Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE In Further Mathematics
Mechanics 1 Paper 9FM0/3C

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2019
Publications Code 9FM0_3C_MS_1906

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $*$ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

- Omission or extra g in a resolution is an accuracy error not method error.

- Omission of mass from a resolution is a method error.

- Omission of a length from a moments equation is a method error.

- Omission of units or incorrect units is not (usually) counted as an accuracy error.

- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

- Any numerical answer which comes from use of $g = 9.8$ should be given to 2 or 3 SF.

- Use of $g = 9.81$ should be penalised once per (complete) question.

  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

  Marks must be entered in the same order as they appear on the mark scheme.

- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.

- Accept column vectors in all cases.

- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

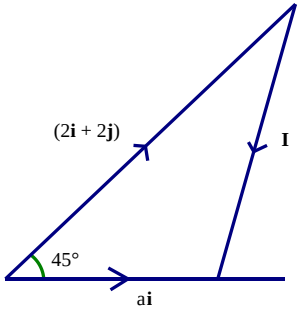
- Mechanics Abbreviations

M(A)	Taking moments about A
N2L	Newton's Second Law (Equation of Motion)
NEL	Newton's Experimental Law (Newton's Law of Impact)
HL	Hooke's Law
SHM	Simple harmonic motion
PCLM	Principle of conservation of linear momentum
RHS, LHS	Right hand side, left hand side

Question	Scheme	Marks	AOs	Notes
1a	Speed after first impact $=\frac{2}{3}u$	B1	3.4	Correct use of impact law, seen or implied. Allow +/-
	Speed after second impact $=\frac{4}{9}u$	B1	3.4	Correct use of impact law a second time, seen or implied. Allow +/-
	Correct method for total time	M1	2.1	Use of $t=\frac{d}{v}$ or equivalent for at least 2 of the 3 parts added
	$T=\frac{d}{u}+\frac{3}{\frac{2}{3}u}+\frac{3-d}{\frac{4}{9}u}$	A1ft A1ft	1.1b 1.1b	Unsimplified expression for T with all 3 terms and at most one error. Follow their speeds. Correct unsimplified expression for T. Follow their speeds
	$=\frac{4d+18+27-9d}{4u}=\frac{45-5d}{4u}$ *	A1*	2.2a	Obtain **given answer** from correct working
		(6)		
1b	• Least T when d is maximum • Furthest distance at highest speed • Highest average speed • Sketch graph of function	B1	2.4	Correct reasoning
	i.e. $d=3$, least $T=\frac{30}{4u}=\frac{15}{2u}$	B1	2.2a	Correct answer only. Any equivalent form. $\left(\frac{7.5}{u}\right)$
		(2)		
		(8 marks)		

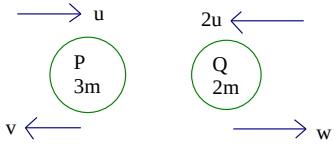
Question	Scheme	Marks	AOs	Notes
2(a)	After hit AB: $\rightarrow 6\cos\alpha\,(=v\cos\beta)\quad(=3.6)$	B1	3.1b	Use model to find component parallel to the wall
	Use of impact law:	M1	3.4	Use model and impact law perpendicular to the wall
	$\uparrow 6e\sin\alpha\ \ (=v\sin\beta)\left(=\frac{24e}{5}\right)(=4.8e)$	A1	1.1b	Correct perpendicular component
	$\tan\beta=\frac{1}{3}=\frac{6e\sin\alpha}{6\cos\alpha}\left(=\frac{24e}{5}\div\frac{18}{5}\right)$	M1	2.1	Use $\frac{1}{3}$ and their components to form equation in e $\left(v=\frac{6\sqrt{10}}{5}=3.79\right)$
	$e=\frac{18}{3\times 24}=\frac{1}{4}$ *	A1*	2.2a	Correct answer from correct exact working
				If only see $e\tan\alpha=\tan\beta$ with no explanation of where it comes from then score 0/5
		(5)		
(b)	After hit BC: $\uparrow\frac{6}{5}$	B1	1.1b	First component correct
	$\rightarrow\frac{1}{2}\times\frac{18}{5}\left(=\frac{9}{5}\right)$	B1	3.4	Second component correct
				Alternative: B1 for speed of impact with $BC=3.79.....$ B1 for path on leaving BC at $56.3...^\circ$ to BC
	Speed $=\frac{3}{5}\sqrt{2^2+3^2}$	M1	1.1b	Use Pythagoras' theorem or trigonometry to find the speed
	$=\frac{3\sqrt{13}}{5}\,(\text{m s}^{-1})$	A1	1.1b	Any equivalent form. 2.2 or better $(2.1633....)$
		(4)		

