



# **Tuesday 20 June 2017 – Afternoon**

# **A2 GCE MATHEMATICS**

4723/01 Core Mathematics 3

#### **QUESTION PAPER**

Candidates answer on the Printed Answer Book.

#### **OCR** supplied materials:

- Printed Answer Book 4723/01
- List of Formulae (MF1)

#### Other materials required:

Scientific or graphical calculator

**Duration:** 1 hour 30 minutes

#### **INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the Printed Answer Book. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer all the guestions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do not write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

## **INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is 72.
- The Printed Answer Book consists of 16 pages. The Question Paper consists of 4 pages.
  Any blank pages are indicated.

# **INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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### Answer all the questions.

A curve has equation  $y = 2 + e^{\frac{1}{2}x}$ . The region *R* is bounded by the curve and by the straight lines x = 0, x = 4 and y = 0. Find the exact volume of the solid obtained when *R* is rotated completely about the *x*-axis.

[5]

2 (i) Use Simpson's rule with four strips to find an approximation to

$$\int_{1}^{9} \ln x \ln(x+4) dx,$$

giving your answer correct to 4 significant figures.

[4]

(ii) Deduce an approximation to

$$\int_{1}^{9} \ln(x^{-1}) \ln(x^{2} + 8x + 16) dx,$$

giving your answer correct to 4 significant figures.

[2]

[3]

[3]

- 3 (i) Sketch the graph of y = |2x 7a|, where a is a positive constant. State the coordinates of the points where the graph meets each axis. [2]
  - (ii) Solve the inequality |2x-7a| < 4a.

(iii) Deduce the largest integer N satisfying the inequality  $|2 \ln N - 10.5| < 6$ .

4 The angle  $\theta$ , where  $90^{\circ} < \theta < 180^{\circ}$ , satisfies the equation

$$3 \sec^2 \theta + 10 \tan \theta = 11$$
.

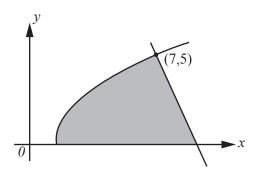
(i) Find the value of  $\tan \theta$ .

(ii) Without using a calculator, determine the value of

(a) 
$$\tan 2\theta$$
, [2]

**(b)** 
$$\cot(2\theta + 135^{\circ})$$
.

5



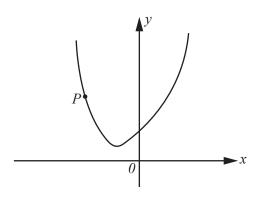
The diagram shows the curve  $y = \sqrt{4x-3}$  and the normal to the curve at the point (7,5). The shaded region is bounded by the curve, the normal and the x-axis. Find the exact area of the shaded region. [8]

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6 (i) Give full details of a sequence of two transformations needed to transform the graph of  $y = \frac{1}{x}$  to the graph of  $y = \frac{3}{x+1}$ .

The function f is defined by  $f(x) = \frac{3}{x+1}$  for  $x \ge 0$ .

- (ii) Determine the range of f. [2]
- (iii) Find an expression for  $f^{-1}(x)$ , and state how the graphs of y = f(x) and  $y = f^{-1}(x)$  are related geometrically.
- (iv) Solve the equation ff(x) = 2.
- 7 (i) It is given that  $y = a^x$  where a is a positive constant. Express x in terms of  $\ln y$  and, by first differentiating x with respect to y, show that  $\frac{dy}{dx} = a^x \ln a$ . [3]
  - (ii)



The diagram shows the curve  $y = x^4 + 4^x$ . At the point P on the curve, the gradient of the curve is -8.

- (a) Show that the x-coordinate of P satisfies the equation  $x = \sqrt[3]{-2 4^{x-1} \ln 4}$ . [3]
- (b) By first using an iterative process based on the equation in part (a) with a starting value of -1, find the coordinates of P. Show the result of each step of the iteration process and give the coordinates of P correct to 2 decimal places.[3]
- 8 (i) Express

 $3 \sin 2\theta \sec \theta + 4 \sin 2\theta \csc \theta$ 

in the form  $R\sin(\theta + \alpha)$ , where R > 0 and  $0^{\circ} < \alpha < 90^{\circ}$ .

(ii) Hence solve the equation

$$3\sin(2\beta + 20^{\circ})\sec(\beta + 10^{\circ}) + 4\sin(2\beta + 20^{\circ})\csc(\beta + 10^{\circ}) = 3$$

for 
$$0^{\circ} < \beta < 360^{\circ}$$
.

- 9 (a) The equation of a curve has the form  $y = \frac{px+q}{x^2+3}$ . Show that the curve has two distinct stationary points for all non-zero values of the constants p and q. [4]
  - **(b)** The equation of a curve has the form  $y = e^{x^2}(ax^2 + b)$ , where a and b are non-zero constants. It is given that  $\frac{d^2y}{dx^2}$  can be expressed in the form  $e^{x^2}(cx^4 + d)$ , where c and d are non-zero constants. Prove that 5a + 2b = 0.

#### **END OF QUESTION PAPER**



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