AQA

A-level
Mathematics

MPC4-Pure Core 4

Mark scheme

June 2018

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

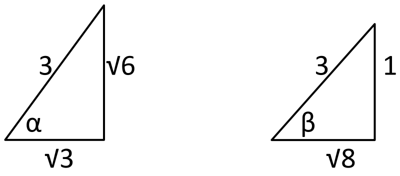
Q1	Solution	Mark	Total	Comment
(a)	$10 + 24x - 12x^2$ $= A(3-x)(1+4x) + B(1+4x) + C(3-x)$	M1		PI by B or C being correct.
	$A = 3$	B1		Could be spotted
	$B = -2$	A1		
	$C = 1$	A1	4	
(b)	$\int 3 - \frac{2}{3-x} + \frac{1}{1+4x}\,dx \;=\; 3x$	B1ft		ft on their value of A.
	$+r\ln(3-x) + s\ln(1+4x)$	M1		
	$+2\ln(3-x) + \frac{1}{4}\ln(1+4x)$	A1ft		ft on their values of B and C.
	$\int_0^2 \mathrm{f(x)}dx = \left(3\times 2 + 2\ln 1 + \frac{1}{4}\ln 9\right)$ $-(3\times 0 + 2\ln 3 + \frac{1}{4}\ln 1)$	M1		Correct use of $F(2) - F(0)$ for **their** $Ax + r\ln(3-x) + s\ln(1+4x)$ form but *0* and ln 1 = *0* can be **PI**
	$= 6 - \frac{3}{2}\ln 3$	A1	**5**	
			9	

(a)	If $A = 3$ clearly comes FIW then **B0**.
(a)	PI by B or C being correct covers case(s) such as the denominator(s) still included at the initial line but not then used when finding B or C.

(a)	By long division $$\begin{array}{r} 3 \\ -4x^2+11x+3 \overline{)\,-12x^2+24x+10} \\ \underline{-12x^2+33x+9} \\ -9x+1 \end{array}$$ $\frac{-9x+1}{(3-x)(1+4x)} = \frac{B}{3-x} + \frac{C}{1+4x}$			
	$-9x + 1 = B(1+4x) + C(3-x)$	(M1)		PI by B or C correct
	$B = -2$	(A1)		
	$C = 1$	(A1)		
	$A = 3$	(B1)	(4)	
	NMS or cover up rule scores			
	$A = 3$	(B1)		
	$B = -2$ **or** $C = 1$	(B2)		**Either** correct
	Both B and C correct	(+B1)	(4)	

(b)	Condone missing brackets with $\ln(3-x)$ and $\ln(1+4x)$ provided recovered when using limits. Beware candidate who changes $-\frac{2}{3-x}$ to $\frac{2}{x-3}$ and gets $2\ln(x-3)$which isn't valid – only award the first **M1** if they clearly use $2\ln\lvert x-3\rvert$ in this case.

