9MA0/02: Pure Mathematics Paper 2 Mark scheme

Question	Scheme	Marks	AOs
1	$\frac{1}{2}r^2(4.8)$	M1	1.1a
	$\frac{1}{2}r^2(4.8) = 135 \Rightarrow r^2 = \frac{225}{4} \Rightarrow r = 7.5$ o.e.	A1	1.1b
	length of minor arc $= 7.5(2\pi - 4.8)$	dM1	3.1a
	$= 15\pi - 36 \quad \{a = 15, b = -36\}$	A1	1.1b
		(4)	
1 Alt	$\frac{1}{2}r^2(4.8)$	M1	1.1a
	$\frac{1}{2}r^2(4.8) = 135 \Rightarrow r^2 = \frac{225}{4} \Rightarrow r = 7.5$ o.e.	A1	1.1b
	length of major arc $= 7.5(4.8)\ \{= 36\}$		
	length of minor arc $= 2\pi(7.5) - 36$	dM1	3.1a
	$= 15\pi - 36 \quad \{a = 15, b = -36\}$	A1	1.1b
		(4)	
			(4 marks)

Question 1 Notes:

M1:	Applies formula for the area of a sector with $\theta = 4.8$; i.e. $\frac{1}{2}r^2\theta$ with $\theta = 4.8$ **Note:** Allow M1 for considering ratios. E.g. $\frac{135}{\pi r^2} = \frac{4.8}{2\pi}$
A1:	Uses a correct equation $\left(\text{e.g. } \frac{1}{2}r^2(4.8) = 135\right)$ to obtain a radius of 7.5
dM1:	Depends on the previous M mark. A complete process for finding the length of the minor arc AB, by either • $(\text{their } r) \times (2\pi - 4.8)$ • $2\pi(\text{their } r) - (\text{their } r)(4.8)$
A1:	Correct exact answer in its simplest form, e.g. $15\pi - 36$ or $-36 + 15\pi$

Question	Scheme	Marks	AOs
2(a)	Attempts to substitute $\cos\theta \approx 1 - \frac{1}{2}\theta^2$ into either $1 + 4\cos\theta$ or $3\cos^2\theta$	M1	1.1b
	$1 + 4\cos\theta + 3\cos^2\theta \approx 1 + 4\left(1 - \frac{1}{2}\theta^2\right) + 3\left(1 - \frac{1}{2}\theta^2\right)^2$		
	$= 1 + 4\left(1 - \frac{1}{2}\theta^2\right) + 3\left(1 - \theta^2 + \frac{1}{4}\theta^4\right)$	M1	1.1b
	$= 1 + 4 - 2\theta^2 + 3 - 3\theta^2 + \frac{3}{4}\theta^4$		
	$= 8 - 5\theta^2$ *	A1*	2.1
		(3)	
(b)(i)	E.g. • Adele is working in degrees and not radians • Adele should substitute $\theta = \frac{5\pi}{180}$ and not $\theta = 5$ into the approximation	B1	2.3
(b)(ii)	$8 - 5\left(\frac{5\pi}{180}\right)^2$ = awrt 7.962, so $\theta = 5°$ gives a good approximation.	B1	2.4
		(2)	
			(5 marks)

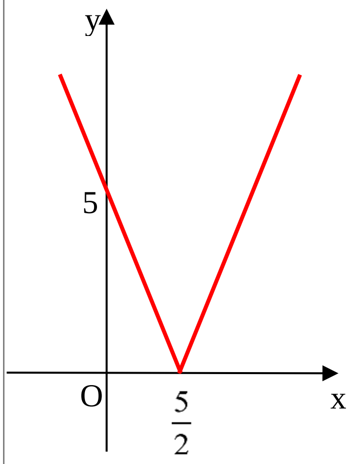
Question 2 Notes:

(a)(i)	
M1:	See scheme
M1:	Substitutes $\cos\theta \approx 1 - \frac{1}{2}\theta^2$ into $1 + 4\cos\theta + 3\cos^2\theta$ and attempts to apply $\left(1 - \frac{1}{2}\theta^2\right)^2$
	Note: It is not a requirement for this mark to write or refer to the term in θ^4
A1*:	Correct proof with no errors seen in working. **Note:** It is not a requirement for this mark to write or refer to the term in θ^4
(a)(ii)	
B1:	See scheme
(b)(i)	
B1:	See scheme
(b)(ii)	
B1:	Substitutes $\theta = \frac{5\pi}{180}$ or $\frac{\pi}{36}$ into $8 - 5\theta^2$ to give awrt 7.962 **and** an appropriate conclusion.

