January 2019

Mark Scheme

Mock Paper (set1)

Pearson Edexcel GCE A Level Mathematics

Pure Mathematics 1 (9MA0/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is awarded.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 100

2. These mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - **bod** – benefit of doubt
   - **ft** – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - **cao** – correct answer only
   - **cso** - correct solution only. There must be no errors in this part of the question to obtain this mark
   - **isw** – ignore subsequent working
   - **awrt** – answers which round to
   - **SC**: special case
   - **o.e.** – or equivalent (and appropriate)
   - **d** or **dep** – dependent
   - **indep** – independent
   - **dp** decimal places
   - **sf** significant figures
   - $\ast$ The answer is printed on the paper or ag- answer given

4. All M marks are follow through.

   A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but answers that don't logically make sense e.g. if an answer given for a probability is >1 or <0, should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

7. Ignore wrong working or incorrect statements following a correct answer.

8. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used. If no such alternative answer is provided but the response is deemed to be valid, examiners must escalate the response for a senior examiner to review.

Question	Scheme	Marks	AOs
1 (a)	$\text{Area}(R) \approx \frac{1}{2} \times 0.5 \times \left[\underline{1 + 2(e^{0.05} + e^{0.2} + e^{0.45}) + e^{0.8}} \right]$	B1	1.1b
		M1	1.1b
	$\left\{ = \frac{1}{4} \times 10.90751301... = 2.726878252... \right\} = 2.73 \text{ (2 dp)}$	A1	1.1b
		(3)	
(b)(i)	$\left\{ \int_0^2 \left(4 + e^{\frac{1}{5}x^2} \right) \mathrm{d}x \right\} = 4(2) + "2.73" = 10.73 \text{ (2 dp)}$	B1ft	2.2a
(b)(ii)	$\left\{ \int_1^3 e^{\frac{1}{5}(x-1)^2} \mathrm{d}x \right\} = "2.73" \text{ (2 dp)}$	B1ft	2.2a
		(2)	
			(5 marks)

Question 1 Notes:

(a)	
B1:	Outside brackets $\frac{1}{2} \times 0.5$ or $\frac{0.5}{2}$ or 0.25 or $\frac{1}{4}$
M1:	For structure of trapezium rule $\left[\underline{\ldots\ldots\ldots\ldots} \right]$.
	No errors are allowed, e.g. an omission of a y-ordinate or an extra y-ordinate or a repeated y-ordinate
A1:	Correct method leading to a correct answer only of 2.73
(b)(i)	
B1ft:	10.73 or a value which is 8 + their answer to part (a)
	Note: Do not allow an answer of 10.6900… which is found directly from integration
(b)(ii)	
B1ft:	2.73 or a value which is the same as their answer to part (a)
	Note: Do not allow an answer of 2.6900… or 2.69 which is found directly from integration

Question	Scheme	Marks	AOs
2	$BC^2 = 7.5^2 + 8.5^2 - 2(7.5)(8.5)\cos(\pi - 1.2)$ $\{\Rightarrow BC = 13.21743597...\}$	M1	1.1b
		A1	1.1b
	Arc length $AB = 7.5(1.2)$ $\{\Rightarrow \text{Arc length } AB = 9\}$	B1	1.1a
	Perimeter $AOCBA$ $= 7.5 + 8.5 + 13.217423597... + 9$	M1	3.1a
	$\{= 38.21743597...\}$ = 38.2 (cm) (1 dp)	A1	1.1b
		(5)	
			(5 marks)

Question 2 Notes:

M1:	Application of cosine rule for BC^2 or BC with any angle
A1:	Correct application of cosine rule for BC^2 or BC using $\pi - 1.2$
B1:	Arc length $AB = 7.5(1.2)$ or 9
M1:	A **complete** strategy for finding the perimeter of the shape $AOCBA$
A1:	38.2 cao

Question	Scheme	Marks	AOs
3	$3x^2 + k = 5x + 2$		
	E.g. $3x^2 - 5x + k - 2 = 0$ or $-3x^2 + 5x + 2 - k = 0$	M1	1.1b
	$\left\{"b^2 - 4ac" < 0 \Rightarrow \right\}$ $25 - 4(3)(k-2) < 0$	M1	1.1b
	$25 - 12k + 24 < 0 \Rightarrow -12k + 49 < 0$		
	Critical value obtained of $\frac{49}{12}$ o.e.	B1	1.1b
	$k > \frac{49}{12}$ o.e.	A1	2.1
		(4)	
		(4 marks)	

Question 3 Notes:

M1:	Forms a one-sided quadratic equation or gathers all terms into a single quadratic expression
M1:	Understands that the given equation has no real roots by applying $"b^2 - 4ac" < 0$ to their one-sided quadratic equation or to their one-sided quadratic expression $\{= 0\}$
B1:	See scheme
A1:	Complete process leading to the correct answer, e.g. • $k > \frac{49}{12}$ • $\frac{49}{12} < k$ • $\left\{k : k > \frac{49}{12}\right\}$ with no errors seen in their mathematical argument

