AQA

GCSE
Mathematics

43651H Paper 1
Mark scheme

4365
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Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

M	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
ft	Follow through marks. Marks awarded for correct working following a mistake in an earlier step.
SC	Special case. Marks awarded for a common misinterpretation which has some mathematical worth.
M dep	A method mark dependent on a previous method mark being awarded.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
oe	Or equivalent. Accept answers that are equivalent. e.g. accept 0.5 as well as $\frac{1}{2}$
[*a*, *b*]	Accept values between *a* and *b* inclusive.
[a, b)	Accept values a ≤ value < b
3.14 …	Accept answers which begin 3.14 e.g. 3.14, 3.142, 3.1416
Q	Marks awarded for quality of written communication
Use of brackets	It is not necessary to see the bracketed work to award the marks.

Examiners should consistently apply the following principles

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a candidate has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the candidate. In cases where there is no doubt that the answer has come from incorrect working then the candidate should be penalised.

Questions which ask candidates to show working

Instructions on marking will be given but usually marks are not awarded to candidates who show no working.

Questions which do not ask candidates to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Candidates often copy values from a question incorrectly. If the examiner thinks that the candidate has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

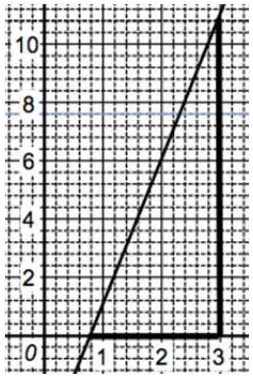
Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Paper 1 Higher Tier

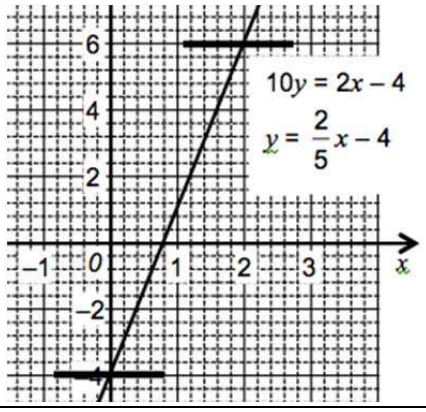
Q	Answer	Mark	Comments
1	$\pm 2w$ or ± 18 or $5w - 3w = 15 + 3$	M1	Terms in w or constant terms collected
	$2w = 18$ or $-2w = -18$ or $\frac{18}{2}$	A1	
	9	A1ft	ft on $2w = a$ where $a \neq 3$ or 15 or $bw = 18$ where $b \neq 5$ or 3
	Additional Guidance		
	$2w = 12$ 6		M1, A0 A1ft
	$8w = 18$ 2.25 or $\frac{18}{8}$ oe		M1, A0 A1ft
	$3w = 12$ 4		M0
	$3w = 18$ 6		M1, A0 A0ft
	Embedded answer of 9		M1, A1, A0
	If only decimal answer given must be accurate to at least 2 dp		

Q	Answer	Mark	Comments
2	1 – (0.2 + 0.3 + 0.15) or 0.65	M1	oe eg 65%
	0.35	A1	oe eg 35%
	Additional Guidance		
	0.2 + 0.3 + 0.15 = 0.2 0.8	Answer follows through	M1 A0
	0.2 + 0.3 + 0.15 = 0.55 1 – 0.55 = 0.25	Method even though answer wrong	M1 A0
	0.2 + 0.3 + 0.15 = 0.55 0.35	No method seen and answer does not follow through	M0 A0
	0.65 0.45		M1 A0
	Answer only of 0.65		M1 A0
	0.2 0.8	No addition seen	M0
	Embedded answer 0.2 + 0.3 + 0.15 + 0.35 = 1		M1, A0
	Embedded answer 0.2 + 0.3 + 0.15 + 0.8 = 1 Answer 0.8		M1, A0