Question	Scheme	Marks	AOs
3 (a)	$\{t = 0,\ \theta = 75 \Rightarrow\ 75 = 25 + A \Rightarrow\ A = 50\}\ \Rightarrow\ \theta = 25 + 50e^{-0.03t}$	B1	3.3
		(1)	
(b)	$\{\theta = 60 \Rightarrow\}\ \Rightarrow 60 = 25 + \text{"}50\text{"}e^{-0.03t}\ \Rightarrow\ e^{-0.03t} = \dfrac{60-25}{\text{"}50\text{"}}$	M1	3.4
	$t = \dfrac{\ln(0.7)}{-0.03} = 11.8891648 = 11.9$ minutes (1 dp)	A1	1.1b
		(2)	
(c)	A valid evaluation of the model, which relates to the large values of t. E.g. • As $20.3 < 25$ then the model is not true for large values of t • $e^{-0.03t} = \dfrac{20.3-25}{\text{"}50\text{"}} = -0.094$ does not have any solutions and so the model predicts that tea in the room will never be $20.3\ °C$. So the model does not work for large values of t • $t = 120\ \Rightarrow\ \theta = 25 + 50e^{-0.03(120)} = 26.36\ldots$ which is not approximately equal to 20.3, so the model is not true for large values of t	B1	3.5a
		(1)	
		(4 marks)	

Question 3 Notes:

(a)	
B1:	Applies $t = 0,\ \theta = 75$ to give the complete model $\theta = 25 + 50e^{-0.03t}$
(b)	
M1:	Applies $\theta = 60$ and their value of A to the model and rearranges to make $e^{-0.03t}$ the subject. **Note:** Later working can imply this mark.
A1	Obtains 11.9 (minutes) with no errors in manipulation seen.
(c)	
B1	See scheme

Question	Scheme	Marks	AOs
4(a)	 Correct graph in quadrant 1 and quadrant 2 with V on the x-axis	B1	1.1b
	States $(0, 5)$ and $\left(\frac{5}{2}, 0\right)$ **or** $\frac{5}{2}$ marked in the correct position on the x-axis **and** 5 marked in the correct position on the y-axis	B1	1.1b
		(2)	
(b)	$\lvert 2x - 5 \rvert > 7$		
	$2x - 5 = 7 \Rightarrow x = \ldots$ **and** $-(2x-5) = 7 \Rightarrow x = \ldots$	M1	1.1b
	{critical values are $x = 6, -1 \Rightarrow$} $x < -1$ or $x > 6$	A1	1.1b
		(2)	
(c)	$\lvert 2x - 5 \rvert > x - \frac{5}{2}$		
	E.g. • Solves $2x - 5 = x - \frac{5}{2}$ to give $x = \frac{5}{2}$ and solves $-(2x-5) = x - \frac{5}{2}$ to also give $x = \frac{5}{2}$ • Sketches graphs of $y = \lvert 2x - 5 \rvert$ and $y = x - \frac{5}{2}$. Indicates that these graphs meet at the point $\left(\frac{5}{2}, 0\right)$	M1	3.1a
	Hence using set notation, e.g. • $\left\{x : x < \frac{5}{2}\right\} \cup \left\{x : x > \frac{5}{2}\right\}$ • $\left\{x \in \mathbb{R}, x \neq \frac{5}{2}\right\}$ • $\mathbb{R} - \left\{\frac{5}{2}\right\}$	A1	2.5
		(2)	
			(6 marks)

Question 4 Notes:	
(a)	
B1:	See scheme
B1:	See scheme
(b)	
M1:	See scheme
A1:	Correct answer, e.g. • $x < -1$ or $x > 6$ • $x < -1 \cup x > 6$ • $\{x: x < -1\} \cup \{x: x > 6\}$
(c)	
M1:	A complete process of finding that $y = \lvert 2x - 5 \rvert$ and $y = x - \frac{5}{2}$ meet at **only** one point. This can be achieved either algebraically or graphically.
A1:	See scheme. **Note:** Final answer must be expressed using set notation.

Question	Scheme	Marks	AOs
5	$3x - 2y = k$ intersects $y = 2x^2 - 5$ at two distinct points		
	Eliminate y and forms quadratic equation = 0 or quadratic expression $\{= 0\}$	M1	3.1a
	$\{3x - 2(2x^2 - 5) = k \Rightarrow\}\ -4x^2 + 3x + 10 - k = 0$	A1	1.1b
	$\{"b^2 - 4ac" > 0 \Rightarrow\}\ 3^2 - 4(-4)(10 - k) > 0$	dM1	2.1
	$9 + 16(10 - k) > 0 \Rightarrow 169 - 16k > 0$		
	Critical value obtained of $\frac{169}{16}$	B1	1.1b
	$k < \frac{169}{16}$ o.e.	A1	1.1b
		(5)	
5 Alt 1	Eliminate y and forms quadratic equation = 0 or quadratic expression $\{= 0\}$	M1	3.1a
	$y = 2\left(\frac{1}{3}(k + 2y)\right)^2 - 5 \Rightarrow y = \frac{2}{9}(k^2 + 4ky + 4y^2) - 5$		
	$8y^2 + (8k - 9)y + 2k^2 - 45 = 0$	A1	1.1b
	$\{"b^2 - 4ac" > 0 \Rightarrow\}\ (8k - 9)^2 - 4(8)(2k^2 - 45) > 0$	dM1	2.1
	$64k^2 - 144k + 81 - 64k^2 + 1440 > 0 \Rightarrow -144k + 1521 > 0$		
	Critical value obtained of $\frac{169}{16}$	B1	1.1b
	$k < \frac{169}{16}$ o.e.	A1	1.1b
		(5)	
5 Alt 2	$\frac{dy}{dx} = 4x,\ m_l = \frac{3}{2} \Rightarrow 4x = \frac{3}{2} \Rightarrow x = \frac{3}{8}$. So $y = 2\left(\frac{3}{8}\right)^2 - 5 = -\frac{151}{32}$	M1 A1	3.1a 1.1b
	$k = 3\left(\frac{3}{8}\right) - 2\left(-\frac{151}{32}\right) \Rightarrow k = \ldots$	dM1	2.1
	Critical value obtained of $\frac{169}{16}$	B1	1.1b
	$k < \frac{169}{16}$ o.e.	A1	1.1b
		(5)	
		(5 marks)	