Question	Scheme	Marks	AOs
4	$\mathrm{f}(x) = \dfrac{12x}{3x+4} \quad x \in \mathbb{R},\ x \geqslant 0$		
(a)	$0 \leqslant \mathrm{f}(x) < 4$	M1	1.1b
		A1	1.1b
		(2)	
(b)	$y = \dfrac{12x}{3x+4} \Rightarrow y(3x+4) = 12x \Rightarrow 3xy + 4y = 12x \Rightarrow 4y = 12x - 3xy$	M1	1.1b
	$4y = x(12-3y) \Rightarrow \dfrac{4y}{12-3y} = x$	M1	2.1
	Hence $\mathrm{f}^{-1}(x) = \dfrac{4x}{12-3x} \quad 0 \leqslant x < 4$	A1	2.5
		(3)	
(c)	$\mathrm{ff}(x) = \dfrac{12\left(\dfrac{12x}{3x+4}\right)}{3\left(\dfrac{12x}{3x+4}\right)+4}$	M1	1.1b
	$= \dfrac{\dfrac{144x}{3x+4}}{\dfrac{36x+12x+16}{3x+4}}$	M1	1.1b
	$= \dfrac{144x}{48x+16} = \dfrac{9x}{3x+1}$ * $\quad \{x \in \mathbb{R},\ x \geqslant 0\}$	A1*	2.1
		(3)	
(d)	$\left\{\mathrm{ff}(x) = \dfrac{7}{2} \Rightarrow\right\} \dfrac{9x}{3x+1} = \dfrac{7}{2} \Rightarrow 18x = 21x + 7 \Rightarrow -3x = 7 \Rightarrow x = ...$	M1	1.1b
	Reject $x = -\dfrac{7}{3}$ As $\mathrm{ff}(x)$ is valid for $x \geqslant 0$, then $\mathrm{ff}(x) = \dfrac{7}{2}$ has no solutions	A1	2.4
		(2)	
(d) Alt 1	$\left\{\mathrm{ff}(x) = \dfrac{7}{2} \Rightarrow\right\} \mathrm{f}(x) = \mathrm{f}^{-1}\left(\dfrac{7}{2}\right) = \dfrac{4\left(\dfrac{7}{2}\right)}{12-3\left(\dfrac{7}{2}\right)}$	M1	1.1b
	$\{\mathrm{f}(x) =\}\ \mathrm{f}^{-1}\left(\dfrac{7}{2}\right) = \dfrac{28}{3}$ As $0 \leqslant \mathrm{f}(x) < 4$ and as $\dfrac{28}{3} > 4$, then $\mathrm{ff}(x) = \dfrac{7}{2}$ has no solutions	A1	2.4
		(2)	
			(10 marks)

Question	Scheme	Marks	AOs
4 (d) Alt 2	Range of $\mathrm{ff}(x)$ is $0 \leqslant \mathrm{ff}(x) < 3$	M1	1.1b
	As $\frac{7}{2} > 3$, then $\mathrm{ff}(x) = \frac{7}{2}$ has no solutions	A1	2.4
		(2)	

Question 4 Notes:

(a)	
M1:	For one "end" fully correct; e.g. accept $\mathrm{f}(x) \geqslant 0$ (**not** $x \geqslant 0$) or $\mathrm{f}(x) < 4$ (**not** $x < 4$); or for both correct "end" values; e.g. accept $0 < \mathrm{f}(x) \leqslant 4$.
A1:	Correct range using correct notation. Accept $0 \leqslant \mathrm{f}(x) < 4$, $0 \leqslant y < 4$, $[0, 4)$, $\mathrm{f}(x) \geqslant 0$ and $\mathrm{f}(x) < 4$
(b)	
M1:	Attempts to find the inverse by cross-multiplying and an attempt to collect all the x-terms (or swapped y-terms) onto one side.
M1:	A fully correct method to find the inverse.
A1:	A correct $\mathrm{f}^{-1}(x) = \frac{4x}{12 - 3x}$, $0 \leqslant x < 4$, o.e. expressed fully in function notation, including the domain, which may be correct or followed through from their part (a) answer for their range of f
Note:	Writing $y = \frac{12x}{3x+4}$ as $y = \frac{4(3x+4) - 16}{3x+4} \Rightarrow y = 4 - \frac{16}{3x+4}$ leads to a correct $\mathrm{f}^{-1}(x) = \frac{1}{3}\left(\frac{16}{4-x} - 4\right)$, $0 \leqslant x < 4$
(c)	
M1:	Attempts to substitute $\mathrm{f}(x) = \frac{12x}{3x+4}$ into $\frac{12\mathrm{f}(x)}{3\mathrm{f}(x)+4}$
M1:	Applies a method of "rationalising the denominator" for their denominator.
A1*:	Shows $\mathrm{ff}(x) = \frac{9x}{3x+1}$ with no errors seen. **Note:** The domain of $\mathrm{ff}(x)$ is not required in this part.
(d)	
M1:	Sets $\frac{9x}{3x+1}$ to $\frac{7}{2}$ and solves to find $x = \ldots$
A1:	Finds $x = -\frac{7}{3}$, rejects this solution as $\mathrm{ff}(x)$ is valid for $x \geqslant 0$ only. Concludes that $\mathrm{ff}(x) = \frac{7}{2}$ has no solutions.

Question 4 Notes Continued:	
(d) **Alt 1**	
M1:	Attempts to find $\mathrm{f}^{-1}\left(\frac{7}{2}\right)$
A1:	Deduces $\mathrm{f}(x)=\mathrm{f}^{-1}\left(\frac{7}{2}\right)=\frac{28}{3}$ and concludes $\mathrm{ff}(x)=\frac{7}{2}$ has no solutions because $\mathrm{f}(x)=\frac{28}{3}$ lies outside the range $0\leqslant \mathrm{f}(x)<4$
(d) **Alt 2**	
M1:	Evidence that the upper bound of $\mathrm{ff}(x)$ is 3
A1:	$0\leqslant \mathrm{ff}(x)<3$ and concludes that $\mathrm{ff}(x)=\frac{7}{2}$ has no solutions because $\frac{7}{2}>3$

Question	Scheme	Marks	AOs
5	Let a point Q have x coordinate $2+h$. So $y_Q = 4(2+h)^2 - 5(2+h)$	B1	1.1b
	$\{P(2,6),\ Q(2+h,\ 4(2+h)^2 - 5(2+h))\}$		
	Gradient $PQ = \dfrac{4(2+h)^2 - 5(2+h) - 6}{2+h-2}$	M1 A1	2.1 1.1b
	$= \dfrac{4(4+4h+h^2) - 5(2+h) - 6}{2+h-2}$		
	$= \dfrac{16+16h+4h^2 - 10 - 5h - 6}{2+h-2}$		
	$= \dfrac{4h^2+11h}{h}$		
	$= 4h+11$	M1	1.1b
	$\dfrac{dy}{dx} = \lim_{h \to 0}\left(4h+11\right) = 11$	A1	2.2a
		(5)	
5 **Alt 1**	Gradient of chord $= \dfrac{4(x+h)^2 - 5(x+h) - (4x^2 - 5x)}{x+h-x}$	B1 M1 A1	1.1b 2.1 1.1b
	$= \dfrac{4(x^2+2xh+h^2) - 5(x+h) - (4x^2-5x)}{x+h-x}$		
	$= \dfrac{4x^2+8xh+4h^2-5x-5h-4x^2+5x}{x+h-x}$		
	$= \dfrac{8xh+4h^2-5h}{h}$		
	$= 8x+4h-5$	M1	1.1b
	$\dfrac{dy}{dx} = \lim_{h \to 0}\left(8x+4h-5\right) = 8x-5$ and so, at P, $\dfrac{dy}{dx} = 8(2)-5 = 11$	A1	2.2a
		(5)	
			(5 marks)

