OCR
Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4751**: Introduction to Advanced Mathematics (C1)

Advanced Subsidiary GCE

Mark Scheme for June 2018

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			$250a^{10}c^{-2}$ or $\frac{250a^{10}}{c^2}$	2 [2]	**B1** for 2 elements correct or **M1** for $\left(5a^2c\right)^3 = 125a^6c^3$	if p12, attached to this image, is blank, put BP on it. If it has an attempt at one or more parts, highlight the qn no on p12, then go to the qn and put a highlight by the side as a reminder that there is extra work and link p12 to this part
2			$y = -4x + 5$ www	3	**M1** for $m = \frac{9 - -3}{-1 - 2}\left[= -4\right]$ soi and **M1** for $y - 9 =$ *their m* $(x + 1)$ or $y + 3 =$ *their m* $(x - 2)$ or for correct ft elimination of fractions in eqn. of line joining two points	may be implicit in eqn eg $\frac{y-9}{-3-9} = \frac{x--1}{-1-2}$
			(0, 5) and (1.25, 0) oe, ft their eqn of line	2 [5]	**B1** for one correct; need not be in coordinate form; isw after acceptable ft ans found	as usual, no need to simplify fractions, but integers may not be left as fractions
3	(i)		$\frac{16}{81}$	2 [2]	**M1** for $\frac{81}{16}$ or for $\left(\frac{4}{9}\right)^2$ or for numerator or denominator correct	M0 for just converting mixed number to fraction or for $\left(\frac{1}{2.25}\right)^2$
3	(ii)		400	2 [2]	**M1** for $\sqrt[3]{8000} = 20$ soi or for $\sqrt[3]{64000000}$	

Question			Answer	Marks	Guidance	
4			$xy + 2x = 6xy + 3y$	M1	for correctly eliminating fractions and expanding brackets	
			$2x - 5xy = 3y$	M1	for correctly collecting x terms on one side and remaining terms on the other and simplifying	ft wrong first step
			$x(2 - 5y) = 3y$	M1	correctly factorising, ft their two or three term side with x terms	
			$[x =] \frac{3y}{2-5y}$ oe as final answer	M1	for correctly dividing by their factor – but **M0** if their factor is only one term since too simple in comparison	award full marks only if working fully correct
					SC for those who work to obtain $y =$: first correct step still gains **M1**; **M0** for collecting y terms on one side and remaining term on the other	ie if ft correct, those who work to obtain $y = \frac{2x}{5x+3}$ earn SC3
					then allow **M1** for correctly factorising, ft their two or three term side with y terms, then **M1** for correctly dividing by their factor to give final answer – must be simplified	
				[4]		

Question			Answer	Marks	Guidance	
5			$3x + 2(4x + 3) = 9$	M1	for subst to eliminate one variable; condone one error; or for multn or divn of one or both eqns to get a pair of coeffts the same, condoning one error	each M1 is for a correct, constructive step
			$11x = 3$	M1	for collecting terms and simplifying; condoning one error ft or for appropriate addn or subtn to eliminate a variable, condoning an error in one term;	for this M mark, ft for equiv difficulty
			(3/11, 45/11 oe)	A2 **[4]**	or $x = 3/11$, $y = 45/11$ oe isw allow **A1** for each coordinate	
6			selecting the term in $\left(\frac{1}{x}\right)^3(-3x)^3$	M1	condone wrong or omitted brackets; may be implied by $(-3)^3$ or by −27 selected	
			${}^6C_3 = 20$ soi	M1	may be part of a fully correct row in Pascal's triangle	mark to advantage of candidate if choice eg if correct row seen but wrong element selected
			−540	A1 **[3]**	allow **B2** for −540 included as part of expansion but not identified as required term – ignore errors in other terms allow **B3** for −540 obtained and identified by multiplying out brackets, ignoring errors in other terms, otherwise M0 for this method	
7	(i)		$17\sqrt{7}$	2 **[2]**	**M1** for $\sqrt{28} = 2\sqrt{7}$ soi or for $\sqrt{175} = 5\sqrt{7}$	