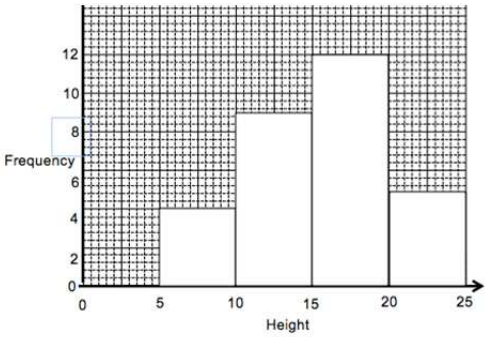
3a	[2.3, 2.5]	B1	Ignore x =

Q	Answer	Mark	Comments
3b	**Alternative method 1**		
	A triangle drawn on graph or a y and **corresponding** x length clearly shown ir stated.	M1	
	their y length ÷ their x length	M1dep	Allow lengths to be ±½ small square ie ± 0.2 vertically or ± 0.1 horizontally
	5	A1	Only award if y length ÷ x length = 5 and does not round to 5 Accept $y = 5x - 4$
	Alternative method 2		
	Substitutes a coordinate value into $y = mx + c$, eg (2, 6)	M1	
	Shows a correct equation, eg $6 = 2m - 4$	M1dep	
	5	A1	Accept $y = 5x - 4$
	Additional Guidance		
	 10.9 ÷ 2.1 = 5		M1 M1dep A0
	$8 = m \times 2.4 - 4$ $4 = 2.4m$ $\frac{8}{3}$		M1 M1dep A0

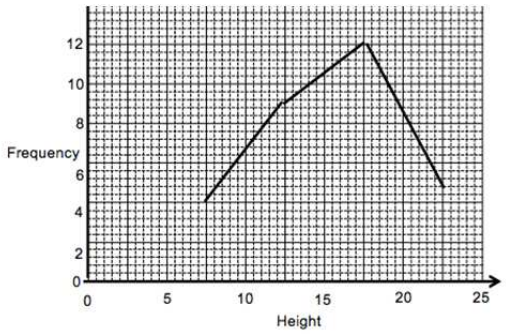
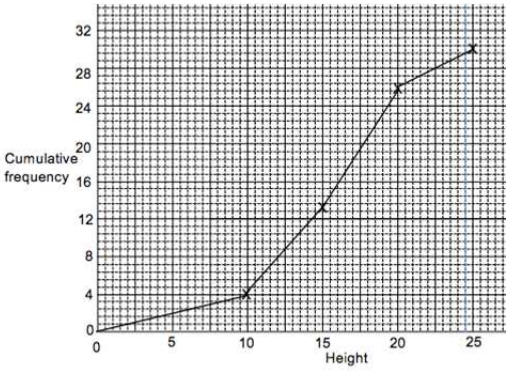
Additional Guidance continues on next page

Q	Answer	Mark	Comments
			M1 M0dep A0

4	$6x - 18 - 4x + 20$	M1	Three correct terms
	$6x - 18 - 4x + 20$	A1	All terms correct
	$2x + 2$ or $2(x + 1)$	A1ft	ft on M and no further errors Do not award if incorrect further work, eg $2x + 2 = 4x$
	Additional Guidance		
	$6x - 18 - 4x - 20$ $2x - 38$		M1, A0 A1ft
	$5x - 18 - 4x + 20$ $x + 2$		M1, A0 A1ft
	$5x - 18 - 4x - 20$ $x - 38$		M0, A0 A0ft

Q	Answer	Mark	Comments
5	Frequency polygon or histogram with equal intervals or cumulative frequency diagram.	B1	
	Vertical axis with equal scales labelled frequency	B1	
	Horizontal axis clearly numbered and labelled height (units not necessary), may start at 5 with zig-zag on axis.	B1	
	Additional Guidance		
	The diagram **must** be an attempt at one of the following. Marks cannot be scored for labels only. If diagram not valid then B0 total.		
		B3	Accept f for frequency Accept height or h for horizontal axis cm need not be stated
			Omissions that lose a mark.
			Histogram Heights out by more than ½ square Missing bars Gaps between bars
			Vertical axis No label Wrong scales
			Horizontal axis No label Wrong scales, eg $5 < h \leq 10$ Starting height at 5 with no 'zig-zag' or other indication