Question 5 Notes:	
M1:	Complete strategy of eliminating x or y and manipulating the resulting equation to form a quadratic equation = 0 or a quadratic expression $\{= 0\}$
A1:	Correct algebra leading to either • $-4x^2 + 3x + 10 - k = 0$ or $4x^2 - 3x - 10 + k = 0$ or a one-sided quadratic of either $-4x^2 + 3x + 10 - k$ or $4x^2 - 3x - 10 + k$ • $8y^2 + (8k - 9)y + 2k^2 - 45 = 0$ or a one-sided quadratic of e.g. $8y^2 + (8k - 9)y + 2k^2 - 45$
dM1:	Depends on the previous M mark. Interprets $3x - 2y = k$ intersecting $y = 2x^2 - 5$ at two distinct points by applying $"b^2 - 4ac" > 0$ to their quadratic equation or one-sided quadratic.
B1:	See scheme
A1:	Correct answer, e.g. • $k < \frac{169}{16}$ • $\left\{k : k < \frac{169}{16}\right\}$
Alt 2	
M1:	Complete strategy of using differentiation to find the values of x and y where $3x - 2y = k$ is a tangent to $y = 2x^2 - 5$
A1:	Correct algebra leading to $x = \frac{3}{8},\ y = -\frac{151}{32}$
dM1:	Depends on the previous M mark. Full method of substituting their $x = \frac{3}{8},\ y = -\frac{151}{32}$ into l and attempting to find the value for k.
B1:	See scheme
A1:	Deduces correct answer, e.g. • $k < \frac{169}{16}$ • $\left\{k : k < \frac{169}{16}\right\}$

Question	Scheme	Marks	AOs
6(a)	$f(x) = (8-x)\ln x,\ x > 0$		
	Crosses x-axis $\Rightarrow f(x) = 0 \Rightarrow (8-x)\ln x = 0$		
	x coordinates are 1 and 8	B1	1.1b
		(1)	
(b)	Complete strategy of setting $f'(x) = 0$ and rearranges to make $x = ...$	M1	3.1a
	$\left\{ \begin{array}{ll} u = (8-x) & v = \ln x \\ \dfrac{du}{dx} = -1 & \dfrac{dv}{dx} = \dfrac{1}{x} \end{array} \right\}$		
	$f'(x) = -\ln x + \dfrac{8-x}{x}$	M1 A1	1.1b 1.1b
	$-\ln x + \dfrac{8-x}{x} = 0 \Rightarrow -\ln x + \dfrac{8}{x} - 1 = 0$ $\Rightarrow \dfrac{8}{x} = 1 + \ln x \Rightarrow x = \dfrac{8}{1+\ln x}$ *	A1*	2.1
		(4)	
(c)	Evaluates both $f'(3.5)$ and $f'(3.6)$	M1	1.1b
	$f'(3.5) = 0.032951317...$ and $f'(3.6) = -0.058711623...$ Sign change and as $f'(x)$ is continuous, the x coordinate of Q lies between $x = 3.5$ and $x = 3.6$	A1	2.4
		(2)	
(d)(i)	$\{x_5 =\}\ 3.5340$	B1	1.1b
(d)(ii)	$\{x_Q =\}\ 3.54$ (2 dp)	B1	2.2a
		(2)	
			(9 marks)

Question 6 Notes:	
(a)	
B1:	Either • 1 and 8 • on Figure 2, marks 1 next to A and 8 next to B
(b)	
M1:	Recognises that Q is a stationary point (and not a root) and applies a complete strategy of setting $f'(x) = 0$ and rearranges to make $x = ...$
M1:	Applies $vu' + uv'$, where $u = 8 - x,\ v = \ln x$ **Note:** This mark can be recovered for work in part (c)
A1:	$(8 - x)\ln x \to\ -\ln x + \dfrac{8 - x}{x}$, or equivalent **Note:** This mark can be recovered for work in part (c)
A1*:	Correct proof with no errors seen in working.
(c)	
M1:	Evaluates both $f'(3.5)$ and $f'(3.6)$
A1:	$f'(3.5) =$ awrt 0.03 and $f'(3.6) =$ awrt -0.06 or $f'(3.6) = -0.05$ (truncated) **and** a correct conclusion
(d)(i)	
B1:	See scheme
(d)(ii)	
B1:	Deduces (e.g. by the use of further iterations) that the x coordinate of Q is 3.54 accurate to 2 dp **Note:** $3.5 \to 3.55119 \to 3.52845 \to 3.53848 \to 3.53404 \to 3.53600 \to 3.53514\ (\to 3.535518...)$

