

Mathematics (MEI)

Advanced GCE

Unit **4762**: Mechanics 2

Mark Scheme for January 2011

Q 1		m a r k	notes
(i)	Let normal reaction be R $\sin \alpha = \sqrt{1-0.8^2} = 0.6$ $R = 2.5 \times 9.8 \times 0.8$ $F_{\max} = 0.85 \times R = 16.66$ Wt cpt down slope is $2.5 \times 9.8 \times 0.6 = 14.7$ $16.66 > 14.7$ so at rest	B1 M1 B1 F1 B1 E1 6	Accept any form and implied Use of $F_{\max} = \mu R$ Expression for R ; may be implied FT their R FT if their F and weight component show given result If g omitted, allow B1M1B0F1B0E1, so 4/6 [Award as follows for use of $\tan \alpha < \mu$: B1 $\tan \alpha = \frac{3}{4}$ E1 $\tan \alpha < \mu$ shown]
(ii)	Let the speeds down the plane be v_A and v_B . PCLM down the plane $1.5 \times 16 = 2.5v_A + 1.5v_B$ so $5v_A + 3v_B = 48$ NEL +ve down the plane $\frac{v_A - v_B}{0 - 16} = -0.4$ $v_A - v_B = 6.4$ $v_A = 8.4$ so 8.4 m s^{-1} down plane $v_B = 2$ so 2 m s^{-1} down plane	M1 A1 M1 A1 E1 F1 6	PCLM Any form NEL. Allow sign errors Any form Condone direction not clear if +8.4 seen Condone direction not clear if +2 seen. SC1 if 2 equations obtained and 8.4 substituted into one to obtain answer 2 (instead of E1F1)
(iii)	$1.5 \times (2 - 16)$ down plane $= -21 \text{ N s}$ down the plane so 21 Ns up the plane	M1 A1 A1 3	Use of $m(\mathbf{v} - \mathbf{u})$ If impulse on A found, treat as MR unless final answer relates this to impulse on B $\pm 21 \text{ N s}$ Direction explicitly commented on

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(iv)	<p>either $(2.5 \times 9.8 \times 0.6 - F_{\max}) \times t = 2.5(0 - 8.4)$</p> <p>so $t = 10.7142 \dots 10.7 \text{ s (3 s. f.)}$</p> <p>or Using N2L down the plane $a = -0.784$</p> <p>using $v = u + at$, $t = 10.7142 \dots 10.7 \text{ s (3 s. f.)}$</p> <p>or $0.5 \times 2.5 \times 8.4^2 + (14.7 - 16.66)x = 0$ $x = 45$</p> <p>$T = 10.7142 \dots 10.7 \text{ (3 s. f.)}$</p>	<p>M1 B1 A1 A1</p> <p>M1 A1 M1 A1</p> <p>M1 A1 M1 A1</p> <p>4</p>	<p>Using Impulse-momentum (must use 8.4) . sufficient to consider one term on LHS</p> <p>Either side correct</p> <p>Allow only sign errors</p> <p>cao</p> <p>Using N2L ; sufficient to consider one force term</p> <p>Allow sign errors</p> <p>Using appropriate <i>suvat</i> must use <i>a</i> or <i>-a</i> found by use of N2L and $u = 8.4$</p> <p>cao</p> <p>Use energy with 8.4, sufficient to consider one non-KE term</p> <p>Using appropriate <i>suvat</i></p> <p>cao</p>
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Q 2		m a r k	notes
(a)	<div style="text-align: center;"> $\begin{array}{ccc} v \text{ m s}^{-1} & V \text{ m s}^{-1} & \mathbf{i} \rightarrow \\ \leftarrow & \rightarrow & \\ \text{C } 0.004 \text{ kg} & \square & \text{B } 0.060 \text{ kg} \end{array}$ </div> <p>Energy: $\frac{1}{2} \times 0.004 \times v^2 + \frac{1}{2} \times 0.060 \times V^2 = 0.8$ $v^2 + 15V^2 = 400$</p> <p>PCLM in \mathbf{i} direction: $0.06V - 0.004v = 0$ $v = 15V$ Solving $(15V)^2 + 15V^2 = 400$ so $V^2 = \frac{400}{240} = \frac{5}{3}$ and $\mathbf{V} = \sqrt{\frac{5}{3}}\mathbf{i}$ $\mathbf{v} = -15\sqrt{\frac{5}{3}}\mathbf{i} (= -\sqrt{375}\mathbf{i})$</p>	<p>M1 A1 M1 A1 M1 A1 F1 A1</p> <p style="text-align: center;">8</p>	<p>Use of KE in two terms in an equation. Any form</p> <p>PCLM. Accept sign errors. Any form Valid method for elimination of v or V from a linear and a quadratic</p> <p>Accept $1.29099\dots\mathbf{i}$ Accept no direction Accept $-19.3649\dots\mathbf{i}$ Accept no direction Second answer follows from first (Relative) directions indicated - accept diagram. Both speeds correct.</p>
(b) (i)	<p>W is work done by resistances on car $\frac{1}{2} \times 800 \times (12^2 - 30^2) = -800 \times 9.8 \times 20 + W$</p> <p>$W = -145\,600$ so 145 600 J done by car against resistances</p>	<p>M1 B1 A1 A1</p> <p style="text-align: center;">4</p>	<p>Use of WE. Must have KE, W and GPE. Allow $-W$ Both KE terms. Accept sign error All correct with W or $-W$</p> <p>cao</p>

