3) or equivs; as final answers; and no others
			3

2	Draw graph showing reflection in a horizontal axis	M1	
	Draw graph showing translation	M1	parallel to x -axis, in either direction; independent of first M1; not earned if curve still passes through O but ignore other coordinates given at this stage
	Draw (more or less) correct graph which must at least reach the negative x -axis, if not cross it, at left end of curve	A1	but ignoring no or wrong stretch in y -dir'n; condone graph existing only for $x < 0$; consider shape of curve and ignore coordinates given
	State $(-5, 24)$ and $(-3, 0)$ wherever located	B1	4 or clearly implied by sketch; allow for coordinates whatever sketch looks like; allow if in solution with no sketch
			4

3	<u>Either:</u> State or imply $8\pi r$ as derivative	B1	or equiv
	Attempt to connect 12 and their derivative	M1	numerical or algebraic; using multiplication or division
	Obtain $8\pi \times 150 \times 12$ and hence 45000 or 14400π or 14000π	A1	3 or equiv; or greater accuracy (45239); condone absence of units or use of wrong units
	<u>Or:</u> Use $r = 12t$ to show $S = 576\pi t^2$	B1	
	Attempt $\frac{dS}{dt}$ and substitute for t	M1	
	Obtain $1152\pi \times \frac{150}{12}$ and hence 45000 or 14400π or 14000π	A1	(3) or equiv; or greater accuracy (45239); condone absence of units or use of wrong units
			3

4 (i)	Obtain $R = 25$ Attempt to find value of α	B1 M1	allow $\sqrt{625}$ or value rounding to 25 implied by correct answer or its complement; allow sin/cos muddles; allow use of radians for this mark; condone $\sin \alpha = 7$, $\cos \alpha = 24$ in the working
	Obtain 16.3°	A1	3 or greater accuracy 16.260...; must be degrees now; allow 16° here

(ii)	Show correct process for finding one answer Obtain $(28.69 - 16.26$ and hence) 12.4°	M1 A1	even if leading to answer outside 0 to 360 or greater accuracy 12.425... or anything rounding to 12.4
	Show correct process for finding second answer Obtain $(151.31 - 16.26$ and hence) 135° or 135.1°	M1 A1	even if further incorrect answers produced 4 or greater accuracy 135.054...; and no other between 0 and 360
	[SC: No working shown and 2 correct angles stated		B1 only in part (ii)]
			7

5	Integrate to obtain form $k(3x - 2)^{\frac{1}{2}}$	M1	any non-zero constant k ; or equiv involving substitution
	Obtain correct $4(3x - 2)^{\frac{1}{2}}$	A1	or (unsimplified) equiv such as $\frac{6(3x - 2)^{\frac{1}{2}}}{3 \times \frac{1}{2}}$
	Apply limits and attempt solution for a	M1	assuming integral of form $k(3x - 2)^n$; taking solution as far as removal of root; with subtraction the right way round; if sub'n used, limits must be appropriate
	Obtain $a = 9$	A1	(this answer written down with no working scores 0/4 so far but all subsequent marks are available)
	State or imply formula $\int \frac{36\pi}{3x - 2} dx$	B1	or (unsimplified) equiv; condone absence of dx; allow B1 retroactively if π absent here but inserted later
	Integrate to obtain form $k \ln(3x - 2)$	*M1	any constant k including π or not; condone absence of brackets
	Obtain $12\pi \ln(3x - 2)$ or $12 \ln(3x - 2)$	A1√	following their integral of form $\int \frac{k}{3x - 2} dx$
	Apply limits the correct way round	M1	dep *M; use of limit 1 is implied by absence of second term; allow use of limit a
	Obtain $12\pi \ln 25$ (or $24\pi \ln 5$)	A1	9 or exact equiv but not with $\ln 1$ remaining; condone answers such as $\pi 12 \ln 25$ and $12 \ln 25\pi$
			9

6 (i)	Attempt use of quotient rule	M1	or equiv; allow numerator wrong way round but needs minus sign in numerator; for M1 condone 'minor' errors such as sign slips, absence of square in denominator, and absence of some brackets
	Obtain $\frac{3(x^3 - 4x^2 + 2) - (3x + 4)(3x^2 - 8x)}{(x^3 - 4x^2 + 2)^2}$	A1	or equiv; allow A1 if brackets absent from $3x + 4$ term or from $3x^2 - 8x$ term but not from both
	Equate numerator to 0 and attempt simplification	M1	at least as far as removing brackets, condoning sign or coeff slips; or equiv
	Obtain $-6x^3 + 32x + 6 = 0$ or equiv and hence $x = \sqrt[3]{\frac{16}{3}x + 1}$	A1 4	AG; necessary detail needed (i.e. at least one intermediate step) and following first derivative with correct numerator

(ii)	Obtain correct first iterate having used initial value 2.4	B1	showing at least 3 dp (2.398 or 2.399 or greater accuracy 2.39861...)
	Apply iterative process	M1	to obtain at least 3 iterates in all; implied by plausible, converging sequence of values; having started with any initial non-negative value
	Obtain at least 3 correct iterates from their starting point	A1	allowing recovery after error
	Obtain 2.398	A1	value required to exactly 3 dp
	Obtain -1.552	A1 5	value required to exactly 3 dp; allow if apparently obtained by substitution of 2.4; answers only with no iterates shown gets 0/5
	[2.4 → 2.3986103 → 2.3981808 → 2.3980480]		

7 (i)	State $\ln(x^2 + 8) = 8$ Attempt solution involving e^8 Obtain $\sqrt{e^8 - 8}$	B1 M1 A1	or equiv such as $x^2 + 8 = e^8$ by valid (exact) method at least as far as $x^2 = \dots$ 3 or exact equiv; and no other answer