Question	Scheme	Marks	AOs
7(a)	$\frac{dp}{dt} \propto p \Rightarrow \frac{dp}{dt} = kp$	B1	3.3
	$\int \frac{1}{p}\,dp = \int k\,dt$	M1	1.1b
	$\ln p = kt \; \{+ c\}$	A1	1.1b
	$\ln p = kt + c \Rightarrow p = e^{kt+c} = e^{kt}e^{c} \Rightarrow p = ae^{kt}$ *	A1 *	2.1
		(4)	
(b)	$p = ae^{kt} \Rightarrow \ln p = \ln a + kt$ and evidence of understanding that either • gradient $= k$ or $"M" = k$ • vertical intercept $= \ln a$ or $"C" = \ln a$	M1	2.1
	gradient $= k = 0.14$	A1	1.1b
	vertical intercept $= \ln a = 3.95 \Rightarrow a = e^{3.95} = 51.935 = 52$ (2 sf)	A1	1.1b
		(3)	
(c)	e.g. • $p = ae^{kt} \Rightarrow p = a(e^{k})^{t} = ab^{t}$, • $p = 52e^{0.14t} \Rightarrow p = 52(e^{0.14})^{t}$	B1	2.2a
	$b = 1.15$ which can be implied by $p = 52(1.15)^{t}$	B1	1.1b
		(2)	
(d)(i)	Initial area (i.e. "52" mm^2) of bacterial culture that was first placed onto the circular dish.	B1	3.4
(d)(ii)	E.g. • Rate of increase per hour of the area of bacterial culture • The area of bacterial culture increases by "15%" each hour	B1	3.4
		(2)	
(e)	The model predicts that the area of the bacteria culture will increase indefinitely, but the size of the circular dish will be a constraint on this area.	B1	3.5b
		(1)	
			(12 marks)

Question 7 Notes:	
(a)	
B1:	Translates the scientist's statement regarding proportionality into a differential equation, which involves a constant of proportionality. e.g. $\frac{dp}{dt} \propto p \Rightarrow \frac{dp}{dt} = kp$
M1:	Correct method of separating the variables p and t in their differential equation
A1:	$\ln p = kt$, with or without a constant of integration
A1*:	Correct proof with no errors seen in working.
(b)	
M1:	See scheme
A1:	Correctly finds $k = 0.14$
A1:	Correctly finds $a = 52$
(c)	
B1:	Uses algebra to correctly deduce either • $p = ab^t$ from $p = ae^{kt}$ • $p = "52"(e^{"0.14"})^t$ from $p = "52"e^{"0.14"t}$
B1:	See scheme
(d)(i)	
B1:	See scheme
(d)(ii)	
B1:	See scheme
(e)	
B1:	Gives a correct long-term limitation of the model for p. (See scheme).

Question	Scheme	Marks	AOs
8(a)	$\frac{dV}{dt} = 160\pi$, $V = \frac{1}{3}\pi h^2(75 - h) = 25\pi h^2 - \frac{1}{3}\pi h^3$		
	$\frac{dV}{dh} = 50\pi h - \pi h^2$	M1	1.1b
		A1	1.1b
	$\left\{\frac{dV}{dh} \times \frac{dh}{dt} = \frac{dV}{dt} \Rightarrow\right\} \left(50\pi h - \pi h^2\right)\frac{dh}{dt} = 160\pi$	M1	3.1a
	When $h = 10$, $\left\{\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} \Rightarrow\right\} \frac{160\pi}{50\pi(10) - \pi(10)^2}$ $\left\{= \frac{160\pi}{400\pi}\right\}$	dM1	3.4
	$\frac{dh}{dt} = 0.4$ (cm s^{-1})	A1	1.1b
		(5)	
(b)	$\frac{dh}{dt} = \frac{300\pi}{50\pi(20) - \pi(20)^2}$	M1	3.4
	$\frac{dh}{dt} = 0.5$ (cm s^{-1})	A1	1.1b
		(2)	
			(7 marks)

Question 8 Notes:

(a)

M1: Differentiates V with respect to h to give $\pm\alpha h \pm \beta h^2$, $\alpha \neq 0$, $\beta \neq 0$

A1: $50\pi h - \pi h^2$

M1: Attempts to solve the problem by applying a complete method of $\left(\text{their } \frac{dV}{dh}\right) \times \frac{dh}{dt} = 160\pi$

M1: Depends on the previous M mark.

Substitutes $h = 10$ into their model for $\frac{dh}{dt}$ which is in the form $\frac{160\pi}{\left(\text{their } \frac{dV}{dh}\right)}$

A1: Obtains the correct answer 0.4

(b)

M1: Realises that rate for of $160\pi \text{ cm}^3\text{s}^{-1}$ for $0 \leqslant h \leqslant 12$ has no effect when the rate is increased to $300\pi \text{ cm}^3\text{s}^{-1}$ for $12 < h \leqslant 24$ and so substitutes $h = 20$ into their model for $\frac{dh}{dt}$ which is in the form $\frac{300\pi}{\left(\text{their } \frac{dV}{dh}\right)}$

