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# A-LEVEL Mathematics

MM2B Mechanics 2B  
Mark scheme

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6360

June 2018

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Version/Stage: 1.0 Final

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

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### Annotations key

Annotation	Use
?	Unclear
^	Omission
BOD	Benefit of doubt
Cross	Incorrect
FT	Follow through
H wavy	Highlight relevant working
ISW	Ignore subsequent working
On page comment	Clarifies issue
SEEN	Only used on blank pages
Tick	Correct

Q	Solution	Mark	Total	Comment
<b>1 (a)(i)</b>	Total weight is 20kg. Taking moments about y axis $7 \times 60 + 6 \times 20 + 3 \times 60 + 4 \times 80 = 20x$ $x = \frac{1040}{20}$ $= 52$ Distance from AD is 52 cm	<b>B1</b>  <b>M1</b>  <b>A1</b>	  <b>3</b>	B1: Total weight CAO  M1: At least three correct multiplications on LHS.  A1: Correct distance CAO.
<b>(ii)</b>	Taking moments about x axis $7 \times 40 + 6 \times 60 + 3 \times 30 + 4 \times 20 = 20y$ $y = \frac{810}{20}$ $= 40.5$ Distance from AB is 40.5 cm	<b>M1</b>  <b>A1</b>	  <b>2</b>	M1: At least three correct multiplications on LHS.  A1: Correct distance CAO.
<b>(b)</b>	If lamina hangs in equilibrium , C of G must be vertically below X Hence distance of C of G from y axis is 60 cm. Moments about AD; $1040 + m \times 120 = (20 + m) 60$ $60m = 160$ Mass is $\frac{8}{3}$ kg	<b>M1</b>  <b>M1A1</b>  <b>A1</b>	    <b>4</b>	M1: Uses 60 or other appropriate distance in their calculations. No need to see this explicitly stated.  M1: At least two terms correct in a 3/4 term moment equation correct. A1: Correct moment equation. A1: Correct mass. CAO Accept 2.67.
	<b>Total</b>		<b>9</b>	

Q	Solution	Mark	Total	Comment
2 (a)	Initial KE is $\frac{1}{2} \times 21 \times 2^2$ = 42 J	M1 A1	2	M1: Correct expression for KE. A1: Correct value for KE
(b)(i)	Energy gained by moving to point B is mgh = $21 \times 8 \times g$ = 1646.4 Total KE at B is $1646.4 + 42$ = 1688.4 = 1690 J	M1 A1 A1F	3	M1: For height change 8. A1: Correct change in PE. A1F: Correct sum of their energies.
(ii)	KE at point B is 1688.4  $\frac{1}{2} \times m \times v^2 = 1688.4$  $v^2 = \frac{1688.4}{10.5}$ = 12.68... Speed is $12.7 \text{ ms}^{-1}$	M1 A1F	2	M1: Seeing $\frac{1}{2}mv^2$ equated to their answer to (b)(i). A1: Correct speed for their answer to (b)(i).
(c)	Work done = $F \times s = \text{change in KE}$ $21g\mu s = 1688.4$ $\mu = \frac{1688.4}{21 \times g \times 18}$ = 0.45578.. = 0.456  OR  $a = -4.4666$  $-21g\mu = 21 \times (-4.4666)$  $\mu = \frac{4.4666}{g} = 0.456$	M1 A1F A1 (M1) (A1) (A1)	3	M1: For $Fs = \text{Change in KE}$ using their answer to (b)(i). A1: Correct equation (using bi). A1: Correct coefficient of friction. CAO  M1: Correct acceleration (Accept -4.48 from 12.7) A1: Correct equation. A1: Correct coefficient of friction. CAO
	<b>Total</b>		<b>10</b>	

