

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**Advanced Subsidiary General Certificate of Education  
Advanced General Certificate of Education**

**MEI STRUCTURED MATHEMATICS**

**4753/1**

Methods for Advanced Mathematics (C3)

Thursday

**8 JUNE 2006**

Morning

1 hour 30 minutes

Additional materials:

8 page answer booklet

Graph paper

MEI Examination Formulae and Tables (MF2)

**TIME** 1 hour 30 minutes

**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

---

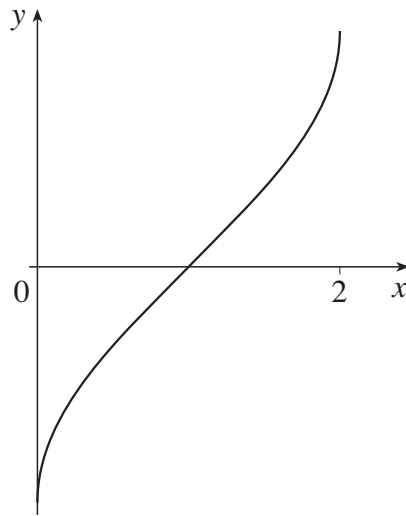
**This question paper consists of 5 printed pages and 3 blank pages.**

## Section A (36 marks)

1 Solve the equation  $|3x - 2| = x$ . [3]

2 Show that  $\int_0^{\frac{1}{6}\pi} x \sin 2x \, dx = \frac{3\sqrt{3} - \pi}{24}$ . [6]

3 Fig. 3 shows the curve defined by the equation  $y = \arcsin(x - 1)$ , for  $0 \leq x \leq 2$ .

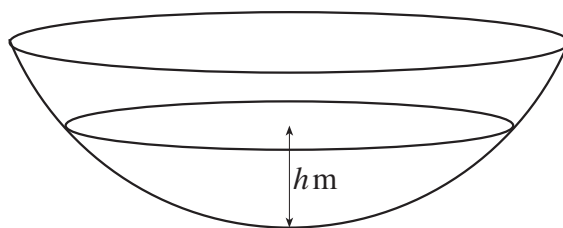


**Fig. 3**

(i) Find  $x$  in terms of  $y$ , and show that  $\frac{dx}{dy} = \cos y$ . [3]

(ii) Hence find the exact gradient of the curve at the point where  $x = 1.5$ . [4]

- 4 Fig. 4 is a diagram of a garden pond.



**Fig. 4**

The volume  $V \text{ m}^3$  of water in the pond when the depth is  $h$  metres is given by

$$V = \frac{1}{3}\pi h^2(3 - h).$$

- (i) Find  $\frac{dV}{dh}$ . [2]

Water is poured into the pond at the rate of  $0.02 \text{ m}^3$  per minute.

- (ii) Find the value of  $\frac{dh}{dt}$  when  $h = 0.4$ . [4]

- 5 Positive integers  $a$ ,  $b$  and  $c$  are said to form a Pythagorean triple if  $a^2 + b^2 = c^2$ .

- (i) Given that  $t$  is an integer greater than 1, show that  $2t$ ,  $t^2 - 1$  and  $t^2 + 1$  form a Pythagorean triple. [3]

- (ii) The two smallest integers of a Pythagorean triple are 20 and 21. Find the third integer.

Use this triple to show that not all Pythagorean triples can be expressed in the form  $2t$ ,  $t^2 - 1$  and  $t^2 + 1$ . [3]

- 6 The mass  $M$  kg of a radioactive material is modelled by the equation

$$M = M_0 e^{-kt},$$

where  $M_0$  is the initial mass,  $t$  is the time in years, and  $k$  is a constant which measures the rate of radioactive decay.

- (i) Sketch the graph of  $M$  against  $t$ . [2]

- (ii) For Carbon 14,  $k = 0.000121$ . Verify that after 5730 years the mass  $M$  has reduced to approximately half the initial mass. [2]

The half-life of a radioactive material is the time taken for its mass to reduce to exactly half the initial mass.

- (iii) Show that, in general, the half-life  $T$  is given by  $T = \frac{\ln 2}{k}$ . [3]

- (iv) Hence find the half-life of Plutonium 239, given that for this material  $k = 2.88 \times 10^{-5}$ . [1]

## Section B (36 marks)

- 7 Fig. 7 shows the curve  $y = \frac{x^2 + 3}{x - 1}$ . It has a minimum at the point P. The line  $l$  is an asymptote to the curve.