A1: Obtains the correct answer 0.5

Question	Scheme	Marks	AOs
9(a)	E.g. midpoint PQ $=\left(\frac{-9+15}{2}, \frac{8-10}{2}\right)$	M1	1.1b
	$=(3,-1)$, which is the centre point A, so PQ is the diameter of the circle.	A1	2.1
		(2)	
(a) Alt 1	$m_{PQ}=\frac{-10-8}{15--9}=-\frac{3}{4} \Rightarrow$ PQ: $y-8=-\frac{3}{4}(x--9)$	M1	1.1b
	PQ: $y=-\frac{3}{4}x+\frac{5}{4}$. So $x=3 \Rightarrow y=-\frac{3}{4}(3)+\frac{5}{4}=-1$ so PQ is the diameter of the circle.	A1	2.1
		(2)	
(a) Alt 2	$PQ=\sqrt{(-9-15)^2+(8--10)^2}\ \left\{=\sqrt{900}=30\right\}$ **and either** • $AP=\sqrt{(3--9)^2+(-1-8)^2}\ \left\{=\sqrt{225}=15\right\}$ • $AQ=\sqrt{(3-15)^2+(-1--10)^2}\ \left\{=\sqrt{225}=15\right\}$	M1	1.1b
	e.g. as $PQ=2AP$, then PQ is the diameter of the circle.	A1	2.1
		(2)	
(b)	Uses Pythagoras in a correct method to find either the radius or diameter of the circle.	M1	1.1b
	$(x-3)^2+(y+1)^2=225\ \left(\text{or } (15)^2\right)$	M1	1.1b
		A1	1.1b
		(3)	
(c)	Distance $=\sqrt{(\text{"}15\text{"})^2-(10)^2}$ or $=\frac{1}{2}\sqrt{(2(\text{"}15\text{"}))^2-(2(10))^2}$	M1	3.1a
	$\left\{=\sqrt{125}\right\}=5\sqrt{5}$	A1	1.1b
		(2)	
(d)	$\sin(A\hat{R}Q)=\frac{20}{2(\text{"}15\text{"})}$ or $A\hat{R}Q=90-\cos^{-1}\left(\frac{10}{\text{"}15\text{"}}\right)$	M1	3.1a
	$A\hat{R}Q=41.8103...=41.8°$ (to 0.1 of a degree)	A1	1.1b
		(2)	
		(9 marks)	

Question 9 Notes:	
(a)	
M1:	Uses a correct method to find the midpoint of the line segment PQ
A1:	Completes proof by obtaining $(3, -1)$ and gives a correct conclusion.
(a) Alt 1	
M1:	Full attempt to find the equation of the line PQ
A1:	Completes proof by showing that $(3, -1)$ lies on PQ and gives a correct conclusion.
(a) Alt 2	
M1:	Attempts to find distance PQ and either one of distance AP or distance AQ
A1:	Correctly shows either • $\text{PQ} = 2\text{AP}$, supported by $\text{PQ} = 30,\ \text{AP} = 15$ and gives a correct conclusion • $\text{PQ} = 2\text{AQ}$, supported by $\text{PQ} = 30,\ \text{AQ} = 15$ and gives a correct conclusion
(b)	
M1:	**Either** • uses Pythagoras correctly in order to find the **radius**. Must clearly be identified as the **radius**. E.g. $r^2 = (-9-3)^2 + (8+1)^2$ or $r = \sqrt{(-9-3)^2 + (8+1)^2}$ or $r^2 = (15-3)^2 + (-10+1)^2$ or $r = \sqrt{(15-3)^2 + (-10+1)^2}$ **or** • uses Pythagoras correctly in order to find the **diameter**. Must clearly be identified as the **diameter**. E.g. $d^2 = (15+9)^2 + (-10-8)^2$ or $d = \sqrt{(15+9)^2 + (-10-8)^2}$ **Note:** This mark can be implied by just 30 clearly seen as the **diameter** or 15 clearly seen as the **radius** (may be seen or implied in their circle equation)
M1:	Writes down a circle equation in the form $(x \pm "3")^2 + (y \pm "-1")^2 = (\text{their } r)^2$
A1:	$(x-3)^2 + (y+1)^2 = 225$ or $(x-3)^2 + (y+1)^2 = 15^2$ or $x^2 - 6x + y^2 + 2y - 215 = 0$
(c)	
M1:	Attempts to solve the problem by using the circle property “the perpendicular from the centre to a chord bisects the chord” and so applies Pythagoras to write down an expression of the form $\sqrt{(\text{their } "15")^2 - (10)^2}$.
A1:	$5\sqrt{5}$ by correct solution only
(d)	
M1:	Attempts to solve the problem by e.g. using the circle property “the angle in a semi-circle is a right angle” and writes down either $\sin(\text{A}\hat{\text{R}}\text{Q}) = \dfrac{20}{2(\text{their } "15")}$ or $\text{A}\hat{\text{R}}\text{Q} = 90 - \cos^{-1}\left(\dfrac{10}{\text{their } "15"}\right)$ **Note:** Also allow $\cos(\text{A}\hat{\text{R}}\text{Q}) = \dfrac{15^2 + \left(2\left(5\sqrt{5}\right)\right)^2 - 15^2}{2(15)\left(2\left(5\sqrt{5}\right)\right)}$ $\left\{= \dfrac{\sqrt{5}}{3}\right\}$
A1:	41.8 by correct solution only