Q2	Solution	Mark	Total	Comment

(a)				or Pythagoras
	$\tan\alpha = \frac{\sqrt{6}}{\sqrt{3}} = \sqrt{2}$	B1		AG - must see √6 in this approach
	$\tan\beta = (\pm)\frac{1}{\sqrt{8}}$	M1		Either $\frac{1}{\sqrt{8}}$ or $-\frac{1}{\sqrt{8}}$
	$\tan\beta = -\frac{1}{\sqrt{8}}$	A1	3	ACF: e.g. $-\frac{1}{2\sqrt{2}}$ or $-\frac{\sqrt{2}}{4}$ etc.
(b)	$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$			
	$= \frac{\sqrt{2} - \left(-\frac{1}{\sqrt{8}}\right)}{1+\sqrt{2}\left(-\frac{1}{\sqrt{8}}\right)}$	M1		**Correct** identity with $\tan\alpha = \sqrt{2}$ and **their** $\tan\beta$ value **correctly** substituted.
	$= \frac{5}{2}\sqrt{2}$	A1	2	OE – accept if written as $\frac{5\sqrt{2}}{2}$ etc. NMS scores 0/2.
			5	
	Candidates who find α and/or β as 54.7⁰ and 160.5⁰ and use these to find $\tan\alpha$ and $\tan\beta$ score 0 marks in part (a) but can score in part (b) provided exact values for $\tan\alpha$ and $\tan\beta$ are used - PI by a correct final answer of $\frac{5}{2}\sqrt{2}$.			
(a)	If $\cos\alpha = \frac{\sqrt{3}}{3}$ is replaced by $\cos\alpha = \frac{1}{\sqrt{3}}$ the right-angled triangle will have sides $1, \sqrt{2}$ and $\sqrt{3}$. As an alternative to the right angled triangle they might use appropriate identities e.g. $\sin^2\alpha + \cos^2\alpha = 1 \rightarrow \sin^2\alpha = \frac{6}{9} \rightarrow \sin\alpha = \sqrt{\frac{6}{9}} \rightarrow \tan\alpha = \sqrt{\frac{6}{9}}\Big/\frac{\sqrt{3}}{3}$ OE $\rightarrow \tan\alpha = \sqrt{2}$ **B1** or $1+\tan^2\alpha = \sec^2\alpha \rightarrow 1+\tan^2\alpha = \frac{9}{3} \rightarrow \tan^2\alpha = 2 \rightarrow \tan\alpha = \sqrt{2}$ **B1** or $1+cot^2\beta = cosec^2\beta = \frac{1}{sin^2\beta} \rightarrow cot^2\beta = 8 \rightarrow \tan\beta = (\pm)\frac{1}{\sqrt{8}}$ **M1** $\rightarrow \tan\beta = -\frac{1}{\sqrt{8}}$ OE **A1** Candidates who just state such as $\tan\beta = \tan\left(sin^{-1}\left(\frac{1}{3}\right)\right) = -\frac{1}{\sqrt{8}}$ score **0/2** etc.			
(b)	$\tan(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \frac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta+\sin\alpha\sin\beta} = \frac{\left(\frac{\sqrt{6}}{3}\right)\left(-\frac{\sqrt{8}}{3}\right)-\left(\frac{\sqrt{3}}{3}\right)\left(\frac{1}{3}\right)}{\left(\frac{\sqrt{3}}{3}\right)\left(-\frac{\sqrt{8}}{3}\right)+\left(\frac{\sqrt{6}}{3}\right)\left(\frac{1}{3}\right)}$ **M1** **Correct** identity with **correct** substitution of $\cos\alpha = \frac{\sqrt{3}}{3}$, $\sin\beta = \frac{1}{3}$ and **their** values of $\sin\alpha$ and $\cos\beta$ $\rightarrow \tan(\alpha-\beta) = \frac{5}{2}\sqrt{2}$ **A1**			

Q3	Solution	Mark	Total	Comment

(a)	$(1-9x)^{\frac{2}{3}} = 1+\frac{2}{3}(-9x)+kx^2$	**M1**		or better with $k \neq 0$
	$= 1-6x-9x^2$	**A1**	**2**	Coefficients must be simplified.
(b)(i)	$(64-9x)^{\frac{2}{3}} = 64^{\frac{2}{3}}\left(1-\frac{9}{64}x\right)^{\frac{2}{3}}$	**B1**		ACF – e.g. $16\left(1-\frac{9}{64}x\right)^{\frac{2}{3}}$
	$\left(1-\frac{9}{64}x\right)^{\frac{2}{3}} = 1+\frac{2}{3}\times\left(-\frac{9}{64}x\right)+\frac{2}{3}\times\frac{-1}{3}\left(-\frac{9}{64}x\right)^2\times\frac{1}{2}$	**M1**		OE – condone missing brackets.
	$(64-9x)^{\frac{2}{3}} = 16-\frac{3}{2}x-\frac{9}{256}x^2$	**A1**	**3**	Accept $16\left(1-\frac{3}{32}x-\frac{9}{4096}x^2\right)$
(b)(ii)	Substituting $x=-\frac{1}{3}$ (OE) into their (b)(i)	**M1**		$64-9x=67 \Rightarrow x=-\frac{1}{3}$ or $-\frac{3}{9}$
	$\left(67^{\frac{2}{3}}\right) = 16+\frac{127}{256}$	**A1**	**2**	Must have **correct** expansion in (b)(i) **and** use $x=-\frac{1}{3}$. OE for $\frac{127}{256}$ - e.g. $\frac{254}{512}$ etc. **NMS** is **0/2** since 'Hence'…
			7	