Question 5 Notes:	
B1:	Writes down the y coordinate of a point close to P E.g. For a point Q with x coordinate $2+h$, $\{y_Q\} = 4(2+h)^2 - 5(2+h)$
M1:	Begins the proof by attempting to write the gradient of the chord PQ in terms of h
A1:	Correct expression for the gradient of the chord PQ in terms of h
M1:	Correct process to obtain the gradient of the chord PQ as $\alpha h + \beta$; $\alpha, \beta \neq 0$
A1:	Correctly shows that the gradient of PQ is $4h+11$ and applies a limiting argument to deduce that at the point $\boldsymbol{P}$ on $y = 4x^2 - 5x$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 11$ E.g. $\lim_{h \to 0} (4h+11) = 11$ **Note:** δx can be used in place of h
Alt 1	
B1:	$4(x+h)^2 - 5(x+h)$, seen or implied
M1:	Begins the proof by attempting to write the gradient of the chord in terms of x and h
A1:	Correct expression for the gradient of the chord in terms of x and h
M1:	Correct process to obtain the gradient of the chord as $\alpha x + \beta h + \gamma$; $\alpha, \beta, \gamma \neq 0$
A1:	Correctly shows that the gradient of the chord is $8x+4h-5$ and applies a limiting argument to deduce that when $y = 4x^2 - 5x$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 8x - 5$. E.g. $\lim_{h \to 0} (8x+4h-5) = 8x-5$ Finally, deduces that at the point P, $\frac{\mathrm{d}y}{\mathrm{d}x} = 11$ **Note:** For Alt 1, δx can be used in place of h

Question	Scheme	Marks	AOs
6 (a)	$\left\{u = \mathrm{e}^{\frac{1}{2}x} \text{ or } x = 2\ln u \Rightarrow\right\}$		
	$\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2}\mathrm{e}^{\frac{1}{2}x}$ **or** $\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2}u$ **or** $\frac{\mathrm{d}x}{\mathrm{d}u} = \frac{2}{u}$ **or** $\mathrm{d}x = \frac{2}{u}\mathrm{d}u$ **or** $2\mathrm{d}u = u\mathrm{d}x$, etc.	B1	1.1b
	<u>Criteria 1</u> $\left\{x = 0 \Rightarrow a = \mathrm{e}^0 \text{ and } x = 2 \Rightarrow b = \mathrm{e}^{\frac{1}{2}(2)}\right\}$ $a = 1, b = \mathrm{e}$ **or** evidence of $0 \to 1$ and $2 \to \mathrm{e}$ **<u>Criteria 2 (dependent on the first B1 mark)</u>** $\int \frac{6}{(\mathrm{e}^{\frac{1}{2}x} + 4)}\mathrm{d}x = \int \frac{6}{(u+4)}.\frac{2}{u}\mathrm{d}u = \int \frac{12}{u(u+4)}\mathrm{d}u$		
	Either Criteria 1 or Criteria 2	B1	1.1b
	Both Criteria 1 and Criteria 2 **and** correctly achieves the result $\int_1^{\mathrm{e}} \frac{12}{u(u+4)}\mathrm{d}u$	B1	2.1
		(3)	
(b)	$\frac{12}{u(u+4)} \equiv \frac{A}{u} + \frac{B}{(u+4)} \Rightarrow 12 \equiv A(u+4) + Bu$	M1	1.1b
	$u = 0 \Rightarrow A = 3;\ u = -4 \Rightarrow B = -3$	A1	1.1b
	$\left\{\int \frac{12}{u(u+4)}\mathrm{d}u = \right\} \int \left(\frac{3}{u} - \frac{3}{(u+4)}\right)\mathrm{d}u = 3\ln u - 3\ln(u+4)$	M1 A1ft	3.1a 1.1b
	$\left\{\text{So, } \left[3\ln u - 3\ln(u+4)\right]_1^{\mathrm{e}}\right\}$		
	$= (3\ln \mathrm{e} - 3\ln(\mathrm{e}+4)) - (3\ln 1 - 3\ln 5)$		
	$= 3\ln \mathrm{e} - 3\ln(\mathrm{e}+4) + 3\ln 5$		
	$= 3\ln\left(\frac{5\mathrm{e}}{\mathrm{e}+4}\right)$ *	A1*	2.1
		(5)	
			(8 marks)

Question 6 Notes:	
(a)	
B1:	See scheme
B1:	See scheme
B1:	See scheme
	Note for Criteria 2: Must start from one of • $\int y\,\mathrm{d}x$, with integral sign and $\mathrm{d}x$ • $\int \frac{6}{\mathrm{e}^{\frac{1}{2}x}+4}\,\mathrm{d}x$, with integral sign and $\mathrm{d}x$ • $\int \frac{6}{\mathrm{e}^{\frac{1}{2}x}+4}\frac{\mathrm{d}x}{\mathrm{d}u}\,\mathrm{d}u$, with integral sign and $\frac{\mathrm{d}x}{\mathrm{d}u}\mathrm{d}u$ **and** end at $\int \frac{12}{u(u+4)}\,\mathrm{d}u$, with integral sign and $\mathrm{d}u$, **with no incorrect working**
(b)	
M1:	Writing $\frac{12}{u(u+4)} \equiv \frac{A}{u} + \frac{B}{(u+4)}$, o.e. or $\frac{1}{u(u+4)} \equiv \frac{P}{u} + \frac{Q}{(u+4)}$, o.e. and a complete method for finding the values of both **their** A and **their** B (or **their** P and **their** Q) **Note:** This mark can be implied by writing down $\frac{"A"}{u} + \frac{"B"}{(u+4)}$ with values stated for **their** A and **their** B where either **their** $A=3$ or **their** $B=-3$
A1:	Both **their** $A=3$ and **their** $B=-3$ (or **their** $P=\frac{1}{4}$ and **their** $Q=-\frac{1}{4}$ with a factor of 12 in front of the integral sign)
M1:	Complete strategy for finding $\int \frac{12}{u(u+4)}\,\mathrm{d}u$, which consists of • expressing $\frac{12}{u(u+4)}$ in partial fractions • and integrating $\frac{12}{u(u+4)} \equiv \frac{M}{u} \pm \frac{N}{(u \pm k)}$; $M, N, k \neq 0$; (i.e. ***a two-term partial fraction***) to obtain **both** $\pm\lambda\ln(\alpha u)$ **and** $\pm\mu\ln(\beta(u \pm k))$; $\lambda, \mu, \alpha, \beta \neq 0$
A1ft:	Integration of both terms is **correctly followed through** from **their** M and **their** N
A1*:	Applies limits of e and 1 in u (or applies limits of 2 and 0 in x), subtracts the correct way round and uses laws of logarithms to correctly obtain $3\ln\left(\frac{5\mathrm{e}}{\mathrm{e}+4}\right)$ with no errors seen.