Q 3	Solution	Mark	Total	Comment
(a)	$\mathbf{a} = (12 - 3t^2)\mathbf{i} + 12e^{-2t}\mathbf{j}$	M1A1	2	M1: Either term correct. A1: All correct
(b)(i)	$\mathbf{F} = m\mathbf{a}$ $= (24 - 6t^2)\mathbf{i} + 24e^{-2t}\mathbf{j}$	M1 A1	2	M1: Use of $F = ma$ A1: Correct expression for force.
(ii)	When $t = 0$ , $\mathbf{F} = 24\mathbf{i} + 24\mathbf{j}$ Magnitude is $\sqrt{24^2 + 24^2}$ $= 24\sqrt{2}$ or 33.9	M1 A1	2	M1: Finding magnitude and substituting $t = 0$ . A1: Correct magnitude. Do not only 34.
(c)	When $\mathbf{F}$ acts north, $\mathbf{i}$ component is zero $24 - 6t^2 = 0$ $t = 2$	M1 A1	2	M1: Setting $\mathbf{i}$ component equal to zero. A1: Correct time.
(d)	$\mathbf{r} = \int \mathbf{v} dt$ $= (6t^2 - \frac{1}{4}t^4)\mathbf{i} + 3e^{-2t}\mathbf{j} + \mathbf{c}$  When $t=0$ , $\mathbf{r} = 4\mathbf{i} - 2\mathbf{j}$ , $\therefore \mathbf{c} = 4\mathbf{i} - 5\mathbf{j}$  $\mathbf{r} = (6t^2 - \frac{1}{4}t^4 + 4)\mathbf{i} + (3e^{-2t} - 5)\mathbf{j}$	M1A1  m1A1  A1	5	M1: Either component correct. A1: Both components correct. Condone missing $\mathbf{c}$ .  m1: Must use $t=0$ . Either component of $\mathbf{c}$ correct. A1: Correct $\mathbf{c}$ . A1: Correct position vector.
	<b>Total</b>		<b>13</b>	

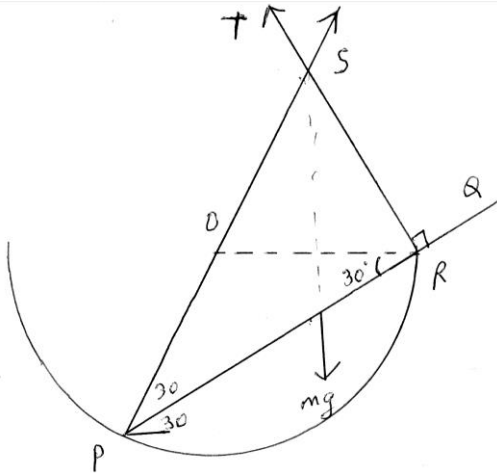
Q	Solution	Mark	Total	Comment
4 (a)	Using $F = \frac{mv^2}{r}$ $F = \frac{900 \times 12^2}{80}$ $= 1620 \text{ N}$	M1  A1	  2	M1: Using $F = \frac{mv^2}{r}$  A1: Correct force.
(b)	Using $F = \mu R$ $1620 = \mu \times 900g$ $\mu = 0.18367..$ $\mu = 0.184$	M1  A1	  2	M1: Use of $F = \mu R$ Condone use of inequality.  A1: Correct coefficient of friction.
	<b>Total</b>		<b>4</b>	