Question			Answer	Marks	Guidance	
7	(ii)		$\frac{36-9\sqrt{2}}{23}$ or $\frac{36+-9\sqrt{2}}{23}$ or $a = 36$, $b = -9$, $c = 23$	3 [3]	**B1** for denominator; **B2** for numerator or **M1** for three terms correct in $30+6\sqrt{2}-15\sqrt{2}+6$	
8	(i)		when n is negative, n^2 is positive and so $n^2 + 6$ is positive	B1	oe with a valid number, or equivalent explanation eg may use $n = 0$	not sufft to say n doesn't have to be positive for $n^2 + 6$ to be positive; but allow B1 for statement such as '$n^2 \geq 0$ whatever n is, so $n^2 + 6$ is always positive' B0 for 'n^2 is always positive so n can be negative'
			$A \Rightarrow B$	B1 [2]	condone $B \Leftarrow A$ **B0** if no attempt at explanation (explanation does not need to gain a mark)	Do not accept $A \rightarrow B$ oe or just $\Rightarrow$ or $\Leftarrow$ with no A and B
	(ii)		the diagonals of a parallelogram also bisect each other, not at 90°	B1	oe for other valid statement/sketch eg 'A is true for parallelograms as well' but **B0** for eg 'parallelograms also have diagonals meeting not at 90°' – need diagonals bisecting each other as well; **B0** if eg square or rhombus or kite or trapezium etc also included as having diagonals bisecting each other, not at 90°	reference merely to 'other shapes' having diagonals bisecting each other but not at 90° is not sufficient; if explanation has words, ignore sketches unless referred to in words; if explanation is by sketches only, they must have diagonals drawn, approx. bisecting each other not at right angles but need not be ruled
			$A \Leftarrow B$	B1 [2]	condone $B \Rightarrow A$ **B0** if no attempt at explanation (condone explanation of why the symbol they give is true or any sketch) - (explanation does not need to gain a mark)	Do not accept $A \leftarrow B$ oe or just $\Rightarrow$ or $\Leftarrow$ with no A and B

Question			Answer	Marks	Guidance	
9			$-a - c = 3$	B1	accept $(-1)^3a$ instead of $-a$	
			$64a + 4c = 108$	B1	accept $(4)^3a$ instead of $64a$	may also be obtained after long division etc
			Correct method for eliminating one variable, condoning one further error	M1	dep on two equations in a and c and at least B1 earned	
			$a = 2$, $c = -5$	A2	**A1** for one correct if M0 but a and c both correct, allow **SC1**	
				[5]		
10	**(i)**		$3(x - 1.5)^2 - 1.75$ oe in fractions, www	B4	**B1** for each of $a = 3$, $b = -1.5$ and **B2** for $c = -1.75$ or **M1** for $5 - 3 \times 1.5^2$ or ft soi or for $5/3 - 1.5^2$ or ft soi	ignore '= 0' if brackets are there, condone missing power of 2
			Line of symmetry is $x =$ – their b	B1	must ft; if correct, $x = 1.5$	for last two B marks, do not allow those starting again since not 'hence'
			min y = their c	B1	must ft; if correct, $y = -1.75$ **B0** for just min pt = (1.5, – 1.75) oe statement needed not just sketch with – 1.75 marked on y-axis	
				[6]		

Question			Answer	Marks	Guidance	
10	(ii)		intersects y-axis at (0, 5)	B1		
			intersects x-axis at $\left(\frac{9\pm\sqrt{21}}{6},0\right)$ or $\left(\frac{3}{2}\pm\sqrt{\frac{7}{12}},0\right)$ or $x=\frac{9\pm\sqrt{21}}{6}$ or $x=\frac{3}{2}\pm\sqrt{\frac{7}{12}}$	B2	**M1** for quadratic formula used or ft from their completing the square in (i), with at most one error; condone answers not in coordinate form; isw after correct x values obtained	
			$\frac{9-\sqrt{21}}{6}<x<\frac{9+\sqrt{21}}{6}$ or ft	B1 **[4]**	ft only for soln using surds; allow ft from wrongly simplified surds; do not accept two separate inequalities	
11	(i)		graph of cubic correct way up	B1	**B0** if stops at x-axis	must not have any ruled sections; no curving back; condone slight 'flicking out' at ends but not approaching another turning point; allow max on y-axis or in 1st or 2nd quadrants; condone some 'doubling' or 'feathering' (deleted work still may show in scans)
			crossing x-axis at −5/2, 1 and 4	B2	on graph or nearby; may be in coordinate form; **M1** for $x^2 - 5x + 4 = (x - 4)(x - 1)$ or for roots 4 and 1 found mark intent for intersections with both axes	allow if no graph, but marked on x-axis condone intercepts for x and / or y given as reversed coordinates
			crossing y-axis at 20	B1 **[4]**	or $x = 0$, $y = 20$ seen if consistent with graph drawn	allow if no graph, but eg B0 for graph with intn on y-axis nowhere near their indicated 20