(c)	An appropriate refinement	B1	3.5c	Two independent refinements relating to the modelling e.g. • Include friction between the floor and the ball • Include friction between the ball and the walls • Give the ball dimensions . • Consider air resistance • Spin / rotation Do not accept comments about mass / gravity / levels / perpendicularity
	A second independent appropriate refinement and no incorrect refinements	B1	3.5c	
		(2)		
			(11 marks)	

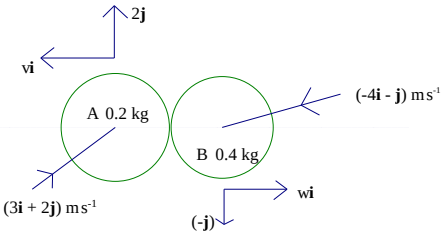
Question	Scheme	Marks	AOs	Notes
3				
	Momentum of P after impulse = a**i** (or a**j**)	B1	2.2a	Correct interpretation of angle of deflection (velocity or momentum a multiple of **i** or **j**)
Either	Use of $\mathbf{I}=m(\mathbf{v}-\mathbf{u})$: $(\mathbf{I}=)0.5(2a\mathbf{i}-(4\mathbf{i}+4\mathbf{j}))(=(a-2)\mathbf{i}-2\mathbf{j})$	M1	3.3	Form vector triangle or equation for **v** or their a**i**
	Use of Pythagoras to form equation in a	M1	3.4	Use trigonometry or Pythagoras' theorem to form equation in a
	$6.25=0.25((2a-4)^2+16)$ $(4a^2-16a+7=0)$	A1ft A1	1.1b 1.1b	Unsimplified equation with at most one error. Follow their a**i** Correct unsimplified equation
Or	$\lambda^2+\mu^2=\frac{25}{4}$	M1		
	$\mathbf{I}=\lambda\mathbf{i}+\mu\mathbf{j}=\frac{1}{2}((x-4)\mathbf{i}-4\mathbf{j})$	M1		
	$\mu=-2$	A1		Dependent on 2nd M (for impulse)
	$\lambda^2=\frac{9}{4}$	A1		

Or	Use of $\mathbf{I}=m(\mathbf{v}-\mathbf{u})$ to form vector triangle	M1	3.3	
	Form equation in their a	M1	3.4	
	$6.25=a^2+8-2a\sqrt{8}\times\frac{1}{\sqrt{2}}$ $\left(4\times6.25=b^2+32-2b\sqrt{32}\times\frac{1}{\sqrt{2}}\text{ for velocity b}\mathbf{i}\right)$ $\left(4a^2-16a+7=0\right)$	A1ft A1	1.1b 1.1b	
	$a=\frac{7}{2},\ \frac{1}{2}\ \Rightarrow\mathbf{I}=\frac{3}{2}\mathbf{i}-2\mathbf{j}$ (Ns)	M1	1.1b	Complete correct method to solve to find a pair of values for λ and μ
	or $\mathbf{I}=-\frac{3}{2}\mathbf{i}-2\mathbf{j}$ (Ns)	A1	1.1b	Two correct pairs of values for λ and μ
	or $\mathbf{I}=-2\mathbf{i}-\frac{3}{2}\mathbf{j}$ (Ns)	M1	2.2a	Use symmetry in complete correct method to find one of the other pairs of values for λ and μ
	or $\mathbf{I}=-2\mathbf{i}+\frac{3}{2}\mathbf{j}$ (Ns)	A1	1.1b	All four correct pairs (They do not need to write out the impulse in full)
		(9)		
			(9 marks)	

Question	Scheme	Marks	AOs	Notes
4(a)	Use of $P = Fv$: $F = \frac{15000}{25} (= 600)$	B1	3.3	600 or equivalent
	Equation of motion:	M1	3.4	Use the model to form the equation of motion If they start with two separate equations each one must be correct.
	$F - (200 + 200 + 25\lambda) = 0$	A1	1.1b	Correct unsimplified equation
	$\lambda = 8$ *	A1*	2.2a	Deduce **given answer** from correct working.
		(4)		
4(b)	Equation of motion:	M1	3.4	Use the model to form the equation of motion for the car (with $v = 10$ used). All terms required. Dimensionally correct. Condone sign error and sin/cos confusion
	$\frac{15000}{10} - 280 - 600g\sin\theta = 600a$	A1 A1	1.1b 1.1b	Unsimplified equation with at most one error. Correct unsimplified equation
	$a = 1.38$ m s^{-2} (1.4)	A1	1.1b	2 or 3 sf only – follows use of 9.8
		(4)		
4(c)	Work energy equation	M1	3.1b	Complete strategy to form the work-energy equation. Condone sin/cos confusion and sign errors
	$\frac{1}{2} \times 150 \times 100 = 200d + 150gd\sin\theta$	A1 A1	1.1b 1.1b	Unsimplified equation with at most one error Correct unsimplified equation for d
	$d = 25.2$ (m) (25)	A1	1.1b	Max 3 sf – follows use of 9.8
		(4)		
		(Total 12 marks)		