(b)(i)	Some candidates omit x in their expansion and just find the coefficients or make other slips. Allow recovery if the correct full expansion is seen later. As a guide, check the answer line first for **B1 M1 A1**.
(b)(i)	**Alt.1 – using (a)** After $(64-9x)^{\frac{2}{3}} = 64^{\frac{2}{3}}\left(1-\frac{9}{64}x\right)^{\frac{2}{3}}$ (**B1**) then using candidate's expansion for (a) with x replaced by $\frac{x}{64}$, $\left(1-\frac{9}{64}x\right)^{\frac{2}{3}} = 1-6\left(\frac{x}{64}\right)-9\left(\frac{x}{64}\right)^2$ (**M1**) $(64-9x)^{\frac{2}{3}} = 16\left(1-\frac{3}{32}x-\frac{9}{4096}x^2\right)$ OE (**A1**) **Alt. 2 – using binomial formula** $(64-9x)^{\frac{2}{3}} = 64^{\frac{2}{3}}+\frac{2}{3}64^{-\frac{1}{3}}(-9x)+\frac{2}{3}.\left(-\frac{1}{3}\right).\frac{1}{2}64^{-\frac{4}{3}}(-9x)^2$ (**M1**) condone missing brackets $= 16-\frac{3}{2}x-\frac{9}{256}x^2$ or $16\left(1-\frac{3}{32}x-\frac{9}{4096}x^2\right)$ OE (**A2**)
(b)(ii)	Accept answer given as $16\ \frac{127}{256}$ OE for final **A1** mark.

Q4	Solution	Mark	Total	Comment
(a)(i)	$18\left(-\frac{2}{3}\right)^3 - 3\left(-\frac{2}{3}\right)^2 - 28\left(-\frac{2}{3}\right) - 12$	**M1**		Correct substitution of $x = -\frac{2}{3}$
	$= 18 \times \left(-\frac{8}{27}\right) - 3\left(\frac{4}{9}\right) - 28\left(-\frac{2}{3}\right) - 12$			or better
	$= 0$ (hence) factor	**A1**	**2**	**Correct** arithmetic **and** conclusion.
(a)(ii)	By factors $6x^2 + bx - 6$ $= 6x^2 - 5x - 6$ $(f(x)) = (3x+2)(3x+2)(2x-3)$ OE	 **M1** **A1** **A1**	 **3**	 'Spotting' $a = 6$ and $c = -6$. **NMS** scores 3/3 if correct
(b)(i)	$18\sin 2\theta\cos\theta - 3\cos 2\theta + 20\sin\theta + 27$			
	$= 18 \times 2\sin\theta\cos\theta\cos\theta$	**B1**		**Correct** identity **used** for $\sin 2\theta$.
	$-3(1 - 2\sin^2\theta) + 20\sin\theta + 27$ $= 36\sin\theta(1 - \sin^2\theta) - 3 + 6\sin^2\theta + 20\sin\theta + 27$	**B1** **M1**		**Any correct** identity **used** for $\cos 2\theta$. Use $\cos^2\theta = 1 - \sin^2\theta$ to obtain a cubic expression in $\sin\theta$ only.
	$= -36\sin^3\theta + 6\sin^2\theta + 56\sin\theta + 24$ $(= -36x^3 + 6x^2 + 56x + 24)$ Equate to 0 and cancel down to the **equation** $18x^3 - 3x^2 - 28x - 12 = 0$	**A1**	**4**	Do not award final **A1** if division or changing signs occurs before equating to 0 or any error seen. Accept in terms of $\sin\theta$.
(b)(ii)	$(\theta =)$ 3.87 , 5.55 CAO	**B1B1**	**2**	-1 for **each** extra sol^n in $0 \le \theta \le 2\pi$
			11	