(ii)	State f only State e^x or e^y Indicate domain is all real numbers	B1 B1 B1	3 or equiv; allow if g, or f and g, chosen however expressed

(iii)	Attempt use of chain rule Obtain $\frac{2 \ln x}{x}$ Obtain $6e^{-3}$	M1 A1 A1	3 whether applied to gf or fg; or equiv such as use of product rule on $(\ln x)(\ln x) + 8$ or equiv 3 or exact equiv but not including ln

(iv)	Attempt evaluation using y attempts Obn $k(\ln 24 + 4 \ln 12 + 2 \ln 8 + 4 \ln 12 + \ln 24)$ Use $k = \frac{2}{3}$ and obtain 20.3	M1 A1 A1	3 with coeffs 1, 4 and 2 occurring at least once each; whether fg or gf any constant k 3 or greater accuracy (20.26...) but must round to 20.3
[Note that use of Simpson's rule between 0 and 4 with two strips, coeffs 1, 4, 1, followed by doubling of result is equiv; SC: Use of Simpson's rule between 0 and 4 with four strips followed by doubling of result - allow 3/3 - answer is 20.2 (20.2327...)]			

- 8 (a) (i) Draw at least two correctly shaped branches, one for $y > 0$, one for $y < 0$ M1 otherwise located anywhere including $x < 0$
 Draw four correct branches M1 now (more or less) correctly located;
 Draw (more or less) correct graph A1 3 with some indication of horiz scale (perhaps only 4π indicated); with asymptotic behaviour shown (but not too fussy about branch drifting slightly away from asymptotic value nor about branch touching asymptote) but branches must not obviously cross asymptotic value; with -1 and 1 shown (or implied by presence of sine curve or by presence of only one of them on a reasonably accurate sketch); no need for vertical (dotted) lines drawn to indicate asymptotic values

- (ii) State expression of form $k\pi + \alpha$ or $k\pi - \alpha$ or $\alpha = k\pi + \beta$ or $\alpha = k\pi - \beta$ M1 any non-zero numerical value of k ; M0 if degrees used
 State $3\pi - \alpha$ A1 2 or unsimplified equiv

- (b) (i) State $\frac{2 \tan \theta}{1 - \tan^2 \theta}$ B1 1 or equiv such as $\frac{t + t}{1 - t \times t}$ or $\frac{2 \tan A}{1 - \tan^2 A}$

- (ii) State or imply $\tan \phi = \frac{1}{4}$ B1 or equiv such as $\frac{1}{\tan \phi} = 4$
 Attempt to evaluate $\tan 2\phi$ or $\cot 2\phi$ M1 perhaps within attempt at complete expression but using correct identity
 Obtain $\tan 2\phi = \frac{8}{15}$ or $\cot 2\phi = \frac{15}{8}$ A1 or (unsimplified) equiv; may be implied
 Attempt to evaluate value of $\tan 4\phi$ M1 perhaps within attempt at complete expression; condone only minor slip(s) in use of relevant identity
 Obtain $\frac{240}{161}$ A1 or (unsimplified) exact equiv; may be implied
 Obtain final answer $\frac{225}{322}$ A1 6 or exact equiv

[SC – (use of calculator and little or no working)]

State or imply $\tan \phi = \frac{1}{4}$ B1; Obtain $\tan 2\phi = \frac{8}{15}$ B1; Obtain $\frac{225}{322}$ B1 (max 3/6)

State or imply $\tan \phi = \frac{1}{4}$ B1; Obtain $\frac{225}{322}$ B2 (max 3/6)

9 (i) (a)	Differentiate to obtain $k_1e^{2x} + k_2e^{-2x}$	M1	any constants k_1 and k_2 but derivative must be different from $f(x)$; condone presence of $+c$
	Obtain $2e^{2x} + 6e^{-2x}$	A1	or unsimplified equiv; no $+c$ now
	Refer to $e^{2x} > 0$ and $e^{-2x} > 0$ or to more general comment about exponential functions	A1	3 or equiv (which might be sketch of $y = f(x)$ with comment that gradient is positive or might be sketch of $y = f'(x)$ with comment that $y > 0$; AG

(b)	Differentiate to obtain $k_3e^{2x} + k_4e^{-2x}$	M1	any constants k_3 and k_4 but second derivative must be different from their first derivative; condone presence of $+c$
	Obtain $4e^{2x} - 12e^{-2x}$	A1	or unsimplified equiv; no $+c$ now
	Attempt solution of $f''(x) > 0$ or of $f(x) > 0$ or of corresponding eqn	M1	at least as far as term involving e^{4x} or e^{-4x}
	Obtain $x > \frac{1}{4} \ln 3$	A1	
	Confirm both give same result	B1	5 AG; necessary detail needed; either by solving the other or by observing that same inequality involved (just noting that $f''(x) = 4f(x)$ is sufficient)

(ii)	Differentiate to obtain $2e^{2x} - 2ke^{-2x}$	B1	or unsimplified equiv
	Attempt to find x -coordinate of stationary pt	M1	equating to 0 and reaching $e^{4x} = \dots$ or equiv
	Obtain $e^{4x} = k$ and hence $\frac{1}{4} \ln k$ or equiv	A1	or equiv such as $e^{2x} = \sqrt{k}$
	Substitute and attempt simplification	M1	using valid processes but allow if only limited progress [note that question can be successfully concluded (without actually finding x) by substitution of $e^{2x} = \sqrt{k}$ and $e^{-2x} = \frac{1}{\sqrt{k}}$]
	Obtain $g(x) \geq 2\sqrt{k}$ or $y \geq 2\sqrt{k}$	A1	5 or similarly simplified equiv with \geq not $>$

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