Additional Guidance continues on nest page

Q	Answer	Mark	Comments
		B3	Accept f for frequency Accept height or h for horizontal axis cm need not be stated. Ignore any lines before and after first and last points. Omissions that lose a mark **Frequency diagram** Points out by more than ½ square Missing or wrong points Plotting at ucb or lcb No lines joining points **Vertical axis** No label Wrong scales **Horizontal axis** No label Wrong scales, eg $5 < h \leq 10$ Starting height at 5 with no 'zig-zag' or other indication
		B3	Accept height or h for horizontal axis cm need not be stated. Ignore any lines before first point (10, 4) Omissions that lose a mark **Cumulative frequency diagram** No lines or smooth curve joining points Points out by more than ½ square Plotting at midpoint or lcb Missing or wrong points **Vertical axis** No label. Must mark c f minimum Wrong scales **Horizontal axis** No label Wrong scales, eg $5 < h \leq 10$ Starting height at 5 with no 'zig-zag' or other indication

Q	Answer	Mark	Comments
6	Side of square = 5 Or 5 × 5 = 25 oe	B1	May be on diagram
	400 ÷ 25	M1	
	16	A1	May be on diagram 16 × 25 = 400 oe is M1, A1
	Yes and 5 and their 16	Q1ft	Strand (iii) Conclusion must be based on length not volume ft their 16 if B1, M1 awarded and correct conclusion
	Additional Guidance		
	Ignore any volume calculations		
	Square = 5 cm 25 × 21 = 400 No		B1 M1 A0 Q1ft

Q	Answer	Mark	Comments

7	**Alternative method 1**		
	BCD = 105	B1	
	DCE = 180 – their 105 or 75	M1	Calculation **must** be shown or correct angle marked on diagram
	CDE = 180 – (their 75 + 30) or 75	M1dep	Calculation **must** be shown or correct angle marked on diagram
	DCE = 75 and *CDE* = 75 and 'two angles equal'	Q1	Strand (ii) Must score B1M2 and have no incorrect angles or calculations seen
	Additional Guidance		
	C = 105 *C* = 180 – 105 = 65 *D* = 180 – (65 + 30) = 85		B1 M1 M1dep Q0
	BCD = 75 *DCE* = 180 – 75 = 105 *CDE* = 180 – (105 + 30) = 45		B0 M1 M1dep Q0
	BCD = 105 *DCE* = 65 *CDE* = 85 (no method shown)		B1 M0 M0dep Q0

Alternative methods continued on the next page

Q	Answer	Mark	Comments
7 cont	**Alternative method 2**		
	ABC = 180 – 105 or 75 or *ADC* = 180 – 105 = 75	M1	Calculation **must** be shown or correct angle marked on diagram
	DCE = their 75	M1dep	their 75 must be the same as their *ABC* or their *ADC*
	CDE = 180 – (their 75 + 30) or 75	M1dep	Calculation **must** be shown or correct angle marked on diagram
	DCE = 75 and *CDE* = 75 and 'two angles equal'	Q1	Strand (ii) Must score M3 and have no incorrect angles or calculations seen
	Additional Guidance		
	B = 180 – 105 = 75 *C* = 105 *D* = 180 – (105 + 30) = 45		M1 M0dep M0dep Q0
	ABC (or *ADC*) = 180 – 105 = 65 *DCE* = 65 *CDE* = 85 (no method shown)		M1 M1dep M0dep Q0
	ABC (or *ADC*) = 180 – 105 = 75 *DCE* = 75 *CDE* = 180 – (75 + 30) = 65		M1 M1dep M1dep Q0

Alternative methods continued on the next page

Q	Answer	Mark	Comments
7 cont	**Alternative method 3**		
	BCD = 105	B1	
	CDE = their 105 – 30 or 75	M1	Calculation **must** be shown or correct angle marked on diagram
	DCE = 180 – (their 75 + 30) or 75	M1dep	Calculation **must** be shown or correct angle marked on diagram
	DCE = 75 and *CDE* = 75 and 'two angles equal'	Q1	Strand (ii) Must score B1M2 and have no incorrect angles or calculations seen
	Additional Guidance		
	C = 105 *D* = 105 – 30 = 65 *C* = 180 – (65 + 30) = 85		B1 M1 M1dep Q0
	BCD = 75 *CDE* = 75 – 30 = 45 *DCE* = 180 – (45 + 30) = 105		B0 M1 M1dep Q0
	BCD = 105 *CDE* = 65 *DCE* = 85 (no method shown)		B1 M0 M0dep Q0