(a)(i)	Must see arithmetic for **A1** (minimum acceptable shown in scheme) $= -\frac{144}{27} - \frac{12}{9} + \frac{56}{3} - 12$ is 'better' For **A1** we must see $= 0$ and conclusion. Might come first - e.g. if $f\left(-\frac{2}{3}\right) = 0$ then $(3x+2)$ is a factor etc. $f\left(-\frac{2}{3}\right) = 18\left(-\frac{2}{3}\right)^3 - 3\left(-\frac{2}{3}\right)^2 - 28\left(-\frac{2}{3}\right) - 12 = 12 - 12 = 0$ is not sufficient arithmetic for **A1**.
(a)(ii)	For guidance, terms in x^2 give $-3 = 12 + 3b$ or terms in x give $-28 = -18 + 2b$ so $b = -5$. Using long division by $(3x+2)$, Quotient = $6x^2 - 5x + c$ **M1** $= 6x^2 - 5x - 6$ **A1** then $f(x) = (3x+2)(2x-3)(3x+2)$ **A1** There must be **NO** remainder seen or implied by wrong arithmetic to earn the final **A1**. Candidates who try to find and use the other (linear) factor of $(2x-3)$ rather than using $(3x+2)$ obtained in (a)(i) score By factors, $9x^2 + bx + 4$ **M1** $= 9x^2 + 12x + 4$ **A1** $f(x) = (2x-3)(3x+2)(3x+2)$ **A1** or, by long division, $9x^2 + 12x + c$ **M1** $= 9x^2 + 12x + 4$ **A1** $f(x) = (2x-3)(3x+2)(3x+2)$ **A1**
(b)(ii)	If (a)(ii) would score **NR** or **no** work seen the marks for (a)(ii) if earned can be scored here.
(b)(ii)	If **0/2** scored award **B1** for **both** solutions correct to **greater** accuracy 3.871320… and 5.553457… (rounded correctly or truncated) but still apply -1 for any excess solution(s) in the interval $0 \le \theta \le 2\pi$. Condone answers given as $x = \cdots$ rather than $\theta = \cdots$ Ignore other factor (even if wrong) if it doesn't lead to any solutions.

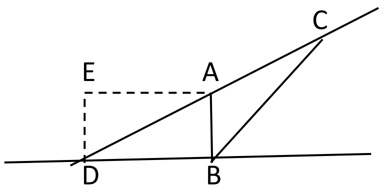
Q5	Solution	Mark	Total	Comment
(a)(i)	4500	**B1**	**1**	
(a)(ii)	1220	**B1**	**1**	
(a)(iii)	$4500\mathrm{e}^{-\frac{1}{20}t} < 1500$			
	$\frac{1}{20}t > \ln 3$ or $-\frac{1}{20}t < \ln\frac{1}{3}$ or better	**M1**		Correctly converting from exponential to logarithmic form
	22	**A1**	**2**	Allow 21.97… **NMS** scores **B2** for 22 or 21.97…
(b)	$(Q = 4P \ \Rightarrow) \quad 3000\mathrm{e}^{-\frac{1}{40}t} = 4(4500)\mathrm{e}^{-\frac{1}{20}t}$ OE	**M1**		Setting up a correct equation but **M0** if logs **not** used later.
	$\frac{t}{40} = \ln 6$ or $-\frac{t}{40} = \ln\frac{1}{6}$ OE	**A1**		e.g. $\ln 3000 - \frac{t}{40} = \ln 18000 - \frac{t}{20}$
	72	**A1**	**3**	CAO
(c)(i)	$3000e^{-\frac{1}{40}T} - 4500e^{-\frac{1}{20}T} = 300$	**M1**		Setting up a correct equation – could include both *x* and *T* (or *t*).
	$\left(x = e^{-\frac{1}{40}T}\right) \quad \Rightarrow \quad 3000x - 4500x^2 = 300$ $15x^2 - 10x + 1 = 0$	**A1**	**2**	Correct quadratic in x (ACF) - apply ISW for wrong cancelling or rearranging.
(c)(ii)	$(x) \quad = \frac{10 \pm \sqrt{40}}{30}$ (0.12… or 0.54…)	**dM1**		
	$T = 24.3(41\ \ldots)$	**A1**		Allow 24
	$= 170$ (days)	**A1**	**3**	Accept October 18th if 170 not seen.
			12	