Question	Scheme	Marks	AOs	Notes
5(a)				
	Use of CLM	M1	3.1a	Use of CLM. All terms required. Must be dimensionally correct. Condone sign errors
	$3mu-4mu=2mw-3mv \quad (-u=-3v+2w)$	A1	1.1b	Correct unsimplified equation
	Use of impact law	M1	3.4	Use of impact law. Must be dimensionally correct and used correctly. Condone sign errors
	$w+v=3ue$	A1	1.1b	Correct unsimplified equation Signs consistent with CLM equation
	Correct strategy to form equation in w and find critical value of $e\in(0,1)$ $(5w=u(9e-1))$	M1	3.1a	Correct overall strategy to find the critical value of e in (0,1) in e eg by using CLM and impact law to form equation or inequality in w and solve for e.
	$w>0:e>\frac{1}{9}$	A1	1.1b	One inequality for e correct Condone $e\geq\frac{1}{9}$
	Complete strategy to justify the range of values of e $(5v=u(1+6e))$ $v>0$: true for all e	M1	3.1a	Correct strategy to find the range of possible value of e. i.e find second speed and form second inequality
	Therefore $\frac{1}{9}<e\leq 1$	A1	2.2a	Correct final conclusion
		(8)		

Question	Scheme	Marks	AOs	Notes
5(b)	Final KE = 25% of initial KE	M1	3.1a	Use KE to form equation in e. 25% should be used correctly Condone if mass cancelled throughout
	$\frac{1}{2}\times 2m\times\frac{u^2(9e-1)^2}{25}=\frac{1}{4}\times\frac{1}{2}\times 2m\times 4u^2 \quad \left(\text{or } w=\frac{1}{2}\times 2u\right)$	A1ft	1.1b	Correct unsimplified equation – follow their w
	$\Rightarrow (9e-1)^2=25,\ e=\frac{2}{3}$ only	A1	1.1b	Or equivalent. Correct conclusion ISW after correct answer.
		(3)		
			(11marks)	

Question	Scheme	Marks	AOs	Notes
6(a)				
	Perpendicular to line of centres: $2\mathbf{j}$	B1	3.4	Use the model to find the component perpendicular to the line of centres. Correct value seen or implied
	CLM parallel to the line of centres	M1	3.1b	Use of CLM parallel to line of centres. Need all terms and dimensionally correct. Condone sign errors
	$0.2\times3-0.4\times4=0.4w-0.2v \quad (-5=2w-v)$	A1	1.1b	Correct unsimplified equation.
	Impact law parallel to the line of centres	M1	3.4	Correct use of impact law parallel to the line of centres. Condone sign errors
	$7e=v+w \quad \Rightarrow 3=v+w$	A1	1.1b	Correct equation with $\frac{3}{7}$ used.
	Complete strategy to find $\mathbf{v}_A$	M1	3.1b	Complete strategy to find components parallel and perpendicular to line of centres, eg by using CLM and impact law
	$\mathbf{v}_A=-\frac{11}{3}\mathbf{i}+2\mathbf{j} \ \left(\text{m s}^{-1}\right)$ follow their $2\mathbf{j}$	A1ft	1.1b	$\mathbf{v}_A$ correct, follow their $a\mathbf{j}$ for $2\mathbf{j}\left(a\neq0\right)$
		(7)		
6(b)	Magnitude of impulse on A: $0.2\left(\frac{11}{3}-\left(-3\right)\right)$	M1	3.1b	Evidence of use of $m\left(v-u\right)$ parallel to the line of centres
	$=0.2\left(\frac{11}{3}+3\right)=\frac{4}{3}$ (Ns)	A1	1.1b	1.3 (Ns) or better
		(2)		