Alternative methods continued on the next page

Q	Answer	Mark	Comments
7 cont	**Alternative method 4**		
	DCE or *CDE* = (180 – 30) ÷ 2 or 75	M1	Calculation **must** be shown or one correct angle marked on diagram
	CDE **and** *DCE* = their 75	M1dep	
	DCB = 180 – their 75 or 105	M1dep	Calculation **must** be shown or correct angle marked on diagram
	DCE = 75 and *CDE* = 75 and *DCB* = 105 and 'opposite angles of parallelogram equal'	Q1	Strand (ii) Must score M3 and have no incorrect angles or calculations seen
	Additional Guidance		
	(180 – 30) ÷ 2 = 65 *C* = 65 and *D* = 65 *C* = 115 (no method shown)		M1 M1dep M0dep Q0
	(180 – 30) ÷ 2 = 75 *DCE* = 75 and *CDE* = 75 *DCB* = 180 – 75 = 105		M1 M1dep M1dep Q0

Q	Answer	Mark	Comments
8	2 × (30 + 70) or 200	M1	
	their 200 ÷ 4 or 50	M1 dep	100 ÷ 2 is M2
	their 50 × their 50 or 2500 or 30 × 70 or 2100	M1	their 50 must follow M1, M1dep
	400	A1	
	Additional Guidance		
	Perimeter = 100 Side of square = 25 2100 – 625 1475		M0 M0dep M1 A0
	Side of square = $\sqrt{100}$ = 10 2100 – 100 2000		M0 M0dep M1 A0
	30 × 70 = 2400 50 × 50 = 2500 2500 – 2400 = 100		M1 M1dep M1 A0
	Side of square = 25 30 × 70 = 2400 625		M0 M0dep M1 A0
	30 × 70 = 2100 2100 × 2 = 4200		3rd M0

9	$6n + 3$ or $3(2n + 1)$	B2	oe B1 for $6n$ Accept $6 \times n$ or $n \times 6$ but not $n6$ B1 for $n6 + 3$ Accept any letter

10	$(x – 10)(x + 10)$ or $(x + 10)(x – 10)$	B1	

Q	Answer	Mark	Comments
11a	1.6×10^{11}	B2	B1 for 2 000 000 × 80 000 B1 for 160 000 000 000 B1 for 16 × 10^{10}
11b	25	B2	B1 2 000 000 ÷ 80 000 or 200 ÷ 8 B1 2.5 × 10 B1 0.25 × 10^2 B1 for $\frac{2}{8} \times 10^2$
12	**Alternative method 1**		
	$2(2x + 3) + (4x - 1)$	M1	oe one sign or arithmetic error if expanded straight away
	$8x + 5$	A1	
	their $(8x + 5) = 20$	M1	oe Not dependent but their $8x + 5$ must be linear
	$\frac{15}{8}$	A1ft	oe ft on 1st and 2nd M and one error. Decimals must be to at least 2dp, eg 1.875 or 1.88, 1.9 is A0
	Alternative method 2		
	$2x + 3 + 2x - \frac{1}{2}$	M1	3 terms correct
	$4x + 2\frac{1}{2}$	A1	
	their $(4x + 2\frac{1}{2}) = 10$	M1	oe Not dependent but their $4x + 2\frac{1}{2}$ must be linear
	$\frac{15}{8}$	A1ft	oe ft on 1st and 2nd M and one error.

Alternative method continues on next page

Q	Answer	Mark	Comments

12 cont	**Alternative method 3**			
	$\frac{4x-1}{2} = 10 - 3 - 2x$	M1		
	$4x - 1 = 14 - 4x$	M1		
	$8x = 15$	A1		
	$\frac{15}{8}$	A1ft	oe ft on 1st and 2nd M and one error. SC1 Answer from T&I	
	Additional guidance			
	$2(2x + 3) + 4x - 1$ $4x + 3 + 4x - 1$ $8x + 2 = 20$ $2\frac{1}{4}$		1 error	M1 A0 M1 A1ft
	$2(2x + 3) + 4x - 1$ $4x + 3 + 4x - 1$ $8x + 2 = 20$ 2.75		2 errors	M1 A0 M1 A0ft
	$2(2x + 3) + 4x - 1$ $8x + 5$ $8x + 5 = 10$ $\frac{5}{8}$			M1 A1 M0 A0
	$2(2x + 3) + 4x - 1$ $2x + 6 + 4x - 1$ $6x + 5 = 10$ $\frac{5}{6}$			M1 A0 M0 A0
	$2x + 3 + 2x - \frac{1}{2}$ $4x + 3\frac{1}{2} = 10$ $\frac{13}{8}$		1 error	M1 A0 M1 A1ft