(a)(iii)	Condone use of = instead of < or poor/incorrect use of inequality signs for **M1** but for **A1** we must see 22 (or 21.97…). **M1** mark is PI by such as $\frac{1}{20}t = 1.09\ \ldots$ OE.
(b)	Misinterpretation of P = 4Q scores 0/3 as it should lead to a negative value for t. **A1** mark is PI by such as $\frac{t}{40} = 1.79\ \ldots$ OE
(c)(i)	The **M1** awarded for setting up an equation in T (or t) is automatically earned if they miss this line out and immediately write down the **correct** quadratic in x. Allow = replaced by an inequality for the **M1** mark but the expression in x must be an equation for **A1**.
(c)(ii)	For **dM1** mark ft on any wrong quadratic – check method used or answers given.

Q6	Solution	Mark	Total	Comment
(a)	$(k =) \quad \pi$	B1	1	
(b)	$\frac{\mathrm{d}}{\mathrm{d}x}(\cos 3y) = -3\sin 3y \frac{\mathrm{d}y}{\mathrm{d}x}$	B1		
	$\frac{\mathrm{d}}{\mathrm{d}x}(y\sin^2 3x) = \frac{\mathrm{d}y}{\mathrm{d}x}\sin^2 3x + ky\sin 3x\cos 3x$	M1		Second term could be kysin6x
	$= \frac{\mathrm{d}y}{\mathrm{d}x}\sin^2 3x + 6y\sin 3x\cos 3x$	A1		or $\frac{\mathrm{d}y}{\mathrm{d}x}\sin^2 3x + 3y\sin 6x$ OE
	$\frac{\mathrm{d}}{\mathrm{d}x}(x+k) = 1$	B1		
	$\frac{\mathrm{d}y}{\mathrm{d}x}(-3\sin 3y + \sin^2 3x) = 1 - 6y\sin 3x\cos 3x$	M1		Factor out correctly $\frac{\mathrm{d}y}{\mathrm{d}x}$ from their **two** terms.
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1 - 6y\sin 3x\cos 3x}{-3\sin 3y + \sin^2 3x}$	A1	6	OE
(c)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1 - 6\left(\frac{3\pi}{2}\right)\sin\left(\frac{3\pi}{2}\right)\cos\left(\frac{3\pi}{2}\right)}{-3\sin\left(\frac{9\pi}{2}\right) + \sin^2\left(\frac{3\pi}{2}\right)}$	M1		Using $x = \frac{\pi}{2}$ and $y = \frac{3\pi}{2}$. PI by $m = -\frac{1}{2}$ from a correct derivative. If $\frac{\mathrm{d}y}{\mathrm{d}x}$ is wrong we must see evidence of **correct** substitution.
	$y = -\frac{1}{2}x + \frac{7\pi}{4}$	A1	2	OE: e.g. $y = \frac{7\pi}{4} - 0.5x$ but must be $y = mx + c$ form with c **exact**. **Must be from correct derivative.**
			9	