Question	Scheme	Marks	AOs
10 (a)	$x > \ln\left(\frac{4}{3}\right)$	B1	2.2a
		(1)	
(b)	Attempts to apply $\int y\frac{dx}{dt}dt$	M1	3.1a
	$\left\{\int y\frac{dx}{dt}dt = \right\} = \int\left(\frac{1}{t+1}\right)\left(\frac{1}{t+2}\right)dt$	A1	1.1b
	$\frac{1}{(t+1)(t+2)} \equiv \frac{A}{(t+1)} + \frac{B}{(t+2)} \Rightarrow 1 \equiv A(t+2) + B(t+1)$	M1	3.1a
	$\{A=1,\ B=-1 \Rightarrow\}$ gives $\frac{1}{(t+1)} - \frac{1}{(t+2)}$	A1	1.1b
	$\left\{\int\left(\frac{1}{(t+1)} - \frac{1}{(t+2)}\right)dt = \right\}\ \ln(t+1) - \ln(t+2)$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = [\ln(t+1) - \ln(t+2)]_0^2 = (\ln 3 - \ln 4) - (\ln 1 - \ln 2)$	M1	2.2a
	$= \ln 3 - \ln 4 + \ln 2 = \ln\left(\frac{(3)(2)}{4}\right) = \ln\left(\frac{6}{4}\right)$		
	$= \ln\left(\frac{3}{2}\right)$ *	A1*	2.1
		(8)	
(b) Alt 1	Attempts to apply $\int y dx = \int \frac{1}{e^x - 2 + 1}dx = \int \frac{1}{e^x - 1}dx$, with a substitution of $u = e^x - 1$	M1	3.1a
	$\left\{\int y dx\right\} = \int\left(\frac{1}{u}\right)\left(\frac{1}{u+1}\right)du$	A1	1.1b
	$\frac{1}{u(u+1)} \equiv \frac{A}{u} + \frac{B}{(u+1)} \Rightarrow 1 \equiv A(u+1) + Bu$	M1	3.1a
	$\{A=1,\ B=-1 \Rightarrow\}$ gives $\frac{1}{u} - \frac{1}{(u+1)}$	A1	1.1b
	$\left\{\int\left(\frac{1}{u} - \frac{1}{(u+1)}\right)du = \right\}\ \ln u - \ln(u+1)$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = [\ln u - \ln(u+1)]_1^3 = (\ln 3 - \ln 4) - (\ln 1 - \ln 2)$	M1	2.2a
	$= \ln 3 - \ln 4 + \ln 2 = \ln\left(\frac{(3)(2)}{4}\right) = \ln\left(\frac{6}{4}\right)$		
	$= \ln\left(\frac{3}{2}\right)$ *	A1 *	2.1
		(8)	
			(9 marks)

Question	Scheme	Marks	AOs
10 (b) Alt 2	Attempts to apply $\int y\,dx = \int \frac{1}{e^x - 2 + 1}\,dx = \int \frac{1}{e^x - 1}\,dx$, with a substitution of $v = e^x$	M1	3.1a
	$\left\{\int y\,dx\right\} = \int \left(\frac{1}{v-1}\right)\left(\frac{1}{v}\right)dv$	A1	1.1b
	$\frac{1}{(v-1)v} \equiv \frac{A}{(v-1)} + \frac{B}{v} \Rightarrow 1 \equiv Av + B(v-1)$	M1	3.1a
	$\{A = 1,\ B = -1 \Rightarrow\}$ gives $\frac{1}{(v-1)} - \frac{1}{v}$	A1	1.1b
	$\left\{\int \left(\frac{1}{(v-1)} - \frac{1}{v}\right)dv = \right\}\ \ln(v-1) - \ln v$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = \left[\ln(v-1) - \ln v\right]_2^4 = (\ln 3 - \ln 4) - (\ln 1 - \ln 2)$	M1	2.2a
	$= \ln 3 - \ln 4 + \ln 2 = \ln\left(\frac{(3)(2)}{4}\right) = \ln\left(\frac{6}{4}\right)$		
	$= \ln\left(\frac{3}{2}\right)$ *	A1 *	2.1
		(8)	

Question 10 Notes:	
(a)	
B1:	Uses $x = \ln(t+2)$ with $t > -\frac{2}{3}$ to deduce the correct domain, $x > \ln\left(\frac{4}{3}\right)$
(b)	
M1:	Attempts to solve the problem by either • a parametric process or • a Cartesian process with a substitution of either $u = e^x - 1$ or $v = e^x$
A1:	Obtains • $\int\left(\frac{1}{t+1}\right)\left(\frac{1}{t+2}\right)dt$ from a parametric approach • $\int\left(\frac{1}{u}\right)\left(\frac{1}{u+1}\right)du$ from a Cartesian approach with $u = e^x - 1$ • $\int\left(\frac{1}{v-1}\right)\left(\frac{1}{v}\right)dv$ from a Cartesian approach with $v = e^x$
M1:	Applies a strategy of attempting to express either $\frac{1}{(t+1)(t+2)}$, $\frac{1}{u(u+1)}$ or $\frac{1}{(v-1)v}$ as partial fractions
A1:	Correct partial fractions for their method
M1:	Integrates to give either • $\pm\alpha\ln(t+1) \pm \beta\ln(t+2)$ • $\pm\alpha\ln u \pm \beta\ln(u+1)$; $\alpha, \beta \neq 0$, where $u = e^x - 1$ • $\pm\alpha\ln(v-1) \pm \beta\ln v$; $\alpha, \beta \neq 0$, where $v = e^x$
A1:	Correct integration for their method
M1:	Either • Parametric approach: Deduces and applies limits of 2 and 0 in t and subtracts the correct way round • Cartesian approach: Deduces and applies limits of 3 and 1 in u, where $u = e^x - 1$, and subtracts the correct way round • Cartesian approach: Deduces and applies limits of 4 and 2 in v, where $v = e^x$, and subtracts the correct way round
A1*:	Correctly shows that the area of R is $\ln\left(\frac{3}{2}\right)$, with no errors seen in their working