Additional Guidance continues on next page

Q	Answer	Mark	Comments
12 cont	$2x + 3 + 4x - 1 = 20$ $6x + 2 = 20$ $6x = 18$ 3		M0 A0 M1 A0ft
	$2x + 3 + 2x - 2 = 10$ $4x + 1 = 10$ $4x = 9$ $2\frac{1}{4}$		M1 A0 M1 A1ft

	Alternative method 1		
13	Proportion/percentage fish in lake = $\frac{3}{60}$ or 5%	M1	Could be shown as ratio
	400 = 5% or $\frac{5}{100} \times 8000$ or their 5% of 8000	M1dep	oe
	100% = 400 × 20 (= 8000) or 400	A1	Calculation of 400 must be clearly shown
	Alternative method 2		
	Proportion/percentage fish in lake = $\frac{400}{8000}$ or 5%	M1	Could be shown as ratio
	$\frac{5}{100} \times 60$ or their 5% of 60	M1dep	$\frac{3}{60}$ oe
	3	A1	Both 5%

Alternative method continues on next page

Q	Answer	Mark	Comments
13 cont	**Alternative method 3**		
	400 ÷ 3 or 8000 ÷ 60	M1	
	8000 ÷ 60 and 400 ÷ 3	M1dep	
	Both 133.333….	A1	Must show equivalence clearly
	Alternative method 4		
	8000 ÷ 400 or 60 ÷ 3	M1	
	60 ÷ 3 and 8000 ÷ 40	M1dep	
	Both equal 20	A1	Must show 8000 ÷ 400 = 20 clearly
	Additional Guidance		
	400 ÷ 3 = 133.3333 8000 ÷ 60 = 800 ÷ 6 = 400 ÷ 3 = 133.333		M1 M1dep A1
	400 ÷ 3 = 133.3333 8000 ÷ 60 = 133.333		M1 M1dep A0
	3 ÷ 60 = 1 ÷ 20 = 0.2 0.2 × 8000 = 400		M1 M1dep A0
	3 ÷ 60 = 0.05 0.05 × 8000 = 400 She is correct		M1 M1dep A0
	3 ÷ 60 = 0.05 0.05 × 8000 = 0.5 × 800 = 5 × 80 = 400 She is correct		M1 M1dep A1
	60 ÷ 3 = 20 8000 ÷ 400 = 80 ÷ 4 = 20		M1 M1dep A1
	60 ÷ 3 = 20 8000 ÷ 400 = 20		M1 M1dep A0
	3 : 60 = 1 : 20 = 2 : 40 = 4 : 80 = 400 : 8000		M1 M1dep A1
	$\frac{400 \times 60}{3} = \frac{^{8}\cancel{24}000}{\cancel{3}}$ = 8000		M1 M1dep A1

Q	Answer	Mark	Comments
14	(Number of girls =) $\frac{360}{36} \times 5$ or 50	M1	oe Check diagram for working
	Blue eyed girls = 3×5 or $\frac{108}{360} \times$ their 50 or 15	M1	
	(Number of boys =) 2×4^2 ($\times\pi$) or 32 or $\left(\frac{4}{5}\right)^2 \times 50$ $(\times \pi)$	M1	oe
	Blue eyed boys = their $32 \div 4$ or 8	M1dep	Dependent on 3rd M
	23	A1	**Must** see 32 and 50

15	23, 22, 25 or 24, 21, 25	B3	B2 for 24, 22, 25 B2 for 23, 21, 25 B2 for 23.5, 21.5, 25 B2 for 23, 22, 26 B2 for 24, 21, 26 B2 for 23, 21, 26 B1 for 24, 22, 26 B1 for division by 10 seen or implied.
	Additional Guidance		
	NB dividing by year will show $215 \div 10$		B0