(b)	Condone a spurious $\frac{\mathrm{d}y}{\mathrm{d}x}$ on LHS for the first 4 marks but penalise if included in finding $\frac{\mathrm{d}y}{\mathrm{d}x}$.
(b)	Alternative for product rule for an attempt to change $\sin^2 3x$ to $\frac{1}{2}(1-\cos 6x)$ – must involve $\cos 6x$ - $\frac{\mathrm{d}}{\mathrm{d}x}\left(y.\frac{1}{2}(1-\cos 6x)\right) = \frac{\mathrm{d}y}{\mathrm{d}x} \times$ their $\frac{1}{2}(1-\cos 6x) + ky\sin 6x$ **M1** $= \frac{\mathrm{d}y}{\mathrm{d}x}.\frac{1}{2}(1-\cos 6x) + 3y\sin 6x$ **A1** They would then earn the second **M1** for factoring out three terms involving $\frac{\mathrm{d}y}{\mathrm{d}x}$ (even if identity used is incorrect) then $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1-3y\sin 6x}{\frac{1}{2}(1-\cos 6x) - 3\sin 3y}$ OE **A1**
(b)	If not scored in (b) the final **M1 A1** marks can be earned if completed in part (c).

7	Solution	Mark	Total	Comment
(a)	$u = 7 + 2x^2$ gives $\frac{\mathrm{du}}{\mathrm{dx}} = 4x$	M1		OE – e.g. $\mathrm{du} = 4x\ \mathrm{dx}$ etc.
	$\int \frac{x}{(7+2x^2)^2}\ \mathrm{d}x = \int \frac{1}{4u^2}\ \mathrm{d}u \ = -\frac{1}{4u}$	dM1		Integral all in u of the form $\int \frac{k}{u^2}\ \mathrm{d}u$ leading to $\pm\frac{k}{u}$.
	$= -\frac{1}{4(7+2x^2)} \quad (+c)$	A1	3	OE – e.g. $-\frac{1}{4}\ (7 + 2x^2)^{-1}$
(b)	$\int e^{-4y}\mathrm{d}y = \int \frac{3x}{(7+2x^2)^2}\ \mathrm{d}x$	B1		Correct separation seen and notation including integral signs and dy & dx.
	LHS $= -\frac{1}{4}e^{-4y}$	B1		
	RHS $= -\frac{3}{4(7+2x^2)}$	B1ft		ft on 3 x (a) from a correct integrand
	$-\frac{1}{4}e^{-4y} = -\frac{3}{4(7+2x^2)} + C$			
	$x = 2$ and $y = 0$ to find C $\left(= -\frac{1}{5}\right)$	M1		Used correctly in an expression of the form $pe^{-4y} = \frac{q}{7+2x^2} + C$
	$-\frac{1}{4}e^{-4y} = -\frac{3}{4(7+2x^2)} - \frac{1}{5}$ OE	A1		
	$y = -\frac{1}{4}\ln\left(\frac{3}{7+2x^2} + \frac{4}{5}\right)$	A1	6	ACF - e.g. $-\frac{1}{4}\ln\left(\frac{43+8x^2}{5(7+2x^2)}\right)$ or $\frac{1}{4}\ln\left(\frac{5(7+2x^2)}{43+8x^2}\right)$ etc.
			9	
(a)	Let $u = 2x^2$, $\frac{\mathrm{du}}{\mathrm{dx}} = 4x$ OE **M1** $\int \frac{x}{(7+2x^2)^2}\ \mathrm{d}x = \int \frac{1}{4(7+u)^2}\ \mathrm{d}u \ = -\frac{1}{4(7+u)}$ **dM1** $= -\frac{1}{4(7+2x^2)} \quad (+c)$ **A1**			
(a)	By 'inspection' $\int \frac{x}{(7+2x^2)^2}\ \mathrm{d}x = \frac{k}{7+2x^2}$ **M1 dM1** $= -\frac{1}{4(7+2x^2)} \quad (+\,c)$ **A1**			
(b)	If the 3 is taken to the LHS, marks are $\int \frac{1}{3}e^{-4y}\mathrm{d}y = \int \frac{x}{(7+2x^2)^2}\ \mathrm{d}x$ **B1** $\rightarrow -\frac{1}{12}e^{-4y}$ **B1** $= -\frac{1}{4(7+2x^2)} \quad (+c)$ **B1ft** Using $x = 2$ and $y = 0$ to find C from and expression of the form $pe^{-4y} = \frac{q}{7+2x^2} + C$ **M1** $\left(C = -\frac{1}{15}\right)$ $-\frac{1}{12}e^{-4y} = -\frac{1}{4(7+2x^2)} - \frac{1}{15}$ **A1** $\rightarrow$ $y = -\frac{1}{4}\ln\left(\frac{3}{7+2x^2} + \frac{4}{5}\right)$ **A1** OE			
(b)	If any correct solution in the form $y = f(x)$ is seen then award the final **A** mark and apply ISW.			
(b)	Be very generous with placement of dy and dx provided they aren't placed in front of any function.			
(b)	**SC** If the candidate finds a **correct** value for c from a **correct simplified exponential** form but then makes a slip when writing out their solution award the first **A1** at the correct value of c stage.			