Question	Scheme	Marks	AOs
11	Arithmetic sequence, $T_2 = 2k$, $T_3 = 5k - 10$, $T_4 = 7k - 14$		
	$(5k - 10) - (2k) = (7k - 14) - (5k - 10) \Rightarrow k = \ldots$	M1	2.1
	$\{3k - 10 = 2k - 4 \Rightarrow\}\quad k = 6$	A1	1.1b
	$\{k = 6 \Rightarrow\}$ $T_2 = 12$, $T_3 = 20$, $T_4 = 28$. So $d = 8$, $a = 4$	M1	2.2a
	$S_n = \frac{n}{2}\left(2(4) + (n-1)(8)\right)$	M1	1.1b
	$= \frac{n}{2}\left(8 + 8n - 8\right) = 4n^2 = (2n)^2$ which is a square number	A1	2.1
		(5)	
		(5 marks)	

Question 11 Notes:

M1:	Complete method to find the value of k
A1:	Uses a correct method to find $k = 6$
M1:	Uses their value of k to deduce the common difference and the first term $(\neq T_2)$ of the arithmetic series.
M1:	Applies $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$ with their $a \neq T_2$ and their d.
A1:	Correctly shows that the sum of the series is $(2n)^2$ and makes an appropriate conclusion.

Question	Scheme	Marks	AOs
12	Complete process to find at least one set of coordinates for P. The process must include evidence of • differentiating • setting $\frac{dy}{dx}=0$ to find $x = ...$ • substituting $x = ...$ into $\sin x + \cos y = 0.5$ to find $y = ...$	M1	3.1a
	$\left\{\cancel{\frac{dy}{dx}} = \right\}$ $\cos x - \sin y\frac{dy}{dx} = 0$	B1	1.1b
	Applies $\frac{dy}{dx}=0$ (e.g. $\cos x = 0$ or $\frac{\cos x}{\sin y} = 0 \Rightarrow \cos x = 0$) $\Rightarrow x = ...$	M1	2.2a
	giving at least one of either $x = -\frac{\pi}{2}$ or $x = \frac{\pi}{2}$	A1	1.1b
	$x = \frac{\pi}{2} \Rightarrow \sin\left(\frac{\pi}{2}\right) + \cos y = 0.5 \Rightarrow \cos y = -\frac{1}{2} \Rightarrow y = \frac{2\pi}{3}$ or $-\frac{2\pi}{3}$	M1	1.1b
	So in specified range, $(x,\ y) = \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and $\left(\frac{\pi}{2}, -\frac{2\pi}{3}\right)$, by cso	A1	1.1b
	$x = -\frac{\pi}{2} \Rightarrow \sin\left(-\frac{\pi}{2}\right) + \cos y = 0.5 \Rightarrow \cos y = 1.5$ has no solutions, and so there are exactly 2 possible points P.	B1	2.1
		(7)	
			(7 marks)

Question 12 Notes:

M1:	See scheme
B1:	Correct differentiated equation. E.g. $\cos x - \sin y\frac{dy}{dx} = 0$
M1:	Uses the information "the tangent to C at the point P is parallel to the x-axis" to deduce and apply $\frac{dy}{dx}=0$ and finds $x = ...$
A1:	See scheme
M1:	For substituting one of their values from $\frac{dy}{dx}=0$ into $\sin x + \cos y = 0.5$ and so finds $x = ...,\ y = ...$
A1:	Selects coordinates for P on C satisfying $\frac{dy}{dx}=0$ and $-\frac{\pi}{2} \leqslant x < \frac{3\pi}{2}$, $-\pi < y < \pi$ i.e. finds $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and $\left(\frac{\pi}{2}, -\frac{2\pi}{3}\right)$ and no other points by correct solution only
B1:	Complete argument to show that there are exactly 2 possible points P.