Question	Scheme	Marks	AOs
7	$3\sin\theta - 4\cos\theta \equiv R\sin(\theta-\alpha);\ R>0,\ 0<\alpha<90°$		
(a)	$\tan\alpha = \frac{4}{3}$ o.e.	M1	1.1b
	Either $R=5$ or $\alpha =$ awrt 53.13	B1	1.1b
	$5\sin(\theta - 53.13°)$	A1	1.1b
		(3)	
(b)(i)	$G_{\max} = 17 + "5" = 22$ (°C)	B1ft	3.4
		(1)	
(b)(ii)	$G = 17 + 3\sin(15t)° - 4\cos(15t)°;\ 0 \leqslant t \leqslant 17$		
	$20 = 17 + "5"\sin(15t - "53.13")$	M1	3.4
	$\sin(15t - "53.13") = \frac{3}{"5"}$ or $\sin(\theta - "53.13") = \frac{3}{"5"}$	M1	1.1b
	After midday solution $\Rightarrow 15t - "53.13" = 180 - 36.86989...$ $\Rightarrow t = \frac{143.1301... + "53.13"}{15}$	M1	3.1b
	$\Rightarrow t = 13.0840... \Rightarrow$ Time $= 6:05$ p.m. or $18:05$	A1	3.2a
		(4)	
			(8 marks)

Question 7 Notes:	
(a)	
M1:	For either $\tan\alpha = \frac{4}{3}$ or $\tan\alpha = \frac{3}{4}$ or $\tan\alpha = -\frac{4}{3}$ or $\tan\alpha = -\frac{3}{4}$
B1:	At least one of either $R = 5$ (condone $R = \sqrt{25}$) or $\alpha =$ awrt 53.13
A1:	$5\sin(\theta - 53.13°)$
(b)(i)	
B1ft:	Either 22 or follow through "17 + their R from part (a)"
(b)(ii)	
M1:	Realisation that the model $G = 17 + 3\sin(15t)° - 4\cos(15t)°$ can be rewritten as $G = 17 + "5"\sin(15t - "53.13")$ and applies $G = 20$
M1:	Rearranges their equation to give either $\sin(15t - "53.13") = \frac{3}{"5"}$ or $\sin(\theta - "53.13") = \frac{3}{"5"}$ **Note:** This mark can be implied by either • $15t - "53.13" = 36.86989...$ or $143.1301...$ • $\theta - "53.13" = 36.86989...$ or $143.1301...$
M1:	Uses the model in a complete strategy to find a value for t which is greater than 7 e.g. p.m. solution occurs when $15t - "53.13" = 180 - 36.86989...$ and so rearranges to give $t = ...$, where t is greater than 7
A1:	Finds the p.m. solution of either $6:05$ p.m. or $18:05$ when the greenhouse temperature is predicted by the model to be $20°C$

Question	Scheme	Marks	AOs
8 (i)	E.g. $y^2 - 4y + 7 = (y-2)^2 - 4 + 7$	M1	2.1
	$= (y-2)^2 + 3 \geqslant 3$, as $(y-2)^2 \geqslant 0$ and so $y^2 - 4y + 7$ is positive for all real values of y	A1	2.2a
		(2)	
(ii)	For an explanation or statement to show when (Bobby's) claim $e^{3x} \geqslant e^{2x}$ fails. This could be e.g. • when $x = -1$, $e^{-3} < e^{-2}$ **or** e^{-3} is not greater than or equal to e^{-2} • when $x < 0$, $e^{3x} < e^{2x}$ **or** e^{3x} is not greater than or equal to e^{2x}	M1	2.3
	Followed by an explanation or statement to show when (Bobby's) claim $e^{3x} \geqslant e^{2x}$ is true. This could be e.g. • $\boldsymbol{x = 2}$, $e^6 \geqslant e^4$ **or** e^6 is greater than or equal to e^4 • when $x \geqslant 0$, $e^{3x} \geqslant e^{2x}$ **and** a correct conclusion. E.g. • (Bobby's) claim is sometimes true	A1	2.4
		(2)	
(ii)	Assuming $e^{3x} \geqslant e^{2x}$, then $\ln(e^{3x}) \geqslant \ln(e^{2x}) \Rightarrow 3x \geqslant 2x \Rightarrow x \geqslant 0$	M1	2.3
Alt 1	Correct algebra, using logarithms, leading from $e^{3x} \geqslant e^{2x}$ to $x \geqslant 0$ **and** a correct conclusion. E.g. (Bobby's) claim is sometimes true	A1	2.4
(iii)	Assume that n^2 is even and n is odd. So $n = 2k+1$, where k is an integer.	M1	2.1
	$n^2 = (2k+1)^2 = 4k^2 + 4k + 1$ So n^2 is odd which contradicts n^2 is even. So (Elsa's) claim is true.	A1	2.4
		(2)	
(iv)	For an explanation or statement to show when (Ying's) claim "the sum of two different irrational numbers is irrational" fails This could be e.g. • $\pi, 9-\pi$; sum $= \pi + 9 - \pi = 9$ is not irrational	M1	2.3
	Followed by an explanation or statement to show when (Ying's) claim "the sum of two different irrational numbers is irrational" is true. This could be e.g. • $\pi, 9+\pi$; sum $= \pi + 9 + \pi = 2\pi + 9$ is irrational and a correct conclusion. E.g. • (Ying's) claim is sometimes true	A1	2.4
		(2)	
			(8 marks)