Q	Solution	Mark	Total	Comment
5(a)	$\text{Power} = F \times v$ $32000 = (k \times 40) \times 40$ $= 1600k$ $k = 20$	<p><b>M1</b></p> <p><b>A1</b></p>	<b>2</b>	<p>M1: Use of <math>P = Fv</math></p> <p>A1: Using an equation that leads to <math>k = 20</math>. AG</p>
(b)(i)	<p>Accelerating forces</p> $= 600g \times \frac{1}{10} - 20v$ $\rightarrow 600 \frac{dv}{dt} = 60g - 20v$ $\frac{dv}{dt} = \frac{g}{10} - \frac{v}{30}$ $\frac{dv}{dt} = \frac{3g-v}{30}$	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p>	<b>3</b>	<p>M1: Resolving to find component of weight in equation. Must see <math>\sin\theta</math> or <math>\times \frac{1}{10}</math>.</p> <p>A1: Use of <math>F = ma</math></p> <p>A1: Correct answer from correct working. AG</p>
(ii)	$\int \frac{dv}{3g-v} = \frac{1}{30} \int dt$ $-\ln(3g-v) = \frac{1}{30}t + c$ <p>When <math>t = 0, v = 18 \Rightarrow c = -\ln(3g-18)</math></p> $\frac{1}{30}t = \ln \frac{3g-18}{3g-v}$ $t = 30 \ln \frac{3g-18}{3g-v}$	<p><b>M1</b></p> <p><b>A1A1</b></p> <p><b>B1</b></p> <p><b>A1</b></p>	<b>5</b>	<p>M1: Separation of variables. A1: Correct LHS. A1: Correct RHS. Condone missing constant. B1: Correct constant. (-2.4336... or AWRT -2.4 or <math>-\ln(11.4)</math> or <math>e^c = \frac{5}{57}</math>) or <math>e^{-c} = \frac{57}{5}</math> A1: Correct final answer. Accept <math>t = 30 \ln \left( \frac{11.4}{3g-v} \right)</math> or <math>t = -30 \ln \left( \frac{3g-v}{11.4} \right)</math> or <math>t = 30 \ln \left( \frac{57}{5(3g-v)} \right)</math> oe Do not accept <math>t = 73.008 - 30 \ln(3g-v)</math></p>
(iii)	<p>When <math>v = 22 \Rightarrow t = 30 \ln \frac{3g-18}{3g-22}</math></p> <p>When <math>v = 18 \quad t=0</math></p> <p>Time taken is <math>= 30 \ln \frac{3g-18}{3g-22}</math></p> <p><math>= 12.964.. \text{ sec}</math></p> <p><math>= 13.0 \text{ sec}</math></p>	<p><b>M1</b></p> <p><b>B1</b></p> <p><b>A1</b></p>	<b>3</b>	<p>M1: Substituting <math>v = 22</math> (must be a log or exp term)</p> <p>B1: Using <math>v = 18</math> when <math>t=0</math>. PI</p> <p>A1: Correct time. Condone 13</p>
<b>Total</b>			<b>13</b>	

Q	Solution	Mark	Total	Comment
<b>6 (a)</b>	$T_A = \frac{mu^2}{a} - mg$ $T_B = \frac{mv^2}{a} + mg$ $\frac{mu^2 - mag}{mv^2 + mag} = \frac{T_A}{T_B} = \frac{5}{7}$ $5v^2 + 5ag = 7u^2 - 7ag \quad [1]$ C of E ; $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + 2mag$ $v^2 = u^2 + 4ag$ Hence from [1] $5u^2 + 25ag = 7u^2 - 7ag$ $32ag = 2u^2$ $u = 4\sqrt{ag}$	<b>M1</b>  <b>A1</b>    <b>M1A1</b>    <b>M1</b>  <b>A1</b>	          <b>6</b>	M1: Resolving correctly at A or B.  A1: Correct $T_A$ and $T_B$    M1: Use of conservation of energy. Accept any height. A1: Correct equation.  M1: Any correct equation linking $ag$ and $u^2$ A1: Correct $u$ . Accept $\sqrt{16ag}$
<b>(b)</b>	$v^2 = 20ag$ $v = 2\sqrt{5ag}$ Ratio $u : v = 2 : \sqrt{5}$ or $2\sqrt{5} : 5$	<b>M1</b>  <b>A1</b>	    <b>2</b>	M1: Correct $v$ . A1: Any correct ratio in any correct form. Do not accept decimals.
	<b>Total</b>		<b>8</b>	



Q	Solution	Mark	Total	Comment
7(a)	EPE in AC is zero. EPE in BC is $\frac{240 \times 2^2}{6}$ = 160J Total EPE is 160 J	B1   B1	2	B1: Zero EPE implied for one string. Must be stated. B1: Shows how to obtain 160. AG
(b)	$\frac{1}{2} \times 8v^2 + \text{EPE [of AC]} + \text{EPE [of BC]}$ + work done by friction = 160 Work done by friction = $8g\mu x$ EPE of AC = $\frac{160 \times x^2}{4}$ EPE of BC = $\frac{240 \times (2-x)^2}{6}$ $4v^2 + \frac{160 \times x^2}{4} + \frac{240 \times (2-x)^2}{6} + 8g\mu x = 160$ $4v^2 + 40x^2 + 40(4 - 4x + x^2) + 8g\mu x = 160$ $v^2 = 40x - 20x^2 - 2g\mu x$ $v = (40x - 20x^2 - 2g\mu x)^{0.5}$	M1  B1  B1 A1  A1	5	M1: Energy equation with correct KE and including 160 and at least one EPE. B1: Correct friction term. B1: Both EPEs correct. A1: All terms correct with correct signs. A1: Correct expression for v.
(c)	Differentiation of any quadratic [wrt $t$ or wrt $x$ ] $2v \frac{dv}{dt} = \frac{dx}{dt} (40 - 40x - 2g\mu)$ At max speed $40 - 40x - 2g\mu = 0$ $x = 1 - \frac{g\mu}{20}$ OR Max speed of v is when $x = -\frac{b}{2a}$ $x = \frac{40 - 2g\mu}{2 \times 40} = 1 - \frac{g\mu}{20}$ OR (using forces) $\frac{160x}{2} + 8g\mu = \frac{240(2-x)}{3}$ $x = 1 - \frac{g\mu}{20}$	M1  A1  (M1) (A1)  (M1) (A1)	9	M1: Derivative equated to zero. A1: Correct expression. M1: Uses equal roots of a quadratic. A1: Correct expression. M1: Correct equation for zero resultant force. A1: Correct expression.
	<b>Total</b>	<b>9</b>	<b>9</b>	