Q 2		m a r k	notes
(ii)	<p>either The slope is $18 \times 25 = 450$ m long $\frac{800 \times 9.8 \times 20 + 750 \times 450}{25}$</p> <p>= 19 772 W</p> <p>or The angle of the slope is $\arcsin (1/22.5)$ $\left(800 \times 9.8 \times \frac{1}{22.5} + 750 \right) \times 18$</p> <p>= 19 772 W</p>	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>5</p>	<p>Use of $P = (\text{Work done}) / (\text{elapsed time})$ used for at least one work done term</p> <p>WD is force \times distance used for at least one force</p> <p>Allow only sign errors both terms</p> <p>cao.</p> <p>Use of $P = Fv$ used for at least one term</p> <p>Attempt at weight component</p> <p>Allow only sign errors both terms</p> <p>cao.</p>
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Q 3		m a r k	notes
(i)	Horizontal $X - 50 = 0$ Vertical: $R - Y - 45 = 0$	B1 B1 2	Any form Any form
(ii)	a. c. moments about A $1 \times R = 3 \times 45$ so $R = 135$ so $135 - Y - 45 = 0$ and $Y = 90$	M1 E1 E1 3	Clearly shown Shown
(iii)	In analysis below all internal forces are taken as tensions	B1 B1 2	Correct arrow pairs for all internal forces Correct labels

Q 4		m a r k	notes
(i)	(2, 2.5)	B1 1	Condone writing as a vector
(ii)	<p>By symmetry, $\bar{y} = 2.5$</p> <p>For \bar{x}: $\left(5h + \frac{1}{2} \times 5 \times 6\right) \bar{x} = 5h \times \left(-\frac{h}{2}\right) + \frac{1}{2} \times 5 \times 6 \times 2$</p> <p>so $(5h + 15) \bar{x} = -2.5h^2 + 30$</p> <p>so $5(h + 3) \bar{x} = 2.5(12 - h^2)$</p> <p>and $\bar{x} = \frac{12 - h^2}{2(h + 3)}$</p>	B1 M1 A1 A1 A1 E1 6	<p>Some justification needed</p> <p>These next 4 marks may be obtained from correct FT of their “2” from (i)</p> <p>1st term RHS correct (allow sign error)</p> <p>Either other term correct</p> <p>All correct</p> <p>Clearly shown, including signs.</p>
(iii)	<p>Need $\bar{x} > 0$</p> <p>So $\frac{12 - h^2}{2(h + 3)} > 0$</p> <p>Hence $12 - h^2 > 0$</p> <p>Since $h > 0$, $0 < h < 2\sqrt{3}$</p>	M1 B1 A1 3	<p>Allow $\bar{x} \geq 0$ or = 0</p> <p>$2\sqrt{3}$ or $-2\sqrt{3}$ oe seen</p> <p>Accept only +ve root mentioned. WWW for signs</p> <p>Accept $h < 2\sqrt{3}$ as answer strict inequality for final A mark</p>

Q 4		m a r k	notes
Q4 (iv)	continued When $h = 3$, $\bar{x} = 0.25$ Let mag of vert force be T N a.c moments about axis thro' O $T \times 6 - 15 \times 0.25 = 0$ so $T = 0.625$ so 0.625 N	B1 M1 A1 3	Could be scored in (v) If moments about another point need all relevant forces. Allow sign errors. Condone use of 15g cao
(v)	Let magnitude of force be U N a.c. moments about axis thro' D $U \cos 30 \times 5 - 15 \times (3 + 0.25) = 0$ $U = 11.25833\dots$ so 11.3 N (3 s. f.)	M1 B1 A1 A1 4	Each term must be a moment. If moments about another point need all relevant forces. Condone use of 15g . moment of U ($5U \cos 30$ or ...) oe (3 + 0.25) oe cao
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