Question 8 Notes:	
(i)	
M1:	Attempts to • complete the square **or** • find the minimum by differentiation **or** • draw a graph of $\mathrm{f}(y) = y^2 - 4y + 7$
A1:	Completes the proof by showing $y^2 - 4y + 7$ is positive for all real values of y with no errors seen in their working.
(ii)	
M1:	See scheme
A1:	See scheme
(ii)	
Alt 1	
M1:	Assumes $e^{3x} \geqslant e^{2x}$, takes logarithms and rearranges to make x the subject of their inequality
A1:	See scheme
(iii)	
M1:	Begins the proof by negating Elsa's claim and attempts to define n as an odd number
A1:	Shows $n^2 = 4k^2 + 4k + 1$, where n is correctly defined and gives a correct conclusion
(iv)	
M1:	See scheme
A1:	See scheme

Question	Scheme	Marks	AOs
9 (a)	$\dfrac{\sin x}{1-\cos x}+\dfrac{1-\cos x}{\sin x}$		
	$=\dfrac{\sin^2 x+(1-\cos x)^2}{(1-\cos x)\sin x}$	M1	2.1
	$=\dfrac{\sin^2 x+1-2\cos x+\cos^2 x}{(1-\cos x)\sin x}$	A1	1.1b
	$=\dfrac{1+1-2\cos x}{(1-\cos x)\sin x}$	M1	1.1b
	$=\dfrac{2-2\cos x}{(1-\cos x)\sin x}=\dfrac{2(1-\cos x)}{(1-\cos x)\sin x}=\dfrac{2}{\sin x}=2\operatorname{cosec} x \quad \{k=2\}$	A1	2.1
		(4)	
(b)	$\left\{\dfrac{\sin x}{1-\cos x}+\dfrac{1-\cos x}{\sin x}=1.6\Rightarrow\right\}\ 2\operatorname{cosec} x=1.6\Rightarrow\operatorname{cosec} x=0.8$ As $\operatorname{cosec} x$ is undefined for $-1<\operatorname{cosec} x<1$ then the given equation has no real solutions.	B1	2.4
		(1)	
(b) Alt 1	$\left\{\dfrac{\sin x}{1-\cos x}+\dfrac{1-\cos x}{\sin x}=1.6\Rightarrow\right\}\ 2\operatorname{cosec} x=1.6\Rightarrow\sin x=1.25$ As $\sin x$ is only defined for $-1\leqslant\sin x\leqslant 1$ then the given equation has no real solutions.	B1	2.4
		(1)	
	(5 marks)		

Question 9 Notes:

(a)	
M1:	Begins proof by applying a complete method of rationalising the denominator **Note:** $\dfrac{\sin^2 x}{(1-\cos x)\sin x}+\dfrac{(1-\cos x)^2}{(1-\cos x)\sin x}$ is a valid attempt at rationalising the denominator
A1:	Expands $(1-\cos x)^2$ to give the correct result $\dfrac{\sin^2 x+1-2\cos x+\cos^2 x}{(1-\cos x)\sin x}$
M1:	Evidence of applying the identity $\sin^2 x+\cos^2 x\equiv 1$
A1:	Uses $\sin^2 x+\cos^2 x\equiv 1$ to show that $\dfrac{\sin x}{1-\cos x}+\dfrac{1-\cos x}{\sin x}\equiv 2\operatorname{cosec} x$ with no errors seen
(b)	
B1:	See scheme
(b) Alt 1	
B1:	See scheme

Question	Scheme	Marks	AOs
10	$V = 4\pi h(h+6) = 4\pi h^2 + 24\pi h \quad 0 \leqslant h \leqslant 25\,;\ \frac{\mathrm{d}V}{\mathrm{d}t} = 80\pi$		
(a)	Time $= \frac{4\pi(24)(24+6)}{80\pi} = \frac{2880\pi}{80\pi} = 36$ (s) *	B1 *	3.4
		(1)	
(b)	When $t = 8,\ V = 80\pi(8) = 640\pi \Rightarrow 640\pi = 4\pi h(h+6)$	M1	3.1a
	$160 = h(h+6) \Rightarrow h^2 + 6h - 160 = 0 \Rightarrow (h+16)(h-10) = 0 \Rightarrow h = ...$	M1	1.1b
	$\{h = -16, \text{reject}\},\ h = 10$	A1	1.1b
	$\frac{\mathrm{d}V}{\mathrm{d}h} = 8\pi h + 24\pi$	M1	1.1b
		A1	1.1b
	$\left\{\frac{\mathrm{d}V}{\mathrm{d}h} \times \frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \Rightarrow\right\}\ (8\pi h + 24\pi)\frac{\mathrm{d}h}{\mathrm{d}t} = 80\pi$	M1	3.1a
	When $h = 10$, $\left\{\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \div \frac{\mathrm{d}h}{\mathrm{d}t} =\right\}\ \frac{80\pi}{(8\pi(10) + 24\pi)}\ \left\{= \frac{80\pi}{124\pi}\right\}$	M1	3.4
	When $h = 10$, $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{10}{13}$ (cm s^{-1}) or awrt 0.769 (cm s^{-1})	A1	1.1b
		(8)	
		(9 marks)	

Question 10 Notes:

(a)	
B1*:	Uses the model to show that it takes 36 seconds to fill the bowl from empty to a height of 24 cm
(b)	
M1:	Complete strategy to find the value of h when $t = 8$
M1:	Uses $\frac{\mathrm{d}V}{\mathrm{d}t} = 80\pi$ to deduce the volume of water in the bowl, V, after 8 seconds and sets this result to $4\pi h(h+6)$
A1:	Finds $h = 10$
M1:	Differentiates V with respect to h to give $\pm\alpha h \pm \beta;\ \alpha, \beta \neq 0$
A1:	$8\pi h + 24\pi$
M1:	A complete strategy of forming an equation relating $\frac{\mathrm{d}h}{\mathrm{d}t}$ to 80π E.g. applies $\left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right) \times \frac{\mathrm{d}h}{\mathrm{d}t} = 80\pi$
M1:	Substitutes their $h = \text{"10"}$ into their model for $\frac{\mathrm{d}h}{\mathrm{d}t}$ which is in the form $\frac{80\pi}{\left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}h}\right)}$, where their h has been found from solving a quadratic equation in h
A1:	$\frac{10}{13}$ or awrt 0.769

Question	Scheme	Marks	AOs
11 (i)	$\{y = a^x \Rightarrow\}\ \ \ln y = \ln a^x \ \Rightarrow\ \ln y = x\ln a \ \Rightarrow\ \dfrac{1}{y}\dfrac{\mathrm{d}y}{\mathrm{d}x} = \ln a$	M1	2.1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = y\ln a \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = a^x \ln a$ *	A1*	1.1b
		(2)	
(i) **Alt 1**	$\{y = a^x \Rightarrow\}\ \ y = \mathrm{e}^{x\ln a} \ \Rightarrow\ \dfrac{\mathrm{d}y}{\mathrm{d}x} = (\ln a)\mathrm{e}^{x\ln a}$	M1	2.1
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = a^x \ln a$ *	A1*	1.1b
		(2)	
(ii)	$\dfrac{\mathrm{d}}{\mathrm{d}y}(2\tan y) = 2\sec^2 y$	M1	1.1b
	$\{x = 2\tan y \Rightarrow\}\ \dfrac{\mathrm{d}x}{\mathrm{d}y} = 2\sec^2 y$ or $1 = (2\sec^2 y)\dfrac{\mathrm{d}y}{\mathrm{d}x}$	A1	1.1b
	$\dfrac{\mathrm{d}x}{\mathrm{d}y} = 2(1+\tan^2 y)$ or $1 = 2(1+\tan^2 y)\dfrac{\mathrm{d}y}{\mathrm{d}x}$	M1	1.1b
	E.g. $\dfrac{\mathrm{d}x}{\mathrm{d}y} = 2\left(1+\left(\dfrac{x}{2}\right)^2\right) \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y} = 2\left(1+\dfrac{x^2}{4}\right) \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y} = 2+\dfrac{x^2}{2}$ $\Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y} = \dfrac{4+x^2}{2} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2}{4+x^2}$	A1	2.1
		(4)	
(ii) **Alt 1**	$\{x = 2\tan y \Rightarrow\}\ \ y = \arctan\left(\dfrac{x}{2}\right) \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\left(1+\left(\dfrac{x}{2}\right)^2\right)} \times \left(\dfrac{1}{2}\right)$	M1 M1 A1	1.1b 1.1b 1.1b
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{2\left(1+\dfrac{x^2}{4}\right)} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\left(2+\dfrac{x^2}{2}\right)} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\left(\dfrac{4+x^2}{2}\right)}$ $\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2}{4+x^2}$	A1	2.1
		(4)	
			(6 marks)

Question 11 Notes:	
(i)	
M1:	Applies the natural logarithm to both sides of $y = a^x$, applies the power law of logarithms and applies implicit differentiation to the result
A1*:	Shows $\frac{\mathrm{d}y}{\mathrm{d}x} = a^x \ln a$, with no errors seen
(i) **Alt 1**	
M1:	Rewrites $y = a^x$ as $y = \mathrm{e}^{x\ln a}$ and writes $\frac{\mathrm{d}y}{\mathrm{d}x} = c\,\mathrm{e}^{x\ln a}$, where c can be 1
A1*:	Shows $\frac{\mathrm{d}y}{\mathrm{d}x} = a^x \ln a$, with no errors seen
(ii)	
M1:	Evidence of $2\tan y$ being differentiated to $2\sec^2 y$
A1:	Differentiates correctly to show that $x = 2\tan y$ gives $\frac{\mathrm{d}x}{\mathrm{d}y} = 2\sec^2 y$ or $1 = (2\sec^2 y)\frac{\mathrm{d}y}{\mathrm{d}x}$
M1:	Applies $\sec^2 y = 1 + \tan^2 y$ to their differentiated expression
A1:	Shows that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{4+x^2}$, with no errors seen
(ii) **Alt 1**	
M1:	Evidence of $\arctan(\lambda x)$; $\lambda \neq 0$ being differentiated to $\lambda\left(\frac{1}{1+(\mu x^2)}\right)$; $\lambda, \mu \neq 0$ **Note:** λ can be 1 for this mark
M1:	Differentiates $y = \arctan(\lambda x)$; $\lambda \neq 0, \lambda \neq 1$ to give an expression of the form $\frac{1}{\left(1+(\lambda x)^2\right)} \times (\lambda)$
A1:	Differentiates $y = \arctan\left(\frac{x}{2}\right)$ correctly to give $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\left(1+\left(\frac{x}{2}\right)^2\right)} \times \left(\frac{1}{2}\right)$, o.e.
A1:	Shows that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{4+x^2}$, with no errors seen