Question			Answer	Marks	Guidance	
11	(ii)		$g(-3) = 2 \times (-3)^3 - 5 \times 9 - 17 \times -3 + 48$ $= -54 - 45 + 51 + 48$ $= 0$	B1	condone $(-3)^3$ instead of -27 etc, but next step of working must be shown correctly or **B1** for correct division of $g(x)$ by $(x + 3)$ with remainder 0 and the conclusion immediately following this (or explicitly connected to it) that $g(-3) = 0$ or that -3 is a root of $g(x)$ oe	B0 for just $x + 3$ is a factor or for $x + 3$ is a root
			$(x + 3)$ used or stated as factor	M1		
			correctly finding other factor as $2x^2 - 11x + 16$	B2	accept $b = -11$ found **M1** for correct division of cubic by $(x + 3)$ as far as obtaining $2x^2 - 11x$ (may be in grid) or for two correct terms of $2x^2 - 11x + 16$ obtained by inspection	
			121 −128 isw or −7	A1	for correct substitution into $b^2 - 4ac$ and obtaining negative (may be seen in formula); no ft from wrong factor	must be correctly simplified to at least the 121 −128 stage
			conclusion no real roots from quadratic factor/equation, so −3 is only real root of $g(x)$	A1 **[6]**	dep on previous A1; must refer back to original request, just 'no real roots' is not sufft they need to mention −3 or say 'so just one real root' or 'no more real roots'	
11	(iii)		[$f(x) =$] $2x^3 - 5x^2 - 17x + 20$ with correct working	B2	**B1** if no working or **M1** for correct working condone inclusion of $+ k$ even if labelled as $f(x)$ instead of $g(x)$	if no working in (iii), check whether the relevant work has already been done in (i). If it has, tick it on the copy in the image zone and allow the mark, but only if $f(x)$ appears/is used in (iii).

Question			Answer	Marks	Guidance	
			$k = 28$ or g(x) is translation of f(x) by $\begin{pmatrix} 0 \\ 28 \end{pmatrix}$	B1 **[3]**	**B0** for just g(x) = f(x) + 28	B1 for $k = 28$ even if stated after no /wrong f(x) obtained
12	**(i)**		radius $\sqrt{50}$ isw wrong conversion to $5\sqrt{2}$ centre $(2, -1)$	B1 B1 **[2]**	**B1** for $5\sqrt{2}$	
12	**(ii)**					NB examiners must use annotation in this part; a tick where each mark is earned is sufficient
			$(x-2)^2 + (2x-9)^2 = 50$	M1	for subst from line into circle eqn; condone one error	eg condone omission of '=50' or having -11 instead of -9
			$5x^2 - 40x + 35$ [= 0]	M1	for simplifying to solvable form; condone one further error	
			$x = 7$ or 1	A1	condone omission of 7 and just using 1	
			B = $(1, -8)$	B1		
			midpt of AB = $\left(\frac{7+their1}{2}, \frac{4+their-8}{2}\right)$ or $(4, -2)$	M1	or length of AB found ft ($\sqrt{180}$ if correct) and Pythagoras used with ½ AB and r	Must use the coordinates of B since 'hence': so M0 for eqn of line through centre perp to AB and intersection with AB used to find mid point of AB or M0 for equation of AB and formula for dist of pt from line used
			distance = $\sqrt{5}$ correctly obtained (answer given)	A1 **[6]**		

Question			Answer	Marks	Guidance	
12	(iii)		$(x-2)^2 + (2x + k + 1)^2 = 50$	M1	condone one error, eg omission of +1, but k must be included	
			$5x^2 + 4kx + k^2 + 2k - 45$ [= 0]	M1	condone one error; accept constant term $(k + 1)^2 - 46$; must be rearranged to '=0' stage unless they go on to complete the square **M0** if wrong eqn used – no ft from original error, only condone one error from working with correct eqn	eg allow M1 for $5x^2 + 4kx + k^2 - 45$ [= 0]
			$b^2 - 4ac = 0$ oe soi	M1	may be earned near end allow for this condition quoted, even if then applied to wrong equation. It is sometimes earned at beginning	0 for just 'discriminant = 0' unless implied by later work
			$(4k)^2 - 4 \times 5 \times (k^2 + 2k - 45)$	M1	for correct substitution ft into $b^2 - 4ac$, dep on first M1 earned; brackets / signs must be correct	can be earned in formula (ignore rest of formula)
			correct simplification to given answer $k^2 + 10k - 225 = 0$.	A1 **[5]**	NB mark working not answer	