8	Solution	Mark	Total	Comment
(a)(i)	B is $(1,5,10)$	**B1**	**1**	Condone as a column vector
(a)(ii)	$\begin{bmatrix}1\\6\\12\end{bmatrix}$	**B1**		$\pm\begin{bmatrix}1\\6\\12\end{bmatrix}$ seen in (a)(ii)
	$\begin{bmatrix}-2\\6\\8\end{bmatrix}$	**B1**		$\pm k\begin{bmatrix}-1\\3\\4\end{bmatrix}$ seen in a(ii)
	$\begin{bmatrix}1\\6\\12\end{bmatrix}.\begin{bmatrix}-2\\6\\8\end{bmatrix} = (1)(-2)+(6)(6)+(12)(8)$	**M1**		Their $\overrightarrow{\boldsymbol{CB}}$ (or $\overrightarrow{\boldsymbol{BC}}$) correctly dotted with their $\overrightarrow{\boldsymbol{AB}}$ (or $\overrightarrow{\boldsymbol{BA}}$ or with the direction vector of l).
	$\sqrt{1^2+6^2+12^2}\sqrt{(-2)^2+6^2+8^2}\cos\theta = \pm130$ **or** $(\cos ABC) = \dfrac{\pm130}{\sqrt{1^2+6^2+12^2}\sqrt{(-2)^2+6^2+8^2}}$	**A1**		OE – scalar product evaluated For RHS only OE – scalar product evaluated
	(acute angle ABC) $= 18.6^0$	**A1**	**5**	CAO 18.6^0
(b)				
	Line through A and C has equation $(\boldsymbol{r}) = \begin{bmatrix}3\\-1\\2\end{bmatrix} + \mu\begin{bmatrix}-3\\0\\-4\end{bmatrix}$	**M1** **A1**		Attempt at line AC - condone one component error. Fully correct
	$\overrightarrow{\boldsymbol{BD}} = \begin{bmatrix}3\\-1\\2\end{bmatrix} + \mu\begin{bmatrix}-3\\0\\-4\end{bmatrix} - \begin{bmatrix}1\\5\\10\end{bmatrix} \quad \left(= \begin{bmatrix}2-3\mu\\-6\\-8-4\mu\end{bmatrix}\right)$	**A1ft**		**ft** on co-ordinates of B from (a)(i) and D (from line AC). Apply ISW if un-simplified form is simplified incorrectly.
	$\overrightarrow{\boldsymbol{BD}}.\overrightarrow{\boldsymbol{AB}} = 0$ $(2-3\mu)(-2)+(-6)(6)+(-8-4\mu)(8) = 0$ $(\mu = -4)$	**dM1**		Their $\overrightarrow{\boldsymbol{BD}}.\overrightarrow{\boldsymbol{AB}}$ evaluated in a **correct** manner and equated to 0.
	D is $(15,-1,18)$	**A1**		Accept as a column vector.
	Finding E by any appropriate method - symmetry, mid-point of AD and BE the same point, $\overrightarrow{\boldsymbol{AE}} = \overrightarrow{\boldsymbol{BD}}$ being used as component vectors.	**M1**		See notes for two possible ways.
	$E\ is\ (17,-7,10)$	**A1**	**7**	Accept as a column vector.
			13	