Question	Scheme	Marks	AOs
12 (a)	$y = ax^2 + c$ $x = 0,\ y = 4 \Rightarrow c = 4$	M1	3.3
	$x = 50,\ y = 24 \Rightarrow 24 = a(50)^2 + 4 \Rightarrow a = \dfrac{20}{50^2} = \dfrac{1}{125}$ or 0.008	M1	3.4
	$y = \dfrac{1}{125}x^2 + 4 \quad \{-50 \leqslant x \leqslant 50\}$	A1	1.1b
		(3)	
(a) Alt 1	$y = ax^2 + bx + c$ $x = 0,\ y = 4 \Rightarrow c = 4$ $x = 50,\ y = 24 \Rightarrow 24 = 2500a + 50b + 4$ $x = -50,\ y = 24 \Rightarrow 24 = 2500a - 50b + 4$	M1	3.3
	$0 = 100b \Rightarrow b = 0$ $24 = 2500a + 4 \Rightarrow a = \dfrac{20}{2500} = \dfrac{1}{125}$ or 0.008	M1	3.4
	$y = \dfrac{1}{125}x^2 + 4 \quad \{-50 \leqslant x \leqslant 50\}$	A1	1.1b
		(3)	
(b)	$x = 50 - 19 = 31 \Rightarrow y = \dfrac{1}{125}(31)^2 + 4$	M1	3.4
	$y = 11.688\ \{< 12\} \Rightarrow$ Lee can safely inspect the defect	A1	2.2b
		(2)	
(b) Alt 1	$12 = \dfrac{1}{125}x^2 + 4 \Rightarrow 8 = \dfrac{1}{125}x^2 \Rightarrow x = \sqrt{1000}$	M1	3.4
	$x = 31.6227766... \Rightarrow$ Distance from tower $= 50 - 31.6227766...$ $= 18.3772234...\ \{< 19\} \Rightarrow$ Lee can safely inspect the defect	A1	2.2b
		(2)	
(c)	E.g. • The thickness/diameter of the cable has not been incorporated into the current model • Weather conditions (e.g. strong winds) may affect the shape of the curve • Walkway may not be completely horizontal	B1	3.5b
		(1)	
			(6 marks)

Question 12 Notes:	
(a)	
M1:	Attempts to use a model of the form $y = ax^2 + c$ **(containing no *x* term)**
M1:	Uses the constraints $x = 0,\ y = 4$ and $x = 50,\ y = 24$ (or $x = -50,\ y = 24$) to find the values for their *c* and for their *a*
A1:	$y = \dfrac{1}{125}x^2 + 4$ (Ignore $-50 \leqslant x \leqslant 50$)
(a) Alt 1	
M1:	Attempts to use a model of the form $y = ax^2 + bx + c$ **and** finds or deduces that $b = 0$
M1:	Uses the constraints $x = 0,\ y = 4;\ x = 50,\ y = 24$ and $x = -50,\ y = 24$ to find the values for their *c*, for their *b* and for their *a*
A1:	$y = \dfrac{1}{125}x^2 + 4$ (Ignore $-50 \leqslant x \leqslant 50$)
(b)	
M1:	Substitutes $x = 50 - 19\ \{= 31\}$ or $x = -50 + 19\ \{= -31\}$ into their quadratic model
A1:	Obtains $y =$ awrt 11.7 and infers from the model that Lee can safely inspect the defect
(b) Alt 1	
M1:	Substitutes $y = 12$ into their quadratic model and rearranges to find $x = ...$
A1:	Obtains distance from tower as awrt 18.4 and infers from the model that Lee can safely inspect the defect
(c)	
B1:	See scheme

Question	Scheme	Marks	AOs
13 (a)	$\sum_{n=1}^{11} \ln(p^n) = \ln p + \ln p^2 + \ln p^3 + \ldots + \ln p^{11}$ $= \ln p + 2\ln p + 3\ln p + \ldots + 11\ln p$ $= \frac{11}{2}(2\ln p + (11-1)\ln p)$ or $\frac{1}{2}(11)(12)\ln p$	M1	3.1a
	$= 66\ln p \quad \{k = 66\}$	A1	1.1b
		(2)	
(b)	$S = \sum_{n=1}^{11} \ln(8p^n) = \ln 8p + \ln 8p^2 + \ln 8p^3 + \ldots + \ln 8p^{11}$ $= 11\ln 8 + 66\ln p$	M1	1.1b
	e.g. • $11\ln 8 + 66\ln p = 11\ln 2^3 + 66\ln p = 33\ln 2 + 66\ln p$ $= 33(\ln 2 + 2\ln p) = 33(\ln 2 + \ln p^2) = 33\ln(2p^2)$ * • $11\ln 8 + 66\ln p = 11\ln 2^3 + 66\ln p = 33\ln 2 + 66\ln p$ $= \ln(2^{33}p^{66}) = \ln\left((2p^2)^{33}\right) = 33\ln(2p^2)$ *	A1*	2.1
		(2)	
(c)	$S < 0 \Rightarrow 33\ln(2p^2) < 0 \Rightarrow \ln(2p^2) < 0$		
	so either $0 < 2p^2 < 1$ or $2p^2 < 1$	M1	2.2a
	$\Rightarrow p^2 < \frac{1}{2}$ and $p > 0 \Rightarrow 0 < p < \frac{1}{\sqrt{2}}$		
	In set notation, e.g. $\left\{p : 0 < p < \frac{1}{\sqrt{2}}\right\}$	A1	2.5
		(2)	
			(6 marks)

Question 13 Notes:	
(a)	
M1:	Attempts to find $\sum_{n=1}^{11} \ln(p^n)$ by using a complete strategy of • applying the power law of logarithms followed by either • applying the correct formula for the sum to n terms of an arithmetic series • applying the correct formula $\frac{1}{2}n(n+1)\ln p$ • summing the individual terms to give $66\ln p$
A1:	$66\ln p$ from correct working
(b)	
M1:	Deduces S or $\sum_{n=1}^{11} \ln(8p^n) = 11\ln 8 +$ (their answer to part (a))
A1*:	and produces a logical argument to correctly show that $S = 33\ln(2p^2)$ with no errors seen
(c)	
M1:	Applies $S < 0$ to give $\ln(2p^2) < 0$ and deduces {e.g. by considering the graph of $y = \ln x$} that either • $0 < 2p^2 < 1$ • $2p^2 < 1$
A1:	Correct answer using set notation. E.g. • $\left\{p : 0 < p < \frac{1}{\sqrt{2}}\right\}$ • $\left\{p : 0 < p < \frac{\sqrt{2}}{2}\right\}$ • $\{p : p > 0\} \cap \left\{p : p < \frac{1}{\sqrt{2}}\right\}$ • $\{p : p > 0\} \cap \left\{p : p < \frac{\sqrt{2}}{2}\right\}$