16	16	B2	B1 for $64^{\frac{1}{3}} = 4$ B1 for $\sqrt[3]{64 \times 64}$ B1 for $\left(64^{\frac{1}{3}}\right)^2$ oe B1 for $\left(64^2\right)^{\frac{1}{3}}$ oe

Q	Answer	Mark	Comments
17	**Alternative method 1**		
	$\sqrt{3}^2 + 2\times\sqrt{3}\times\sqrt{75} + \sqrt{75}^2$	M1	Allow one error
	$3 + 2 \times 15 + 75$	A1	
	Alternative method 2		
	$6\sqrt{3}$ or $\sqrt{3}^2 \times \left(1+\sqrt{25}\right)^2$ or $\sqrt{3}+5\sqrt{3}$	M1	
	3×6^2	A1	
	Alternative method 3		
	$\sqrt{108} = 6\sqrt{3}$	M1	
	$\sqrt{3} + 5\sqrt{3} = \sqrt{3} + \sqrt{75}$	A1	
	Additional Guidance		
	$\left(\sqrt{3} + \sqrt{75}\right)\left(\sqrt{3} + \sqrt{75}\right) = 9 + \sqrt{225} + \sqrt{225} + 75 = 108$		M1, A0
18	$2(x – 1) + x – 3$ or $(x – 3)(x – 1)$	M1	
	$2(x – 1) + x – 3 = (x – 3)(x – 1)$	M1dep	
	$2x – 2 + x – 3 = x^2 – x – 3x + 3$ and $x^2 – 7x + 8 = 0$	A1	oe Do not award if any incorrect anlegra seen in collecting terms
	Additional Guidance		
	Ignore any attempts to solve		
	$2(x – 1) + x – 3 = (x – 3)(x – 1)$ $3x – 5 = x^2 – 4x + 3$ $x^2 – 7x + 8 = 0$		M1, M1dep A1

Q	Answer	Mark	Comments
19	$(2x + 5)(3x + 1) = 8$	M1	
	$6x^2 + 17x – 3 = 0$	A1	
	$(6x – 1)(x + 3)$ or $(ax + c)(bx + d)$ where $ab = 6$ and $cd = 3$ or $\frac{-17 \pm \sqrt{361}}{12}$ or $\frac{17 \pm \sqrt{361}}{12}$ or $\frac{-17 \pm \sqrt{217}}{12}$	M1	An attempt to solve their quadratic if not $6x^2 + 17x + 5 = 0$. Must take as far as a correct factorisation or correct substitution into formula.
	$\frac{1}{6}$	A1ft	If negative value (–3 if correct) given do not award A1 ft their solution if only positive value given and evaluated to 2dp at least
	Additional Guidance		
	$(2x + 5)(3x + 1) = 8$ $6x^2 + 8x + 2x + 5 = 8$ $6x^2 + 10x – 3 = 0$ $\frac{-10 \pm \sqrt{172}}{12}$	M1 A0 M1 A0	
	$(2x + 5)(3x + 1) = 8$ $6x^2 + 5x + 2x + 5 = 8$ $6x^2 + 7x – 3 = 0$ $(2x + 3)(3x – 1) = 0$ $\frac{1}{3}$	M1 A0 M1 A1ft	
20a	130	B1	
20b	95	B1	

Q	Answer	Mark	Comments
20c	*BAD* = 50 or *BOD* (reflex) = 260 or *ADC* = 75	B1	
	BOD = 100	B1	
	OBC = 65 or *ODC* = 65	B1	
	10	B1	
	Additional Guidance		
21	$\frac{150}{360} \times 2 \times \pi \times 6$ or 5π or [15.5, 15.71]	M1	oe
	2 × their 5π or $\frac{300}{360} \times 2 \times \pi \times 6$	M1dep	oe NB $\frac{300}{360} \times 2 \times \pi \times 6$ is M2
	10π or [31, 31.42]	A1	
	their 10π + 18 or [49, 49.42]	A1ft	SC1 18 or 6 + 6 + 3 + 3 seen