OCR

Oxford Cambridge and RSA

Tuesday 20 June 2017 – Afternoon

A2 GCE MATHEMATICS

4723/01 Core Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4723/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

1 A curve has equation $y = 2 + e^{\frac{1}{2}x}$. The region R is bounded by the curve and by the straight lines $x = 0$, $x = 4$ and $y = 0$. Find the exact volume of the solid obtained when R is rotated completely about the x-axis. **[5]**

2 **(i)** Use Simpson's rule with four strips to find an approximation to

$$\int_1^9 \ln x \ln(x+4)\,dx,$$

giving your answer correct to 4 significant figures. **[4]**

(ii) Deduce an approximation to

$$\int_1^9 \ln(x^{-1})\ln(x^2+8x+16)\,dx,$$

giving your answer correct to 4 significant figures. **[2]**

3 **(i)** Sketch the graph of $y = |2x - 7a|$, where a is a positive constant. State the coordinates of the points where the graph meets each axis. **[2]**

(ii) Solve the inequality $|2x - 7a| < 4a$. **[3]**

(iii) Deduce the largest integer N satisfying the inequality $|2\ln N - 10.5| < 6$. **[2]**

4 The angle θ, where $90° < \theta < 180°$, satisfies the equation

$$3\sec^2\theta + 10\tan\theta = 11.$$

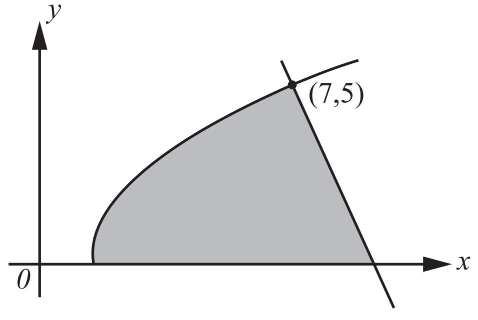
(i) Find the value of $\tan\theta$. **[3]**

(ii) Without using a calculator, determine the value of

(a) $\tan 2\theta$, **[2]**

(b) $\cot(2\theta + 135°)$. **[3]**

5



The diagram shows the curve $y = \sqrt{4x-3}$ and the normal to the curve at the point $(7, 5)$. The shaded region is bounded by the curve, the normal and the x-axis. Find the exact area of the shaded region. **[8]**

6 **(i)** Give full details of a sequence of two transformations needed to transform the graph of $y = \frac{1}{x}$ to the graph of $y = \frac{3}{x+1}$. **[2]**

The function f is defined by $\mathrm{f}(x) = \frac{3}{x+1}$ for $x \geqslant 0$.

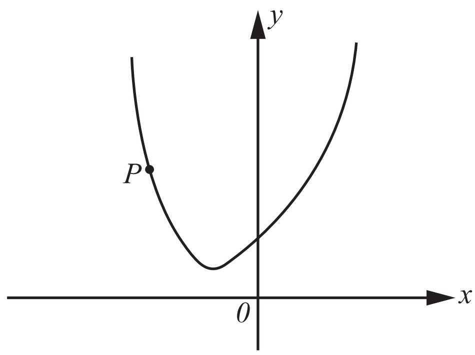
(ii) Determine the range of f. **[2]**

(iii) Find an expression for $\mathrm{f}^{-1}(x)$, and state how the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ are related geometrically. **[3]**

(iv) Solve the equation $\mathrm{ff}(x) = 2$. **[3]**

7 **(i)** It is given that $y = a^x$ where a is a positive constant. Express x in terms of $\ln y$ and, by first differentiating x with respect to y, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = a^x \ln a$. **[3]**

(ii)



The diagram shows the curve $y = x^4 + 4^x$. At the point P on the curve, the gradient of the curve is -8.

(a) Show that the x-coordinate of P satisfies the equation $x = \sqrt[3]{-2 - 4^{x-1}\ln 4}$. **[3]**

(b) By first using an iterative process based on the equation in part **(a)** with a starting value of -1, find the coordinates of P. Show the result of each step of the iteration process and give the coordinates of P correct to 2 decimal places. **[3]**

8 **(i)** Express

$$3\sin 2\theta \sec\theta + 4\sin 2\theta \operatorname{cosec}\theta$$

in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. **[5]**

(ii) Hence solve the equation

$$3\sin(2\beta + 20°)\sec(\beta + 10°) + 4\sin(2\beta + 20°)\operatorname{cosec}(\beta + 10°) = 3$$

for $0° < \beta < 360°$. **[5]**

9 **(a)** The equation of a curve has the form $y = \dfrac{px+q}{x^2+3}$. Show that the curve has two distinct stationary points for all non-zero values of the constants p and q. **[4]**

(b) The equation of a curve has the form $y = e^{x^2}(ax^2+b)$, where a and b are non-zero constants. It is given that $\dfrac{d^2y}{dx^2}$ can be expressed in the form $e^{x^2}(cx^4+d)$, where c and d are non-zero constants. Prove that $5a+2b=0$. **[5]**

END OF QUESTION PAPER