Question	Scheme	Marks	AOs
14	$y = 4xe^{-2x} \Rightarrow \left\{\begin{matrix} u = 4x & v = e^{-2x} \\ \frac{du}{dx} = 4 & \frac{dv}{dx} = -2e^{-2x} \end{matrix}\right\}, \left\{\begin{matrix} u = 4x & \frac{du}{dx} = 4 \\ \frac{dv}{dx} = e^{-2x} & v = -\frac{1}{2}e^{-2x} \end{matrix}\right\}$		
	$\frac{dy}{dx} = 4e^{-2x} - 8xe^{-2x}$	M1	2.1
		A1	1.1b
	At $P(1, 4e^{-2})$, $m_T = 4e^{-2} - 8e^{-2} = -4e^{-2} \Rightarrow m_N = \frac{-1}{-4e^{-2}}$ or $\frac{1}{4}e^2$	M1	1.1b
	l: $y - 4e^{-2} = \frac{e^2}{4}(x-1)$ and $y = 0 \Rightarrow -4e^{-2} = \frac{e^2}{4}(x-1) \Rightarrow x = ...$	M1	3.1a
	$\{y = 0 \Rightarrow x = 1 - 16e^{-4}\}$		
	$\int 4xe^{-2x}dx = -2xe^{-2x} - \int -2e^{-2x}\,dx$	M1	2.1
		A1	1.1b
	$= -2xe^{-2x} - e^{-2x}$	A1	1.1b
	<u>Criteria</u> • $\left[-2xe^{-2x} - e^{-2x}\right]_0^1 = \left(-2e^{-2} - e^{-2}\right) - (0-1)$ $\{= 1 - 3e^{-2}\}$ • Area triangle = $\frac{1}{2}\left(16e^{-4}\right)\left(4e^{-2}\right)$ $\{= 32e^{-6}\}$	M1	2.1
	Area$(R) = 1 - 3e^{-2} - 32e^{-6}$ or $\frac{e^6 - 3e^4 - 32}{e^6}$	M1	3.1a
		A1	1.1b
		(10)	
			(10 marks)

Question 14 Notes:	
M1:	Begins the process to find where l intersects the x-axis by differentiating $y = 4xe^{-2x}$ using the product rule
A1:	$\frac{dy}{dx} = 4e^{-2x} - 8xe^{-2x}$, which can be simplified or un-simplified
M1:	A correct method to find the value for the gradient of the normal using $m_N = \frac{-1}{\text{their } m_T}$
M1:	Complete strategy to find where l intersects the x-axis i.e. Applying $y - 4e^{-2} = m_N(x-1)$, (where $m_N \neq$ their m_T) followed by setting $y = 0$ and rearranging to give $x = ...$
M1:	Begins the process of finding the area under the curve by applying integration by parts in the correct direction to give $\pm\alpha xe^{-2x} \pm \int \beta e^{-2x}\{dx\}$; $\alpha, \beta \neq 0; \alpha < 4$
A1:	$4xe^{-2x} \to -2xe^{-2x} - \int -2e^{-2x}\{dx\}$, which can be simplified or un-simplified
A1:	$4xe^{-2x} \to -2xe^{-2x} - e^{-2x}$, which can be simplified or un-simplified
M1:	At least one of the two listed criteria
M1:	Both criteria satisfied, followed by a complete strategy of subtracting the areas to find $\text{Area}(R)$
A1:	Correct exact answer. E.g. $1 - 3e^{-2} - 32e^{-6}$ or $\frac{e^6 - 3e^4 - 32}{e^6}$, o.e.

Question	Scheme	Marks	AOs
15	$\overrightarrow{OA} = \begin{pmatrix} -3 \\ 2 \\ 7 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} 3 \\ -1 \\ p \end{pmatrix}, \overrightarrow{BC} = \begin{pmatrix} 0 \\ 6 \\ -7 \end{pmatrix}, \overrightarrow{AD} = \begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix};$ p is a constant		
(a)	$\left\{\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = \begin{pmatrix} -3 \\ 2 \\ 7 \end{pmatrix} + \begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix} \Rightarrow \right\} \overrightarrow{OD} = \begin{pmatrix} -1 \\ 7 \\ 3 \end{pmatrix}$	B1	1.1b
		(1)	
(b)	$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = \begin{pmatrix} 3 \\ -1 \\ p \end{pmatrix} + \begin{pmatrix} 0 \\ 6 \\ -7 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ p-7 \end{pmatrix}$	M1	3.1a
	$\overrightarrow{DC} = \overrightarrow{OC} - \overrightarrow{OD} = \begin{pmatrix} 3 \\ 5 \\ p-7 \end{pmatrix} - \begin{pmatrix} -1 \\ 7 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ p-10 \end{pmatrix}$	A1	1.1b
	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 3 \\ -1 \\ p \end{pmatrix} - \begin{pmatrix} -3 \\ 2 \\ 7 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \\ p-7 \end{pmatrix}$ so $\overrightarrow{AB} = 1.5\overrightarrow{DC} \Rightarrow p - 7 = 1.5(p - 10)$	M1	3.1a
	$p - 7 = 1.5p - 15 \Rightarrow 8 = 0.5p \Rightarrow p = 16$	A1	1.1b
		(4)	
			(5 marks)

Question 15 Notes:

(a)	
B1:	$\left\{\overrightarrow{OD}\right\} = \begin{pmatrix} -1 \\ 7 \\ 3 \end{pmatrix}$
(b)	
M1:	Complete strategy for finding the vector $\overrightarrow{DC}$ or $\overrightarrow{CD}$ (e.g. finding $\overrightarrow{OC}$ followed by $\overrightarrow{DC}$)
A1:	For either $\left\{\overrightarrow{DC}\right\} = \begin{pmatrix} 4 \\ -2 \\ p-10 \end{pmatrix}$ or $\left\{\overrightarrow{CD}\right\} = \begin{pmatrix} -4 \\ 2 \\ -p+10 \end{pmatrix}$
M1:	Complete strategy of • finding the vector $\overrightarrow{AB}$ (or $\overrightarrow{BA}$) • discovering that $\overrightarrow{AB}$ (or $\overrightarrow{BA}$) is parallel to $\overrightarrow{DC}$ (or $\overrightarrow{CD}$) and so writes an equation of the form (their **k** component in terms of p of $\pm\overrightarrow{AB}$) = δ(their **k** component in terms of p of $\pm\overrightarrow{DC}$), where $\delta \neq 1$ is a constant
A1:	Correct solution leading to $p = 16$