Question			Answer	Marks	Guidance	
12	(iii)		**method 2**	**or**		
			line perp to $y = 2x + k$ through centre is $y = -\frac{1}{2}x$ oe	M1	condone attempt $y = -\frac{1}{2}x + n$, with $n \neq 0$	M0 for just $y = -\frac{1}{2}x + c$ with no attempt to subst $(2, -1)$ to find c
			finding intersection with $y = 2x + k$ [if correct, $x = -\frac{2}{5}k, y = \frac{1}{5}k$]	M1	allow for finding intn of $y = 2x + k$ and line with grad – ½ but error in constant	
			$\left(2 + \frac{2}{5}k\right)^2 + \left(-1 - \frac{1}{5}k\right)^2 = 50$ oe	M1	for correct substitution ft into circle equation, dep on first M1 earned; brackets / signs must be correct	using distance from centre = radius, or point of intersection being on circle
			correct simplification to given answer $k^2 + 10k - 225 = 0$.	A2	NB mark working not answer; **A1** for correct expansion of brackets or correctly eliminating fractions as first step, working with correct equation only	A1 for $4 + \frac{8k}{5} + \frac{4k^2}{25} + 1 + \frac{2k}{5} + \frac{k^2}{25} = 50$ oe or $(10 + 2k)^2 + (5 + k)^2 = 1250$ oe
				[5]		

Question			Answer	Marks	Guidance	
12	(iii)		**method 3**	**or**		
			line perp to $y = 2x + k$ through centre is $y = -\frac{1}{2}x$ oe	M1	condone attempt $y = -\frac{1}{2}x + n$, with $n \neq 0$	M0 for just $y = -\frac{1}{2}x + c$ with no attempt to subst $(2, -1)$ to find c
			finding intn of their perp line with circle: $(x - 2)^2 + (-\frac{1}{2}x + 1)^2 = 50$ and simplifying to solvable form	M1	allow using line with grad $-\frac{1}{2}$ but error in constant	
			$x = \frac{4 \pm \sqrt{160}}{2}, y = \frac{-4 \text{ m} \sqrt{160}}{4}$ oe	A1		
			using $y = 2x + k$ to obtain k $k = -5 \pm 5\sqrt{10}$ oe if correct	M1	dep on previous Ms; or may use eqn of line gradient 2 through each of these points and compare results with $y = 2x + k$	
			$(k + 5)^2 = 250$ and correct working to obtain given answer $k^2 + 10k - 225 = 0$.	A1	NB mark working not answer; or allow subst of $k = -5 \pm 5\sqrt{10}$ oe into $k^2 + 10k - 225 = 0$ and showing consistent	
				[5]		

Question			Answer	Marks	Guidance	
12	(iii)		**method 4**	**or**		
			using calculus: $2x-4+2y\frac{dy}{dx}+\frac{dy}{dx}=0$ and subst $\frac{dy}{dx}=2$ [if correct, $2x-4+4y+2=0$]	M1	condone one error	
			using $y = 2x + k$, subst and solving: [if correct, $2x-4+4(2k+1)+2=0$ and $x=-\frac{2k}{5}, y=\frac{k}{5}$]	M1	condone one error	[cf method 2: more work to be done by method 4 to get to the stage of finding the point of contact in terms of k]
			$\left(2+\frac{2}{5}k\right)^2+\left(-1-\frac{1}{5}k\right)^2=50$ oe	M1	for correct substitution ft into circle equation, dep on first M1 earned; brackets / signs must be correct	using distance from centre = radius, or point of contact being on circle
			correct simplification to given answer $k^2 + 10k - 225 = 0$.	A2	NB mark working not answer; **A1** for correct expansion of brackets or correctly eliminating fractions as first step, working with correct equation only	A1 for $4+\frac{8k}{5}+\frac{4k^2}{25}+1+\frac{2k}{5}+\frac{k^2}{25}=50$ oe or $(10+2k)^2+(5+k)^2=1250$ oe
				[5]		

Appendix – sample explanations for 8(i)

Explanation	Mark
Even if a number is negative, its square will be positive.	1
$n^2 + 6$ will always be positive (because of the n^2). This being positive does not imply that n is positive.	1
any negative number squared is positive, therefore can be positive or negative	1
B can still be correct even if n was a negative number	1 bod
A can be a negative value and B will still be a positive number	1 bod
B will always be positive but A can be a negative number	1 bod
eg $(-5)^2 = 25$, so B = 31 is positive	1 bod
eg $-3^2 + 6 = 9 + 6 = 15$, so the number can be negative	1 bod
any number squared is positive, so n can be positive or negative	0