8 cont.	Notes
(a)(ii)	OE is such as $130 = \sqrt{1^2+6^2+12^2}\,\sqrt{(-2)^2+6^2+8^2}\cos\theta$ for **A1** (allow $\pm\,130$ on LHS) For guidance $\cos\theta = \pm\frac{130}{\sqrt{181}\sqrt{104}}$ or $\pm\,0.9475$ are two alternatives for the **A1** mark. Candidates who use $\begin{bmatrix}-1\\3\\4\end{bmatrix}$ should get the equivalent of $\pm\frac{65}{\sqrt{181}\sqrt{26}}$ or $\pm\,0.9475$ **<u>Alternative Method– cosine rule</u>** $AB^2 = 2^2+6^2+8^2(=104)$ **(B1)** $BC^2 = 1^2+6^2+12^2(=181)$ **(B1)** $AC^2 = 3^2+0^2+4^2 = 25)$ Cosine rule: $cosABC = \frac{AB^2+BC^2-AC^2}{2.AB.BC} = \frac{104+181-25}{2\sqrt{104}\sqrt{181}}$ **(M1) (A1)** $(\sim 0.9475\ldots)$ $ABC = 18.6^0$ **(A1)** (CAO)
(b)	The line through A and C and hence the co-ordinates of D could be found using vector $\begin{bmatrix}3\\0\\4\end{bmatrix}$ as the direction vector or $C(0,-1,-2)$ as the known point rather than $A(3,-1,2)$ so check answers that may differ to those in the main scheme.
(b)	Having found $\overrightarrow{\boldsymbol{BD}}$ they may use Pythagoras to find D before finding E rather than the dot product. $BD^2 + AB^2 = AD^2$ $(2-3\mu)^2+(-6)^2+(-8-4\mu)^2 + (-2)^2+6^2+8^2 = (-3\mu)^2+0^2+(-4\mu)^2$ $\mu = -4$ **M1** (linear equation in μ and solving) D is $(15,-1,18)$ **A1**
(b)	**<u>Method 1 for finding E - equal vectors</u>** $\overrightarrow{\boldsymbol{DE}} = \overrightarrow{\boldsymbol{BA}} \rightarrow \overrightarrow{\boldsymbol{OE}} - \begin{bmatrix}15\\-1\\18\end{bmatrix} = \begin{bmatrix}2\\-6\\-8\end{bmatrix}$ **M1** correctly formed $\rightarrow \overrightarrow{\boldsymbol{OE}} = \begin{bmatrix}17\\-7\\10\end{bmatrix}$ **A1** Could equally use $\overrightarrow{\boldsymbol{AE}} = \overrightarrow{\boldsymbol{BD}}$. Using symmetry of $\overrightarrow{\boldsymbol{DE}}$ with $\overrightarrow{\boldsymbol{BA}}$ it is also possible to write down the co-ordinates of E so NMS scores **B2**. **<u>Method 2 for finding E – mid-point of AD and BE</u>** If M is mid-point of AD then M is $(9,-1,10)$ **M1** (Since M is MP of BE) E is $(17,-7,10)$ **A1**