Question	Scheme	Marks	AOs
13(a)	$\text{cosec}\,2x + \cot 2x \equiv \cot x, \quad x \neq 90n°, \quad n \in \mathbb{Z}$		
	$\text{cosec}\,2x + \cot 2x = \dfrac{1}{\sin 2x} + \dfrac{\cos 2x}{\sin 2x}$	M1	1.2
	$= \dfrac{1 + \cos 2x}{\sin 2x}$	M1	1.1b
	$= \dfrac{1 + 2\cos^2 x - 1}{2\sin x \cos x} = \dfrac{2\cos^2 x}{2\sin x \cos x}$	M1	2.1
		A1	1.1b
	$= \dfrac{\cos x}{\sin x} = \cot x \quad *$	A1*	2.1
		(5)	
(b)	$\text{cosec}(4\theta + 10°) + \cot(4\theta + 10°) = \sqrt{3}; \quad 0 \leqslant \theta < 180°,$		
	$\cot(2\theta \pm ...°) = \sqrt{3}$	M1	2.2a
	$2\theta \pm ... = 30° \Rightarrow \theta = 12.5°$	M1	1.1b
		A1	1.1b
	$2\theta + 5° = 180° + \text{PV}° \Rightarrow \theta = ...°$	M1	2.1
	$\theta = 102.5°$	A1	1.1b
		(5)	
			(10 marks)

Question 13 Notes:	
(a)	
M1:	Writes $\operatorname{cosec}2x = \dfrac{1}{\sin 2x}$ and $\cot 2x = \dfrac{\cos 2x}{\sin 2x}$
M1:	Combines into a single fraction with a common denominator
M1:	Applies $\sin 2x = 2\sin x \cos x$ to the denominator **and** applies either • $\cos 2x = 2\cos^2 x - 1$ • $\cos 2x = 1 - 2\sin^2 x$ and $\sin^2 x + \cos^2 x = 1$ • $\cos 2x = \cos^2 x - \sin^2 x$ and $\sin^2 x + \cos^2 x = 1$ to the numerator and manipulates to give a one term numerator expression
A1:	Correct algebra leading to $\dfrac{2\cos^2 x}{2\sin x \cos x}$ or equivalent.
A1*:	Correct proof with correct notation and no errors seen in working
(b)	
M1:	Uses the result in part (a) in an attempt to deduce either $2x = 4\theta + 10$ or $x = 2\theta + ...$ and uses $x = 2\theta + ...$ to write down or imply $\cot(2\ \ \pm ...°) = \sqrt{3}$
M1:	Applies $\operatorname{arccot}\left(\sqrt{3}\right) = 30°$ or $\arctan\left(\dfrac{1}{\sqrt{3}}\right) = 30°$ and attempts to solve $2\theta \pm ... = 30°$ to give $\theta = ...$
A1:	Uses a correct method to obtain $\theta = 12.5°$
M1:	Uses $2\theta + 5 = 180 +$ their PV° in a complete method to find the second solution, $\theta = ...$
A1:	Uses a correct method to obtain $\theta = 102.5°$, with no extra solutions given either inside or outside the required range $0 \leqslant \theta < 180°$

Question	Scheme	Marks	AOs
14 (i)	For an explanation or statement to show when the claim $3^x \ldots 2^x$ fails This could be e.g. • when $x = -1$, $\frac{1}{3} < \frac{1}{2}$ or $\frac{1}{3}$ is not greater than or equal to $\frac{1}{2}$ • when $x < 0$, $3^x < 2^x$ or 3^x is not greater than or equal to 2^x	M1	2.3
	followed by an explanation or statement to show when the claim $3^x \ldots 2^x$ is true. This could be e.g. • $x = 2$, $9 \ldots 4$ or 9 is greater than or equal to 4 • when $x \ldots 0$, $3^x \ldots 2^x$ and a correct conclusion. E.g. • so the claim $3^x \ldots 2^x$ is sometimes true	A1	2.4
		(2)	
(ii)	Assume that $\sqrt{3}$ is a rational number So $\sqrt{3} = \frac{p}{q}$, where p and q integers, $q \neq 0$, and the HCF of p and q is 1	M1	2.1
	$\Rightarrow p = \sqrt{3}q \Rightarrow p^2 = 3q^2$	M1	1.1b
	$\Rightarrow p^2$ is divisible by 3 and so p is divisible by 3	A1	2.2a
	So $p = 3c$, where c is an integer From earlier, $p^2 = 3q^2 \Rightarrow (3c)^2 = 3q^2$	M1	2.1
	$\Rightarrow q^2 = 3c^2 \Rightarrow q^2$ is divisible by 3 and so q is divisible by 3	A1	1.1b
	As both p and q are both divisible by 3 then the HCF of p and q is not 1 This contradiction implies that $\sqrt{3}$ is an irrational number	A1	2.4
		(6)	
			(8 marks)

Question 14 Notes:	
(i)	
M1:	See scheme
A1:	See scheme
(ii)	
M1:	Uses a method of proof by contradiction by initially assuming that $\sqrt{3}$ is rational and expresses $\sqrt{3}$ in the form $\frac{p}{q}$, where p and q are correctly defined.
M1:	Writes $\sqrt{3} = \frac{p}{q}$ and rearranges to make p^2 the subject
A1:	Uses a logical argument to prove that p is divisible by 3
M1:	Uses the result that p is divisible by 3, (to construct the initial stage of proving that q is also divisible by 3), by substituting $p = 3c$ into their expression for p^2
A1:	Hence uses a correct argument, in the same way as before, to deduce that q is also divisible by 3
A1:	Completes the argument (as detailed on the scheme) that $\sqrt{3}$ is irrational.
	Note: All the previous 5 marks need to be scored in order to obtain the final A mark.