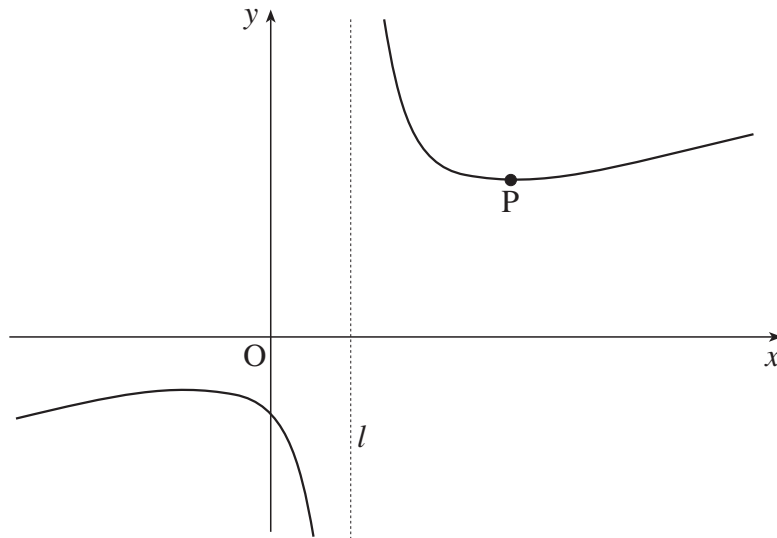


Fig. 7

- (i) Write down the equation of the asymptote  $l$ . [1]
- (ii) Find the coordinates of P. [6]
- (iii) Using the substitution  $u = x - 1$ , show that the area of the region enclosed by the  $x$ -axis, the curve and the lines  $x = 2$  and  $x = 3$  is given by

$$\int_1^2 \left( u + 2 + \frac{4}{u} \right) du.$$

Evaluate this area exactly.

[7]

- (iv) Another curve is defined by the equation  $e^y = \frac{x^2 + 3}{x - 1}$ . Find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$  by differentiating implicitly. Hence find the gradient of this curve at the point where  $x = 2$ . [4]

- 8 Fig. 8 shows part of the curve  $y = f(x)$ , where  $f(x) = e^{-\frac{1}{5}x} \sin x$ , for all  $x$ .

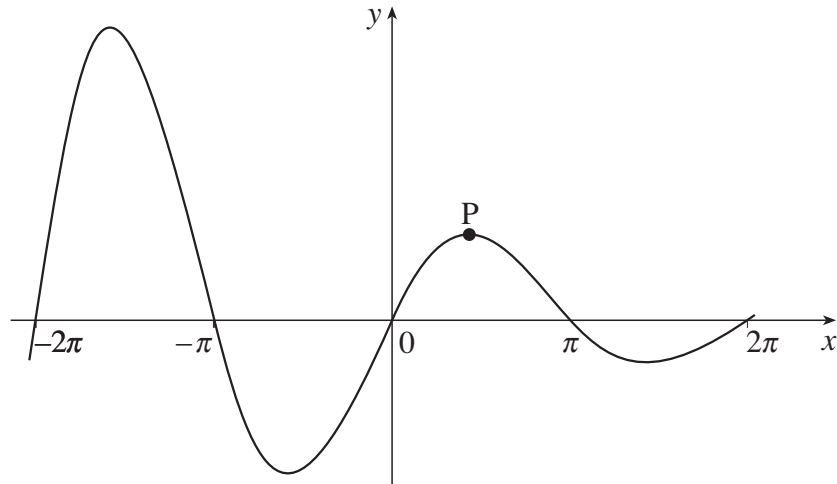


Fig. 8

- (i) Sketch the graphs of

(A)  $y = f(2x)$ ,

(B)  $y = f(x + \pi)$ .

[4]

- (ii) Show that the  $x$ -coordinate of the turning point P satisfies the equation  $\tan x = 5$ .

Hence find the coordinates of P.

[6]

- (iii) Show that  $f(x + \pi) = -e^{-\frac{1}{5}\pi} f(x)$ . Hence, using the substitution  $u = x - \pi$ , show that

$$\int_{\pi}^{2\pi} f(x) dx = -e^{-\frac{1}{5}\pi} \int_0^{\pi} f(u) du.$$

Interpret this result graphically. [You should *not* attempt to integrate  $f(x)$ .]

[8]