Q	Solution	Mark	Total	Comment
8	 <p data-bbox="336 792 647 831"><math>PR = 2r\cos 30^\circ = \sqrt{3}r</math></p> <p data-bbox="236 909 564 943">Resolving along the rod</p> <p data-bbox="236 965 507 999"><math>S \cos 30 = mg \sin 30</math></p> <p data-bbox="236 1021 328 1077"><math>S = \frac{mg}{\sqrt{3}}</math></p> <p data-bbox="236 1167 469 1200">Moments about R</p> <p data-bbox="236 1223 453 1256"><math>S \cdot 2r\cos 30 \cdot \sin 30</math></p> <p data-bbox="236 1279 587 1335"><math>= mg(2r\cos 30 - \frac{1}{2}l) \cos 30</math></p> <p data-bbox="236 1357 592 1391"><math>4rS \sin 30 = mg(4r\cos 30 - l)</math></p> <p data-bbox="236 1491 639 1547"><math>4r \cdot \frac{mg}{\sqrt{3}} \cdot \sin 30 = mg(4r\cos 30 - l)</math></p> <p data-bbox="236 1570 421 1626"><math>\frac{2r}{\sqrt{3}} = 2\sqrt{3}r - l</math></p> <p data-bbox="236 1648 320 1704"><math>\frac{4r}{\sqrt{3}} = l</math></p> <p data-bbox="236 1727 336 1794"><math>r = \frac{\sqrt{3}l}{4}</math></p>	<p data-bbox="794 483 831 517"><b>B1</b></p> <p data-bbox="794 595 831 629"><b>B1</b></p> <p data-bbox="794 786 831 819"><b>B1</b></p> <p data-bbox="794 954 831 987"><b>M1</b></p> <p data-bbox="794 1010 831 1043"><b>A1</b></p> <p data-bbox="794 1189 831 1223"><b>M1</b></p> <p data-bbox="794 1301 831 1335"><b>A1</b></p> <p data-bbox="794 1615 831 1648"><b>A1</b></p> <p data-bbox="794 1783 831 1816"><b>A1</b></p>	<p data-bbox="919 1794 940 1827"><b>9</b></p>	<p data-bbox="995 495 1485 573">B1 for S clearly through the centre of hemisphere</p> <p data-bbox="995 595 1501 674">B1 for force at R clearly perpendicular to rod</p> <p data-bbox="995 786 1302 819">B1: Correct length of PR.</p> <p data-bbox="995 954 1347 987">M1: Resolving to find S or T.</p> <p data-bbox="995 1010 1398 1043">A1: Correct expression for S or T.</p> <p data-bbox="995 1189 1426 1267">M1: Taking moments about P or R. in terms of l, r or PR</p> <p data-bbox="995 1279 1347 1312">Note moments about P gives:</p> <p data-bbox="1034 1335 1469 1413"><math>T \times 2r\cos 30^\circ = mg \times \frac{l}{2} \cos 30^\circ</math></p> <p data-bbox="995 1435 1390 1469">A1: Correct moment equation.</p> <p data-bbox="995 1615 1461 1693">A1: Correct equation containing only r and l.</p> <p data-bbox="995 1760 1342 1794">A1: Correct expression for r.</p>
	<b>Total</b>		<b>9</b>	