Question	Scheme	Marks	AOs	Notes
6(c)	Use of scalar product to find the angle	M1	3.1a	Complete method for finding the required angle. Allow for $\tan^{-1}\frac{3}{2}$ or $\tan^{-1}\frac{2}{3}$ and $\tan^{-1}\frac{6}{11}$ or $\tan^{-1}\frac{11}{6}$
	$\cos\theta = \dfrac{(3\mathbf{i}+2\mathbf{j}).\left(-\frac{11}{3}\mathbf{i}+2\mathbf{j}\right)}{\sqrt{13}\times\sqrt{\frac{157}{9}}}$	A1ft	1.1b	A correct unsimplified expression Follow their $\mathbf{v}_A$. Do not ISW
	$\theta = 118°$	A1	1.1b	Correct answer only. (Q asks for the nearest degree) Do not ISW
	Alternative method: $180° - \tan^{-1}\frac{2}{3} - \tan^{-1}\frac{6}{11}$ Or $\tan^{-1}\frac{3}{2} + \tan^{-1}\frac{11}{6}$			$62°$ probably scores M1A0A0
		(3)		
		(Total 12 marks)		

Question	Scheme	Marks	AOs	Notes
7(a)	From A to B EPE lost = GPE gained	M1	2.1	Use conservation of energy with EPE $= \frac{\lambda x^2}{2a}$. (Condone EPE $= \frac{\lambda x^2}{a}$ here). All three terms required. Must be dimensionally correct. Condone sign errors.
	$\frac{kmg \times 4a^2}{2a} - \frac{kmg \times \frac{a^2}{4}}{2a} = mg \times \frac{5a}{2}$	A1	1.1b	Correct unsimplified equation in k
	$k = \frac{4}{3}$ *	A1*	2.2a	Derive given result from correct working.
		(3)		
7(b)	At A, equation of motion:	M1	3.1a	Use $T = \frac{\lambda x}{a}$ and N2L to form equation of motion. All terms required. Dimensionally correct. Condone sign errors
	$(T - mg =) \frac{4mg \times 2a}{3a} - mg = m \times \text{acceleration}$	A1	1.1b	Correct unsimplified equation
	$\Rightarrow \text{acceleration} = \frac{5g}{3}$	A1	1.1b	Correct only ISW. Condone 1.7g or better. Accept + / -
		(3)		

Question	Scheme	Marks	AOs	Notes
7(c)	Max speed at equilibrium position	M1	3.1a	Maximum speed at equilibrium seen or implied, and correct method to find e
	$\frac{4mge}{3a} = mg, \quad e = \frac{3a}{4}$	A1	1.1b	Correct e
				Alternative: form energy equation for movement through a height of h and differentiate v^2 wrt h to find h for max v M1 $h = \frac{5a}{4}$ A1
	Forms equation using conservation of energy	M1	3.1a	Form energy equation for movement from A to equilibrium position. Need all 4 terms. Correct form for EPE. Dimensionally correct. Condone sign errors. Allow in a, g and e (with e defined)
	$\frac{4mg \times 4a^2}{3 \times 2a} = \frac{4mg \times \frac{9a^2}{16}}{3 \times 2a} + \frac{1}{2}mv^2 + mg \times \frac{5a}{4}$	A1ft A1ft	1.1b 1.1b	Unsimplified equation in their e with at most one error Correct unsimplified equation (using their e) for v
	$v = \frac{5}{2}\sqrt{\frac{ga}{3}}$	A1	1.1b	Any equivalent form. Accept $1.44\sqrt{ag}$ or $1.4\sqrt{ag}$
		(6)		
				SHM is not on this specification, but you might see some candidates using it. See over for SHM alternative for parts (b) and (c)

	At equilibrium, $\frac{4mge}{3a} = mg, \quad e = \frac{3a}{4}$			They need to start by showing that they have SHM in order tto justify using the standard results. No marks scored for this at this stage.
	Equation of motion: $mg - \frac{4mg}{3a}(e + x) = m\ddot{x}$, so $\ddot{x} = -\frac{4g}{3a}x$ Hence SHM			
	(b) Use of $x = \frac{5a}{4}$ and their ω^2	M1		Substitute to find acceleration
	$\ddot{x} = -\frac{4g}{3a} \times \frac{5a}{4} = -\frac{5g}{3}, \quad \lvert\ddot{x}\rvert = \frac{5g}{3}$	A1		Correct only ISW. Condone 1.7g or better
		(2)		
	(c) $\frac{4mge}{3a} = mg,$	M1		This work now scores the two marks provided it is used in part (c)
	$e = \frac{3a}{4}$	A1		
	Use of $v_{max} = \omega a$	M1		Correct method to find max v
	$v_{max} = \sqrt{\frac{4g}{3a}} \times \frac{5a}{4}$	A2ft		Follow their e and ω
	$v_{max} = \frac{5}{2}\sqrt{\frac{ga}{3}}$	A1		Any equivalent form. Accept $1.44\sqrt{ag}$ or $1.4\sqrt{ag}$
		(6)		
			(Total 12 marks)	

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom